MATHEMATICS

(Three hours)

(*Candidates are allowed additional 15 minutes for* ***only*** *reading the paper.*

They must NOT start writing during this time.)

Section A - *Answer* **Question 1** *(compulsory) and* **five** *other questions.*

Section B and Section C - *Answer* **two** *questions from* ***either*** *Section B* ***or*** *Section C.*

All working, including rough work, should be done on the same sheet as, and adjacent to, the rest of the answer.

The intended marks for questions or parts of questions are given in brackets [ ].

Mathematical tables and graph papers are provided.

Slide rule may be used.

SECTION A

Question 1 **[10×3]**

(i) If $A = \begin{pmatrix} 3 & 1 \\ 7 & 5 \end{pmatrix}$, find x and y so that $A^2 + xI_2 = yA$.

(ii) Evaluate: $\tan[2\mathrm{Tan}^{-1}(1/5) - \pi/4]$

(iii) Find the value(s) of k so that the line $3x - 4y + k = 0$ is tangent to the hyperbola $x^2 - 4y^2 = 5$.

(iv) Evaluate:

$$\lim_{x \to 1} \left(\frac{x}{x-1} - \frac{1}{\log x} \right)$$

(v) Evaluate: $\int \frac{xe^x}{(x+1)^2} dx$

(vi) Evaluate $\int_0^{\pi/2} \frac{2\cos x - \sin 2x}{2(1+\sin x)} dx$

(vii) Deepak rolls two dice and gets a sum more than 9. What is the probability that the number on the first die is even?

(viii) You are given the following two lines of regression. Find the regression of Y on X and X on Y and justify your answer:

$3x + 4y = 8; \quad 4x + 2y = 10$

(ix) If w is the cube root of unity, then find the value of $(1-3w+w^2)(1+w-3w^2)$

(x) Solve: $(y + xy)dx + y(1-y^2)dy = 0$

Question 2

(a) Using properties of determinants, prove that: **[5]**

$$\begin{vmatrix} b^2+c^2 & ab & ac \\ ba & c^2+a^2 & bc \\ ca & cb & a^2+b^2 \end{vmatrix} = 4a^2b^2c^2$$

(b) Using matrix method, solve the following system of linear equations: **[5]**

$x - 2y - 2z - 5 = 0;$

$-x + 3y + 4 = 0$ and

$-2x + z - 4 = 0$

Question 3

(a) Verify Rolle's theorem for $f(x) = e^x(\sin x - \cos x)$ on $\left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$ and find the point in the interval where derivative vanishes. **[5]**

(b) Find the equation of the parabola whose vertex is at the point (4, 1) and focus is (6, -3). **[5]**

Question 4

(a) Prove that: $\text{Tan}^{-1}[\text{Sin}^{-1}(1/\sqrt{17}) + \text{Cos}^{-1}(9/\sqrt{85})] = ½$. **[5]**

(b) A, B and C represent three switches in 'on' position and A′, B′ and C′ represent the three switches in 'off' position. Construct a switching circuit representing the polynomial: **[5]**

$(A + B)(B' + C)(A' + C')$.

Using the laws of Boolean Algebra, show that the above polynomial is equivalent to $AB' + AC + B$ and construct an equivalent switching circuit.

Question 5

(a) Using suitable substitution, find $\frac{dy}{dx}$ for $y = \text{Tan}^{-1}\left[\frac{\sqrt{1+x^2}-1}{x}\right]$ **[5]**

(b) Show that the right circular cone of least curved surface area and given volume has an altitude equal to $\sqrt{2}$ times the radius of the base. **[5]**

Question 6

(a) Evaluate:

$$\int \frac{\tan x + \tan^2 x}{1+\tan^2 x} dx$$ **[5]**

(b) Sketch the graphs of y = x(4 - x) and find the area bounded by the curve, x – axis and the lines x = 0 and x = 5. **[5]**

Question 7

(a) Calculate Spearman's rank correlation for the following data: **[5]**

Advertisement cost (₹ in thousands)	39	65	62	90	82	75	25	98	36	78
Sales (₹ in lakhs)	47	53	58	86	62	68	60	91	51	84

(b) Fit a straight line to the following data, treating y as dependent variable: **[5]**

x	14	12	13	14	12
y	22	23	22	24	24

Hence, predict the value of y when x = 16.

Question 8

(a) Bag I has two red and three black balls, bag II has four red and one black ball and bag III has three white and two black balls. A bag is selected at random and a ball is drawn at random. What is the probability of drawing a red ball? **[5]**

(b) A man makes attempts to hit a target. The probability of hitting the target is 3/5. Find the probability that the man hit the target at least two times in five attempts. **[5]**

Question 9

(a) Solve: **[5]**

$$(\text{Tan}^{-1}y - x)\,dy = (1 + y^2)\,dx.$$

(b) If $z = x + iy$, $\omega = (2 - iz) / (2z - i)$ and $|\omega| = 1$, find the locus of z and illustrate it in the complex plane. **[5]**

SECTION B

Question 10

(a) Find the shortest distance between the lines whose vector equations are: **[5]**

$$\vec{r} = (1-t)\hat{i} + (t-2)\hat{j} + (3-2t)\hat{k} \text{ and}$$

$$\vec{r} = (s+1)\hat{i} + (2s-1)\hat{j} - (2s+1)\hat{k}$$

(b) Find the equation of the plane passing through the point (1, 1, -1) and perpendicular to the planes $x + 2y + 3z - 7 = 0$ and $2x - 3y + 4z = 0$. **[5]**

Question 11

(a) Show that $(\vec{a} \times \vec{b})^2 = \begin{vmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{b} \\ \vec{a}.\vec{b} & \vec{b}.\vec{b} \end{vmatrix}$ **[5]**

(b) Using Vector method, prove that the line segment joining the midpoints of two sides of a triangle is parallel to the third side and equal to the half of it. **[5]**

Question 12

(a) A manufacturer has three machines operators A, B and C. The first operator A produces 1% defective items, whereas the other two operators B and C produce 5% and 7% defective items respectively. A is on the job for 50% of the time, B is on the job for 30% of the time and C is on the job for 20% of the time. A defective item is produced. What is the probability that it was produced by A? **[5]**

(b) If a coin is tossed ten times, find the probability of getting: **[5]**

(i) Exactly six heads

(ii) At least six heads.

SECTION C

Question 13

(a) An aeroplane can carry a maximum of 200 passengers. A profit of ₹ 1000 is made on each executive class ticket and a profit of ₹ 600 is made on each economy class ticket. The airline reserves at least 20 seats for executive class. However, at least four times as many passengers prefer to travel by economy class than by the executive class. Determine how many tickets of each type must be sold in order to maximise the profit for the airline. What is the maximum profit earned? **[5]**

(b) How much should be a company set aside at the beginning of each year if it has to buy a machine expected to cost ₹ 2,00,000 at the end of six years, when the rate of interest is 10% per annum, compounded annually. **[5]**

(Use $(1.1)^6 = 1.772$)

Question 14

(a) Given the total cost function for x units of a commodity as: **[5]**

$$C(x) = \frac{x^3}{3} + 3x^2 - 7x + 16, \text{ find:}$$

(i) The marginal cost.

(ii) The average cost.

(iii) Show that the marginal average cost is given by $\{ x\,MC - C(x)\}/ x^2$?.

(b) A bill of ₹ 12,750 was payable 60 days after sight. It was accepted on 4th March 1990 and discounted on 25th March 1990. What was the discounted value of the bill if the rate of interest was 7/2% per annum? **[5]**

Question 15

(a) Calculate the index number for 1979 with 1970 as the base year by using the weighted average of price relative method: **[5]**

Commodity	Weight	Price (in ₹)	
		1970	1979
A	22	2.50	6.20
B	48	3.30	4.40
C	17	6.25	12.75
D	13	0.65	0.90

(b) Calculate the three yearly moving averages of the data given below and plot them on a graph paper: **[5]**

Year	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992	1993	1994
Production (in tonnes)	137	140	134	137	151	121	124	159	157	169	172	150