Total No. of Questions: 12]

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F. E. Examination - 2010

ENGINEERING MATHEMATICS - I

(2003 Course)

Time: 3 Hours]

[Max. Marks : 100

Instructions:

- (1) From section I solve Q.1 or Q. 2, Q. 3 or Q. 4, Q. 5 or Q. 6. From section II solve Q or Q. 8, Q. 9 or Q. 10, Q. 11 or Q. 12.
- (2) Answers to the two sections should be written in separate books.
- (3) Neat diagrams must be drawn wherever necessary.
- (4) Black figures to the right indicate full marks.
- (5) Use of electronic pocket calculator is allowed.
- (6) Assume suitable data, if necessary.

SECTION - I

Q.1) (A) Define normal form of a matrix. Reduce the following matrix to its normal form and hence find its rank, where [06]

$$\mathbf{A} = \begin{bmatrix} 3 & -6 & 4 & -3 & 2 \\ 2 & -4 & 3 & 1 & 0 \\ 0 & 1 & -1 & 3 & 1 \\ 4 & -7 & 4 & -4 & 5 \end{bmatrix}$$

(B) Given the linear transformation Y = AX, where

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 5 \\ 1 & 3 & 3 \end{bmatrix}, \text{ find the co-ordinates}$$

 (x_1, x_2, x_3) corresponding to (2, 0, 5) in Y

[05]

(C) Verify Cayley - Hamilton Theorem for

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

and use it to find A4.

[06]

OR

Q.2) (A) Find the eigen values and the corresponding eigen vectors of the following matrix: [07]

$$A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -5 & 2 \end{bmatrix}$$

- (B) If A is an orthogonal matrix then show that A⁻¹ and A^t are also orthogonal. [04]
- (C) Examine for linear dependence, the system of vectors $X_1 = \{1, 2, 3\}$, $X_2 = \{3, -2, 1\}$, $X_3 = \{1, -6, -5\}$. If dependent, fine relation between them.
- Q.3) (A) Show that the roots of $(x + 1)^6 + (x 1)^6 = 0$ are given

by
$$-i \cot \left(\frac{2r+1}{12}\right)\pi$$
, $r = 0, 1, 2, 3, 4, 5.$ [06]

(B) Find real and imaginary parts of $tanh^{-1}(x + iy)$. [06]

(C) Prove that
$$\log\left(\frac{a+ib}{a-ib}\right) = 2i \tan^{-1}\left(\frac{b}{a}\right)$$
 and hence evaluate $\cos\left[i\log\left(\frac{a+ib}{a-ib}\right)\right]$. [05]

OR

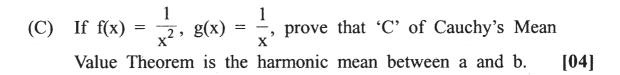
- A square lies above real axis in Argand's Diagram and two $\mathbf{Q.4}$) (A) of its adjacent vertices are the origin and the point 5 + 6i. Find the complex numbers representing other vertices. [06]
 - If tan(x + iy) = i, where x and y are real, prove that x is (B) indeterminate and y is infinite. [06]
 - By considering principal value, express in the form a + ib the (C) expression $(1 + i\sqrt{3})^{(1+i\sqrt{3})}$ [05]
- Find the n^{th} derivative of the function $e^{2x} \sin x \cos x$. (A) Q.5)[05].
 - If $x = \sin \theta$, $y = \sin 2\theta$, then prove that $(1 - x^2)y_{n+2} - (2x+1)xy_{n+1} - (n^2 - 4)y_n = 0$ [06]
 - Prove that: $\frac{b-a}{1+b^2} < (\tan^{-1}b \tan^{-1}a) < \frac{b-a}{1+a^2}$, if a < b. [05]

Q.6) (A) If $y = \frac{\sinh^{-1} x}{\sqrt{1 + x^2}}$, then prove that, $(1 - x^2)y_{n+2} + (2n + 3)xy_{n+1} + (n + 1)^2 y_n = 0.$ (B) If $I_n = \frac{d^n}{dx^n} (x^n \log x)$, prove that $I_n = n I_{n-1} + (n - 1) ! \text{ and hence show that}$ [06]

(B) If
$$I_n = \frac{d^n}{dx^n} (x^n \log x)$$
, prove that

$$I_n = n! \left[\log x + 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n} \right].$$
 [06]

[3761]-11 3 P.T.O.



SECTION - II

Determine the range of convergence of $\mathbf{Q.7}$

$$\sum_{n=1}^{\infty} \frac{n+1}{2n+1} \cdot \frac{(x-3)^n}{2^n}$$

[05]

Discuss the convergence of any one of the following: (B) [04]

(1)
$$\left(\frac{1}{4}\right)^2 + \left(\frac{1\cdot 5}{4\cdot 8}\right)^2 + \left(\frac{1\cdot 5\cdot 9}{4\cdot 8\cdot 12}\right)^2 + \dots$$

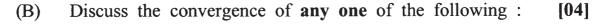
(2)
$$\frac{2 \cdot 1^3 + 5}{4 \cdot 1^5 + 1} + \frac{2 \cdot 2^3 + 5}{4 \cdot 2^5 + 1} + \dots + \frac{2 \cdot n^3 + 5}{4 \cdot n^5 + 1} + \dots$$

- Attempt any two of the following: (C) [80]
 - (1) Expand $\frac{x}{e^x} = 1$ by to x^4 .
 - (2) Arrange in powers of x using Taylor's Theorem $7 + (x + 2) + 3(x + 2)^3 + (x + 2)^4 - (x + 2)^5$

(3) Prove that
$$e^{\cos^{-1}x} = e^{\pi/2} \left[1 - x + \frac{x^2}{2} - \frac{x^3}{3} + \dots \right]$$

Q.8) (A) Solain the range of convergence of the series

$$\sum \frac{\sqrt{n}}{\sqrt{n^2+1}} x^n$$
 [05]



(1)
$$\frac{1}{2} + \frac{2}{3}x + \left(\frac{3}{4}\right)^2x^2 + \left(\frac{4}{5}\right)^3x^3 + \dots$$

(2)
$$\frac{1}{(\log 2)^2} + \frac{1}{(\log 3)^2} + \dots + \frac{1}{(\log n)^2} + \dots$$

(1) Prove that

$$e^{e^{x}} = e \left[1 + x + x^{2} + \frac{5}{6} x^{3} + \frac{5}{8} x^{4} + \dots \right]$$

- (2) Expand $(1 + x)^{(1+x)}$ upto term containing x^3 .

(3) Using Taylor's Theorem show that :
$$\sqrt{1 + x + 2x^2} = 1 + \frac{7}{2} + \frac{7}{8}x^2 - \frac{7}{16}x^3 + \dots$$

(1) Evaluate
$$\lim_{x\to 0} \left[\frac{1}{x^2} - \cot^2 x \right]$$

(2) Evaluate:
$$\lim_{x\to 0} \left(\frac{a^x + b^x + c^x}{3} \right)^{1/x}$$

Evaluate:
$$\lim_{x\to 0} \left[\frac{\sin^{-1}x - x}{x^3} \right]$$

(B) If
$$x = e^r \cos\theta$$
, $y = e^r \sin\theta$ P.T. [04]

$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = e^{-2r} \left[\left(\frac{\partial u}{\partial r}\right)^2 + \left(\frac{\partial u}{\partial \theta}\right)^2 \right]$$

(C) If
$$u = \log (x^3 + y^3 - x^2y - xy^2)$$
 prove that : [05]

$$(1) \quad x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3$$

(2)
$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = -3$$

[08]

Find the values of a, b, c so that **(1)**

$$\lim_{x \to 0} \frac{ae^x - b\cos x + ce^{-x}}{x \sin x} =$$

(2) Evaluate:
$$\lim_{x \to 1} (1 - x^2)^{\frac{1}{\log(1-x)}}$$

(3) Evaluate:
$$\lim_{x\to 0} \log_{ta} x \cos 2x$$

(3) Evaluate:
$$\lim_{x\to 0} \log_{\tan x} \tan 2x$$

(B) If $u = ax + by$, $v = bx - ay$, find the value of [04]

$$\left(\frac{\partial u}{\partial x}\right)_{y} \cdot \left(\frac{\partial x}{\partial v}\right)_{v} \cdot \left(\frac{\partial x}{\partial v}\right)_{x} \left(\frac{\partial v}{\partial y}\right)_{u}$$
(C) Find $\frac{dy}{dx}$ where $v^{x^{y}} = \sin x$.

(C) Find
$$\frac{dy}{dx}$$
 when $y^{x^y} = \sin x$. [05]

Q.11) (A) If u, v, w are the roots of equation

$$(\lambda - x)^3 + (\lambda - y)^3 + (\lambda - z)^3 = 0 \text{ find } \frac{\partial(u, v, w)}{\partial(x, y, z)}$$
 [06]

(B) Prove that error in calculating the power $W = V^2/R$ generated

in the resistor is $\frac{V}{R^2}$ (2R δ V – V δ R). If there are errors of

1% and 2% in measuring the voltage V and resistance R, find % error in calculating of work $W = V^2/R$. [05]

Examine for stationary values (C)

$$f(x, y) = \sin x + \sin y + \sin(x + y)$$
 [05]

OR

Q.12) (A) If
$$v^2 + w^2 = x$$
, $y = w^2 + u^2$, $z = u^2 + v^2$ prove that $JJ' = 1$. [06]

(B) Examine whether the functions

$$u = x + y + z$$

 $v = x^2 + y^2 + z^2$
 $w = x^3 + y^3 + z^3 - 3xyz$

are functionally dependent. If dependent find the relation between them.

[05]

(C) Use Lagranges Method of Undertermined Multipliers to find the maximum and minimum distance of the point (3, 4, 12) from the sphere $x^2 + y^2 + z^2 = 1$. [05]