Total No. of Questions : 12] **[Total No. of Printed Pages : 7**

[3761]-11

F. E. Examination - 2010

ENGINEERING MATHEMATICS - I

(2003 Course)

Time : 3 Hours] **[Max. Marks : 100**

Instructions :

(1) *From section I solve Q.1 or Q. 2, Q. 3 or Q. 4, Q. 5 or Q. 6. From section II solve Q. 7 or Q. 8, Q. 9 or Q. 10, Q. 11 or Q. 12.*

(2) *Answers to the* ***two sections*** *should be written in* ***separate books****.*

(3) *Neat diagrams must be drawn wherever necessary.*

(4) *Black figures to the right indicate full marks.*

(5) *Use of electronic pocket calculator is allowed.*

(6) *Assume suitable data, if necessary.*

SECTION - I

Q.1) (A) Define normal form of a matrix. Reduce the following matrix to its normal form and hence find its rank, where **[06]**

$$A = \begin{bmatrix} 3 & -6 & 4 & -3 & 2 \\ 2 & -4 & 3 & 1 & 0 \\ 0 & 1 & -1 & 3 & 1 \\ 4 & -7 & 4 & -4 & 5 \end{bmatrix}$$

(B) Given the linear transformation $Y = AX$, where

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 5 \\ 1 & 3 & 3 \end{bmatrix}$$, find the co-ordinates

(x_1, x_2, x_3) corresponding to $(2, 0, 5)$ in Y. **[05]**

(C) Verify Cayley - Hamilton Theorem for

$$A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

and use it to find A^4. **[06]**

OR

Q.2) (A) Find the eigen values and the corresponding eigen vectors of the following matrix : **[07]**

$$A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -5 & -2 \end{bmatrix}$$

(B) If A is an orthogonal matrix then show that A^{-1} and A^t are also orthogonal. **[04]**

(C) Examine for linear dependence, the system of vectors $X_1 = (1, 2, 3)$, $X_2 = (3, -2, 1)$, $X_3 = (1, -6, -5)$. If dependent, find relation between them. **[06]**

Q.3) (A) Show that the roots of $(x + 1)^6 + (x - 1)^6 = 0$ are given by $-i \cot\left(\frac{2r+1}{12}\right)\pi$, $r = 0, 1, 2, 3, 4, 5$. **[06]**

(B) Find real and imaginary parts of $\tanh^{-1}(x + iy)$. **[06]**

(C) Prove that $\log\left(\frac{a+ib}{a-ib}\right) = 2i\tan^{-1}\left(\frac{b}{a}\right)$ and hence evaluate $\cos\left[i\log\left(\frac{a+ib}{a-ib}\right)\right]$. **[05]**

OR

Q.4) (A) A square lies above real axis in Argand's Diagram and two of its adjacent vertices are the origin and the point $5 + 6i$. Find the complex numbers representing other vertices. **[06]**

(B) If $\tan(x + iy) = i$, where x and y are real, prove that x is indeterminate and y is infinite. **[06]**

(C) By considering principal value, express in the form $a + ib$ the expression $\left(1 + i\sqrt{3}\right)^{\left(1 + i\sqrt{3}\right)}$ **[05]**

Q.5) (A) Find the n^{th} derivative of the function $e^{2x}\sin x\cos x$. **[05]**

(B) If $x = \sin\theta$, $y = \sin 2\theta$, then prove that

$(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 - 4)y_n = 0$ **[06]**

(C) Prove that : $\frac{b-a}{1+b^2} < \left(\tan^{-1}b - \tan^{-1}a\right) < \frac{b-a}{1+a^2}$, if $a < b$. **[05]**

OR

Q.6) (A) If $y = \frac{\sinh^{-1}x}{\sqrt{1+x^2}}$, then prove that,

$(1 + x^2)y_{n+2} + (2n + 3)xy_{n+1} + (n + 1)^2 y_n = 0.$ **[06]**

(B) If $I_n = \frac{d^n}{dx^n}\left(x^n \log x\right)$, prove that

$I_n = n\, I_{n-1} + (n - 1)!$ and hence show that

$I_n = n!\left[\log x + 1 + \frac{1}{2} + \frac{1}{3} + \ldots\ldots + \frac{1}{n}\right]$. **[06]**

(C) If $f(x) = \frac{1}{x^2}$, $g(x) = \frac{1}{x}$, prove that 'C' of Cauchy's Mean Value Theorem is the harmonic mean between a and b. **[04]**

SECTION - II

Q.7) (A) Determine the range of convergence of

$$\sum_{n=1}^{\infty} \frac{n+1}{2n+1} \cdot \frac{(x-3)^n}{2^n}.$$ **[05]**

(B) Discuss the convergence of **any one** of the following : **[04]**

(1) $\left(\frac{1}{4}\right)^2 + \left(\frac{1\cdot 5}{4\cdot 8}\right)^2 + \left(\frac{1\cdot 5\cdot 9}{4\cdot 8\cdot 12}\right)^2 + \ldots\ldots$

(2) $\frac{2\cdot 1^3 + 5}{4\cdot 1^5 + 1} + \frac{2\cdot 2^3 + 5}{4\cdot 2^5 + 1} + \ldots\ldots + \frac{2\cdot n^3 + 5}{4\cdot n^5 + 1} + \ldots\ldots$

(C) Attempt **any two** of the following : **[08]**

(1) Expand $\frac{x}{e^x - 1}$ up to x^4.

(2) Arrange in powers of x using Taylor's Theorem

$7 + (x+2) + 3(x+2)^3 + (x+2)^4 - (x+2)^5$

(3) Prove that $e^{\cos^{-1}x} = e^{\pi/2}\left[1 - x + \frac{x^2}{2} - \frac{x^3}{3} + \ldots\ldots\right]$

OR

Q.8) (A) Obtain the range of convergence of the series

$$\sum \frac{\sqrt{n}}{\sqrt{n^2+1}}\, x^n$$ **[05]**

(B) Discuss the convergence of **any one** of the following : **[04]**

(1) $\frac{1}{2} + \frac{2}{3}x + \left(\frac{3}{4}\right)^2 x^2 + \left(\frac{4}{5}\right)^3 x^3 + \ldots\ldots$

(2) $\frac{1}{(\log 2)^2} + \frac{1}{(\log 3)^2} + \ldots\ldots + \frac{1}{(\log n)^2} + \ldots\ldots$

(C) Attempt **any two** of the following : **[08]**

(1) Prove that

$$e^{e^x} = e\left[1 + x + x^2 + \frac{5}{6}x^3 + \frac{5}{8}x^4 + \ldots\right]$$

(2) Expand $(1 + x)^{(1+x)}$ upto term containing x^3.

(3) Using Taylor's Theorem show that :

$$\sqrt{1 + x + 2x^2} = 1 + \frac{x}{2} + \frac{7}{8}x^2 - \frac{7}{16}x^3 + \ldots$$

Q.9) (A) Attempt **any two** of the following : **[08]**

(1) Evaluate : $\lim_{x\to 0}\left[\frac{1}{x^2} - \cot^2 x\right]$

(2) Evaluate : $\lim_{x\to 0}\left(\frac{a^x + b^x + c^x}{3}\right)^{1/x}$

(3) Evaluate : $\lim_{x\to 0}\left[\frac{\sin^{-1}x - x}{x^3}\right]$

(B) If $x = e^r \cos\theta$, $y = e^r \sin\theta$ P.T. **[04]**

$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = e^{-2r}\left[\left(\frac{\partial u}{\partial r}\right)^2 + \left(\frac{\partial u}{\partial \theta}\right)^2\right]$$

(C) If $u = \log(x^3 + y^3 - x^2y - xy^2)$ prove that : **[05]**

(1) $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = 3$

(2) $x^2\dfrac{\partial^2 u}{\partial x^2} + 2xy\dfrac{\partial^2 u}{\partial x \partial y} + y^2\dfrac{\partial^2 u}{\partial y^2} = -3$

OR

Q.10) (A) Attempt **any two** of the following : **[08]**

(1) Find the values of a, b, c so that

$$\lim_{x\to 0} \frac{ae^x - b\cos x + ce^{-x}}{x\sin x} = 2.$$

(2) Evaluate : $\lim_{x\to 1} \left(1 - x^2\right)^{1/\log(1-x)}$

(3) Evaluate : $\lim_{x\to 0} \log_{\tan x} \tan 2x$

(B) If $u = ax + by$, $v = bx - ay$, find the value of **[04]**

$$\left(\frac{\partial u}{\partial x}\right)_y \cdot \left(\frac{\partial x}{\partial v}\right)_v \cdot \left(\frac{\partial y}{\partial v}\right)_x \left(\frac{\partial v}{\partial y}\right)_u$$

(C) Find $\dfrac{dy}{dx}$ when $y^{x^y} = \sin x$. **[05]**

Q.11) (A) If u, v, w are the roots of equation

$(\lambda - x)^3 + (\lambda - y)^3 + (\lambda - z)^3 = 0$ find $\dfrac{\partial(u, v, w)}{\partial(x, y, z)}$ **[06]**

(B) Prove that error in calculating the power $W = V^2/R$ generated in the resistor is $\dfrac{V}{R^2}(2R\delta V - V\delta R)$. If there are errors of 1% and 2% in measuring the voltage V and resistance R, find % error in calculating of work $W = V^2/R$. **[05]**

(C) Examine for stationary values

$f(x, y) = \sin x + \sin y + \sin(x + y)$ **[05]**

OR

Q.12) (A) If $v^2 + w^2 = x$, $y = w^2 + u^2$, $z = u^2 + v^2$ prove that $JJ' = 1$. **[06]**

(B) Examine whether the functions

$u = x + y + z$

$v = x^2 + y^2 + z^2$

$w = x^3 + y^3 + z^3 - 3xyz$

are functionally dependent. If dependent find the relation between them. **[05]**

(C) Use Lagranges Method of Undertermined Multipliers to find the maximum and minimum distance of the point (3, 4, 12) from the sphere $x^2 + y^2 + z^2 = 1$. **[05]**