C.S.E. (MAIN)

PHYSICS – 2005

PAPER – I

Time Allowed : Three Hours *Maximum Marks : 300*

Candidates should attempt question Nos. 1 and 5 which are compulsory, and any three *of the remaining questions selecting at least* one *question from each Section.*

Assume suitable data if considered necessary and indicate the same clearly.

All questions carry equal marks.

SECTION–A

1. Answer any *three* of the following : 3 × 20 = 60

(a) How are linear and angular momenta related to each other ? Considering a system of mass points under the influence of forces derived from potentials only, prove that the generalised linear momentum is conserved. Establish further that the angular momentum of the above system is also conserved if the potential is centrally symmetric.

(b) Show that the mass–energy relationship in relativistic kinematics can lead to the equation

$$E^2 = c^2 \rho^2 + m_0^2 c^4$$

where E is the total energy of the particle of rest mass m_0, linear momentum p and moving with a velocity v; c is the velocity of light in free space.

(c) Explain how Einstein's A and B coefficients are related to the phenomena of spontaneous and stimulated emission of radiation, respectively. Derive the relation between A and B. Establish that at very high frequency around X-ray wavelength regime, lasers cannot be made as easily as at low frequencies, e.g. far infra-red regime.

(d) Drawing a neat diagram, discuss how light travels through an optical fibre. Show that the numerical aperture of a commercially available optical fibre is around 0·25. Explain its physical significance.

2. (a) Considering the scattering of α-particles by the atomic nuclei, find out the Rutherford scattering cross-section. Explain the physical significance of the final expression. 40

(b) What are constraints of motion ? Explain with examples the holonomic and non-holonomic constraints. Discuss critically how can one overcome the limits of constraints by introducing generalised coordinates. 20

3. (a) Show that the length L of an object moving with a velocity v is given in the direction of motion by

$$L = L_0\left(1 - v^2/c^2\right)^{1/2},$$

where L_0 is the proper length and c is the velocity of light in free space.

What will be the shape of a spherical ball while moving under relativistic regime ? 20

(b) For a transverse sinusoidal wave of wavelength λ propagating along negative x direction through a string fixed at a point, show that the nodes are located at $x = 0, \lambda/2, \lambda, 3\lambda/2, ...$ while the kinetic energy/unit length at the antinodes is given by

$$E = 2\,\rho A^2 \omega^2 \cos^2\omega t$$

where, ρ, A and ω, are the mass density/unit length, amplitude of transverse displacement and angular frequency of the wave, respectively. 20

(c) What do you understand by paraxial rays ? Show that the effect of translation of a paraxial ray while travelling along a homogeneous medium is represented by a 2×2 matrix if the ray is initially defined by a 2×1 matrix. 20

4. (a) Why does one get three-dimensional image in holography ? Explain with appropriate figures hos can one construct and read a hologram. 20

(b) What is the essential difference between interference and diffraction of light ? How can you achieve Fraunhofer difraction in the laboratory ? Using the concept of Fraunhofer diffraction at a single slit, find out the intensity distribution produced by two slits of equal width. 20

(c) Why does one see two image points for a single object point while viewed through a calcite crystal ? What is this property of the crystal known as ? What is an optic axis of a crystal ?

Explain the meanings of positive and negative crystals with one example for each kind. 20

SECTION–B

5. Answer any *three* of the following : $3 \times 20 = 60$

(a) Prove the relation

$$\nabla^2\left(\frac{1}{|\vec{r} - \vec{r}'|}\right) = -4\pi\delta\left(|\vec{r} - \vec{r}'|\right)$$

and hence show that

$$\phi\left(\vec{r}\right) = \int \frac{p(\vec{r}')}{|\vec{r} - \vec{r}'|} d\vec{r}'$$

is a solution of the Poisson equation

$$\nabla^2\phi(\vec{r}) = -4\pi\rho(\vec{r}).$$

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(b) Show that the electric and magnetic field vectors, $\vec{E}$ and $\vec{B}$, in plane electromagnetic waves are mutually perpendicular in a plane normal to the direction of propagation. How are phases of $\vec{E}$ and $\vec{B}$ related to each other ?

(c) Discuss the differences in the assumptions underlying Einstein and Debye theories of specific heat C_v. Give schematic plots of C_v versus reduced temperature for these therories and elucidate the differences therein. Elaborate the meaning of the "law of corresponding states" for these plots.

(d) Consider the expression

$$C_P - C_V = -T\left(\frac{\partial v}{\partial T}\right)_P^2 \left(\frac{\partial p}{\partial v}\right)_T$$

and give reasoning regarding the values of T when $C_p = C_v$ for water. Also, evaluate $C_p - C_v$ for a vander Waals gas to elaborate that its value is larger for any real gas as compared to an ideal gas.

6. (a) Consider two long and straight current carrying wires placed parallel to each other a certain distance apart. Derive and expression for the force per unit length experienced by these wires. Discuss that the attractive (repulsive) nature of this force is related to the directions of flow of currents in the two wires. 30

(b) Derive approximate expressions for the potential and the radial as well as the azimuthal components of the field due to and electric dipole at points far away from it. Also derive expression and hence describe the effect of a unifrom electric field on a dipole which can rotate freely. 30

7. (a) Write down the macroscopic form of the Maxwell's equations in any isotropic (but inhomogeneous) medium and define the symbols appearing therein. Convert these equations in the integral forms to highlight the laws represented by these equations. 25

(b) Describe physical significance of the displacement current considering the example of current flow through a capacitor. 15

(c) Use the Planck formula for the blackbody radiation

$$u(\omega, T) = \frac{\hbar^2}{\pi^2 c^3} \frac{\omega^3}{\exp(\beta\hbar\omega) - 1}$$

with $\beta = \frac{1}{k_B T}$ to derive Wien's law, Rayleigh-Jeans law and Stefan-Boltzmann law. 20

8. (a) Starting from the expression

$N = \sum_k \langle n_k \rangle$, where $\langle n_k \rangle$ is the average number of particles in the k^{th} quantum state, derive an expression for the average number of particles in the ground state of an ideal Bose gas. 15

(b) Utilize the above expression to elaborate the concept of the Bose—Einstein condensation and discuss that the phenomenon explains qualitatively the properties in the low-temperature phase of liquid ^4He. 25

(c) Show that the chemical potential of a system is an intensive quantity and is a function of temperature and pressure only. 20

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PAPER – II – 2005

Time Allowed : Three Hours *Maximum Marks : 300*

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Assume suitable data if considered necessary and indicate the same clearly. Some constants are given at the end of the questions.

SECTION – A

1. Answer any *three* of the following : 20 × 3 = 60

(a) (i) Write the commutation relations for the position variable x and the momentum components p_x, p_y, and p_z. Explain the physical significance of these relations.

(ii) Calculate the de Broglie wavelength of an electron moving with a kinetic energy of 1 MeV. 10 + 10

(b) (i) How many lines occur in the multiplet arising from $^2P_{3/2} \rightarrow {}^2S_{1/2}$ and $^2P_{1/2} \rightarrow {}^2S_{1/2}$ transitions of alkali metal atoms placed in a weak magnetic field, why ?

(ii) The wavefunction of a particle confined in a cube of volume L^3 is given by

$$\Psi(x,y,z) = \left(\frac{2}{L}\right)^{3/2} \sin\frac{\pi x}{L} \sin\frac{\pi y}{L} \sin\frac{\pi z}{L}$$

Calculate the average values of p_x and p_x^2 in the region $0 < x < L$. 10 + 10

(c) (i) Explain the molecular phenomenon of spontaneous emission between two electronic states of the same multiplicity and differentiate it form that of different multiplicity.

(ii) The constants B and V_0 for a KCl molecule have values $1{\cdot}43 \times 10^{-5}$ eV $8{\cdot}4 \times 10^{12}\ s^{-1}$ respectively. Determine the number of rotational levels between the vibrational levels v = 0 and v = 1. 10 + 10

(d) (i) The ground state wavefunction of a linear simple harmonic oscillator is

$$\psi = A \exp\left(-\frac{\alpha^2 x^2}{2}\right)$$

Calculate the constant A and the average values of x^2 and x. Given that

$$\int_0^\infty e^{-x^2} = \frac{\pi^{1/2}}{2}$$

(ii) How can the pure rotation spectrum of H_2 molecule be observed ? If the bond length of H_2 molecule is 0·07417 nm, what would be the spacing of lines in its spectrum ? 10 + 10

2. (a) Consider a particle of mass *m* in an infinite one dimensional potential well of width *a*. The particle is found in the state given by

$$\psi(x) = c\left[\sin\frac{\pi x}{a} + \frac{1}{2}\sin\frac{2\pi x}{a}\right]$$

(i) Calculate c.

(ii) If a measurement of energy is made, what are the possible results and what are the probabilities for each one of them ?

(b) Discuss WKB approximation and apply the same to determine the transition probability for leakage through a potential barrier. 30 + 30

3. (a) Set up the time-independent Schrodinger equation for an electron moving in Coulomb field, $V(r) = \frac{Ze^3}{4\pi\varepsilon_0 r}$, in polar coordinates. Solve the radial equation to get the energy eigen values.

(b) Describe Stern-Gerlach experiment and discuss its implications. 40 + 20

4. (a) Differentiates between Rayleigh and Raman scatterings. Why is Raman scattering considered to be a breakthrough in molecular spectroscopy ? What are the advantages of using laser light in Raman spectroscopy ?

(b) Obtain an expression for the vibration-rotation energy levels of a diatomic molecule in a given electronic state.

The wave numbers of the vibrational transitions occuring in HF, HCl and HI molecules are 4141·3 cm^{-1}, 2988·9 cm^{-1} and 2309·5 cm^{-1} respectively. Compare the force constants of these three molecules. 30 + 30

SECTION–B

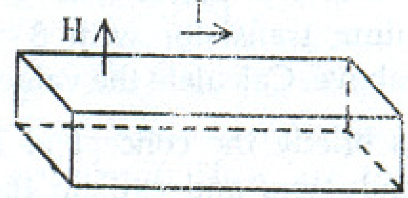
5. Answer any *three* of the following :

(a) (i) Write the Weizacker mass formula and explain the significance of various terms.

(ii) Determine the amount of $^{210}_{84}\rho_0$ necessary to provide a source of α-particles of strength 5 mC_1. The half-life of $^{210}_{84}\rho_0$ is 138 d. 15 + 5

(b) (i) State the quantum numbers I_z, Y and S for the uds quarks and antiquarks. What combination of these leads to the formation of (a) proton and (b) neutron ?

(ii)



An In As semiconductor sample is cut in the form of a small bar of size 1·0 cm × 1·0 cm × 2·0 mm. Its lengthwise resistance is 1·25 Ω. A Hall field of 1·7 V/m develops when a current of 0·12 A is passed lengthwise and a magnetic field of 0·05 T is applied normal to its length as shown above. Calculate the carrier density.

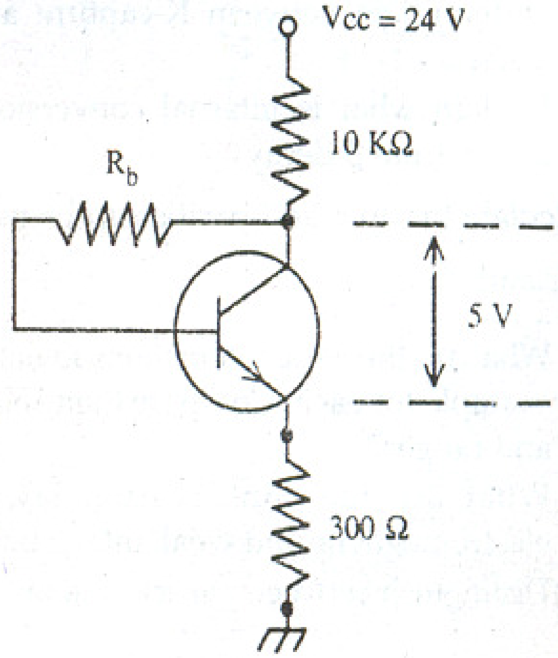
(iii) Determines the reciprocal lattice vectors of an fcc lattice. 6 + 6 +8

(c) (i) Draw the logic circuit for the following Boolean expression : $Y = \overline{A + B} + \overline{C}$

What are the 'Y' values for the input combinations :

(1) 1, 1, 0; (2) 1, 0, 1; (3) 0, 0, 1 ?

(ii)



Define α and β parameters of a transistor. A germanium transistor with $\beta = 45$ is biased as shown above. Calculate the value of R_b. **12 + 8**

(d) (i) Discuss briefly the concept of effective mass in semiconductors and explain the significance of negative effective mass.

(ii) In isolated atoms, the electrons have discrete and definite energies but in solids they have bands of energies. Explain why.

(iii) A proton, deuteron and α-particle accelerated through the same potential difference, on entering a region of uniform magnetic field applied normal to their plane of motion, move in circular orbits. Compare the radii of the orbits described by them.

10 + 5 + 5

6. (a) Describe how parity violation was experimentally detected in $^{60}C_0$ β-decay. How was the observed asymmetry in the distribution of emitted electrons explained ?

(b) (i) Differentiate between K-capture and inverse β-decay.

(ii) Explain what is internal conversion and how it differs from β-decay.

(c) Calculate the spin and parities of the ground states of $^{4}_{2}He$ and $^{67}_{30}Zn$ nuclei. **30 + 20 + 10**

7. (a) (i) What are the basic interactions in nature ? Give one example for each. Compare their relative strengths and ranges.

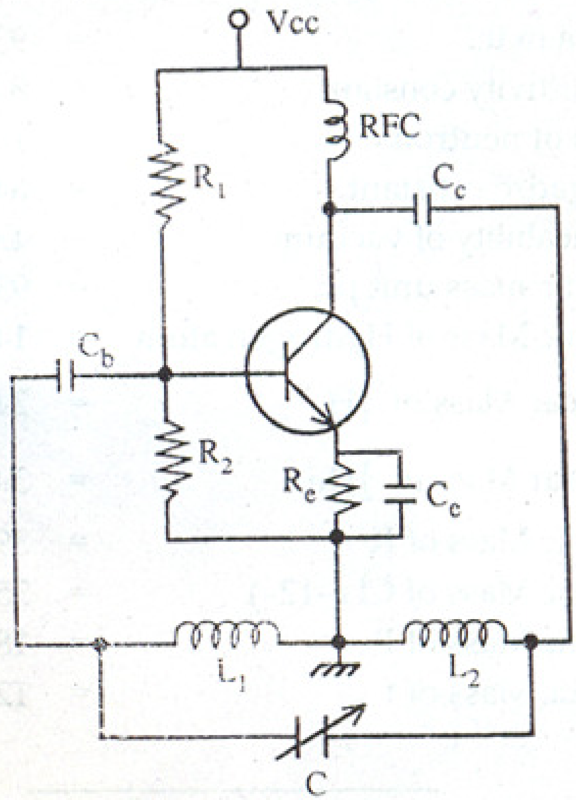
(ii) What are the conservation laws for strong, electromagnetic and weak interactions ?

(iii) Distinguish between particle and anti-particle. How was positron discovered ?

(b) Discuss the motion of an electron in one-dimensional periodic potential and show that it leads to formation of bands of allowed and forbidden states in the electron energy spectrum. How are the insulators, semi-conductors and conductors discriminated on the basis of band structure ? 30 + 30

8. (a) Define (i) input bias current, (ii) input offset current, (iii) input offset voltage, (iv) output offset voltage, (v) power supply rejection ratio and (vi) slew rate for an OPAMP.

(b)



Discuss the functioning of the above oscillator circuit. Obtain the condition for maintenance of oscillations and the expression for the frequency assuming that the resistances of the inductors are negligible. 30 + 30

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CONSTANTS WHICH MAY BE NEEDED

m_e	Mass of electron	=	$9{\cdot}11 \times 10^{-31}$ kg
e	Charge of electron	=	$-1{\cdot}6 \times 10^{-19}$ C
k_B	Boltzmann constant	=	$1{\cdot}38 \times 10^{-23}$ J/K
h	Planck's constant h	=	$6{\cdot}63 \times 10^{-34}$ J-s
	h	=	$1{\cdot}054 \times 10^{-34}$ J-s
C	Velocity of light in vacuum	=	$3{\cdot}0 \times 10^{8}$ ms^{-1}
M_P	Mass of Proton ${}^{1}_{1}H$	=	1·00814511 a.m.u.
	Mass of Helium ${}^{4}_{2}He$	=	4·003873 a.m.u.
	1 a.m.u.	=	931·141 MeV
ε_0	Permittivity constant	=	$8{\cdot}85 \times 10^{-12}$ Fm^{-1}
M_n	Mass of neutron	=	$1{\cdot}675 \times 10^{-27}$ kg
N_a	Avogadro constant	=	$6{\cdot}02 \times 10^{23}$ mol^{-1}
μ_0	Permeability of vacuum	=	$4\pi \times 10^{-7}$
	Atomic mass unit pu	=	931·5 MeV/c^2
	Atomic Mass of Hydrogen atom	=	1·00797 g/Mol
	Nuclear Mass of ${}^{2}_{1}H$	=	2·014102 a.m.u.
	Nuclear Mass of ${}^{3}_{2}He$	=	3·016029 a.m.u.
	Atomic Mass of K	=	39·102 g/Mol
	Atomic Mass of Cl (–12–)	=	35·453 g/Mol
	Atomic Mass of F	=	18·998 g/Mol
	Atomic Mass of I	=	126·904 g/Mol