Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/B.Tech (CSE/IT)/SEM-4/M-401/2010

2010

MATHEMATICS

Time Allotted : 3 Hours   *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

i) The generators of the cyclic group ($Z, +$) are

a) 1, – 1 b) 0, 1

c) 0, – 1 d) 2, – 2.

ii) The mapping $f : R \to R$ given by $f(x) = |x|, x \in R$ is

a) Injective b) Surjective

c) Bijective d) None of these.

iii) Let S be a finite set of n distinct elements. The number of bijective mapping from S to S is

a) n^2 b) $n!$

c) 2^n d) None of these.

iv) If three Boolean variables x, y and z are defined on Boolean Algebra B, then which one of the following is a fundamental product ?

a) $xy'z$ b) xy

c) $xy(x+y)$ d) none of these.

v) If G is binary tree on n vertices, the G has edges

a) $n(n-1)$ b) $n-1$

c) n d) $\frac{n(n-1)}{2}$.

vi) Solution of the recurrence relation $S_n = 2S_{n-1}$ with $S_0 = 1$ is $S_n =$

a) 2^n b) 2^{n-1}

c) 2^{n+1} d) none of these.

vii) A complete graph is

a) regular b) connected simple

c) circuit d) planar graph.

viii) On the set $A = \{1, 2, 3\}$, the relation $R = \{(2, 1), (1, 2), (3, 3)\}$. Then R is

a) symmetric b) reflexive

c) transitive d) not a relation at all.

ix) In the additive group Z_6 the order of the element $[4]$ is

a) 0 b) 2

c) 3 d) 6.

x) Let G be a group and $a \in G$. If $o(a) = 17$, then $o(a^8)$ is

a) 17 b) 16

c) 8 d) 5.

xi) If S and T are two subgroup of a group G, then which of the following is a subgroup ?

a) $S \cup T$ b) $S \cap T$

c) $S - T$ d) $G - S$.

xii) The dual of a planar graph is dual. It is

a) True b) False.

xiii) A binary tree should have at least

a) one vertex b) two vertices

c) three vertices d) four vertices.

xiv) A connected graph is Eulerian iff it has no vertex of odd degree. It is

a) True b) False.

xv) The number of idempotent element in Z is

a) 0 b) 1

c) 2 d) none of these.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Let $G = \{ (a, b) : a \neq 0, b \in R \}$ and * be a binary compostion defined on G by $(a, b) * (c, d) = (ac, bc + d)$. Show that $(G, *)$ is a non Abelian group.

3. Show that for any two subgroups H and K of a group G $H \cap K$ is also a subgroup of G.

4. Let G be a group, if $a, b \in G$ such that $a^4 = e$, the identity element of G and $ab = ba^2$, prove that $a = e$.

5. Prove that every cyclic group is an Abelian group.

6. Show that the mapping $F : (Z, \bullet) \rightarrow (R, \bullet)$ defined by $f(x) = x^2 \; \forall x \in Z$ is a monomorphism but not isomorphism.

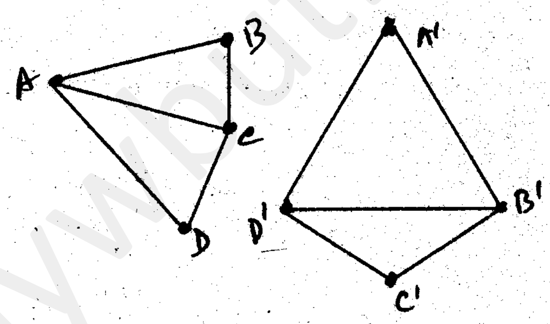
7. If in a ring R with unity, $(xy)^2 = x^2 y^2 \; \forall x, y \in R$, then show that R is a commutative.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

8. a) Examine whether the following two graphs are isomorphic.



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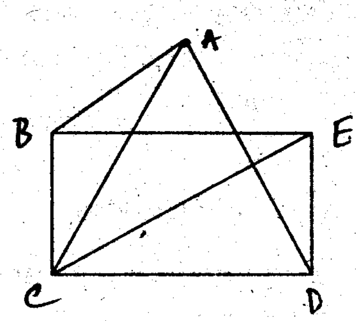
b) Draw the dual of the graph.



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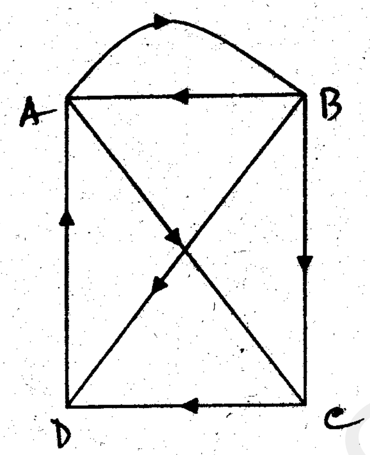
c) Determine the adjacency matrix of the following di-graph :



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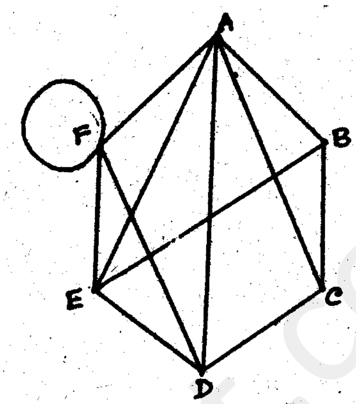
9. a) Construct a simple logic circuit which would satisfy the truth table.

x	y	f
1	1	1
0	1	1
1	0	0
0	0	1

5

b) Prove that a graph G has a spanning tree if and only if G is connected. 5

c) Find the minimal spanning tree by Kruskal's Algorithm from the following graph G :



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10. a) Consider the lattice $L = \{ 1, 2, 3, 4, 6, 12 \}$, the divisors of 12 ordered by divisibility. Find the lower and upper bound of L. Is L a complemented lattice ? 5

b) For any Boolean Algebra, show that.

$$(xy' + xz') + x' = (x' + y + z)(x' + y + z')(x' + y' + z').$$

5

c) Using generating function solve the recurrence relation, $a_n - 7a_{n-1} + 10a_{n-2} = 0$, for $n > 1$ and $a_0 = 3$, $a_1 = 3$. 5

11. a) Prove that the number of vertices in a binary tree is always odd. 5

b) Find the truth table of the Boolean function

$$f = z'xy + xy' + y.$$ 5

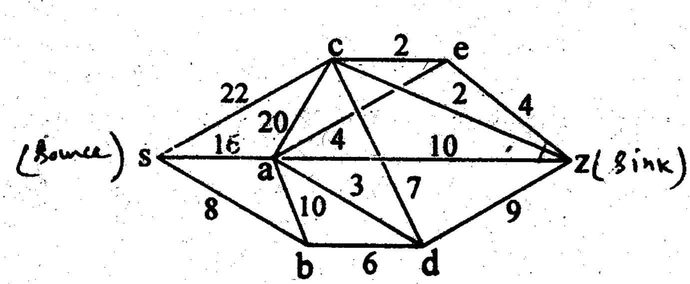
c) Prove that a complete graph with n vertices consist of $\frac{n(n-1)}{2}$ number of edges. 5

12. a) Prove that the identity elements and the inverse of an element in a group is unique. 5

b) Prove that in a group $(G, *)$, $(a * b)^{-1} = b^{-1} * a^{-1}$. 5

c) Prove that the set of matrices $H = \left\{ \begin{pmatrix} x & 0 \\ 0 & x \end{pmatrix} : x \in R, x \neq 0 \right\}$ forms a normal subgroup of $GL(2, R)$, the group of all real non-singular 2×2 matrices under multiplication. 5

13. a) Using Ford-Fulkerson's algorithm, find the maximum flow in the following network :

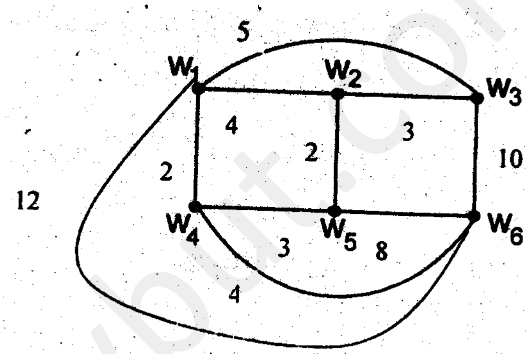


8

b) Using Floyd's algorithm, find the shortest path between

i) w_2 and w_6

ii) w_1 and w_6.



7