T.E. Electronic & telecommunication
Sem V Rev

Con. 6198-10. Random Signal Analysis **GT-6753**

(REVISED COURSE)

(3 Hours) **[Total Marks : 100**

N.B. : (1) Question No. **1** is **compulsory**.
(2) Solve any **four** from **remaining** questions.
(3) Assume **suitable data** wherever **necessary** and **state** it **clearly**.

1. (a) State and Explain central limit theorem. **20**
(b) (i) State Axiomatic definition of probability.
(ii) If X and Y are independent random variables show that $E[XY] = \mu_x \mu_y$ where $\mu_x = E[X]$ and $\mu_y = E[Y]$.
(c) Define strict sense stationary and WSS random process.
(d) Define Markov Chain. State any two application areas of Markov Chains.

2. (a) Define random variable, explain with suitable example. State the conditions for a function to be a random variable. **10**
(b) Let X_1, X_2, ...be a sequence of random variables. **10**
Define : (i) convergence almost everywhere
(ii) convergence in Probability
(iii) convergence in mean square sense
(iv) convergence in Distribution
for the above sequence to a random variable X.

3. (a) Define random process. Give example of a random process. Show few member functions. Define first and second order distribution and density functions for a random process. **10**
(b) The signal Z at the input of an amplifier consists of a signal X to which is added random noise Y. Thus Z = X + Y. If X is also a random variable. **10**
(i) Determine the pdf of Z.
(ii) If X and Y are independent then what is the pdf of Z ?

4. (a) In a communication system, a zero is transmitted with probability 0·4 and a one is transmitted with probability 0·6. Due to noise in the channel, a zero can be received as one with probability 0·1 and as a zero with probability 0·9, similarly one can be received as zero with probability 0·1 and as a one with probability 0·9. Now— **10**
(i) A one is observed, what is the Probability that zero was transmitted.
(ii) A one is observed, what is the Probability that a one was transmitted.
(b) In Medical imaging such as computer tomography the relation between detector reading y and body absorptivity x follows a $y = e^x$ law. Let X be $N(\mu, \sigma^2)$. Compute the pdf of y. **10**

5. (a) Define n-step transition probability for a Markov Chain. Derive Chapman-Kolmogorov equation. 10

(b) Consider the random process— 10

$$X(t) = A \sin (w_0 t + \theta)$$

where A and θ are independent, real-valued random variables and θ is uniformly distributed over $[-\pi, +\pi]$. Find the mean $\mu_x(t)$ and Autocorrelation function $R_{xx}(t_1, t_2)$ of X(t).

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6. (a) A WSS random process x(t) with autocorrelation— 10

$$R_{xx}(\tau) = Ae^{-a|\tau|}$$

where A and a are real positive constants, is applied to the input of an LTI system with impulse response $h(t) = e^{-bt}u(t)$, where b is a real positive constant. Find the autocorrelation of the output Y(t) of the system.

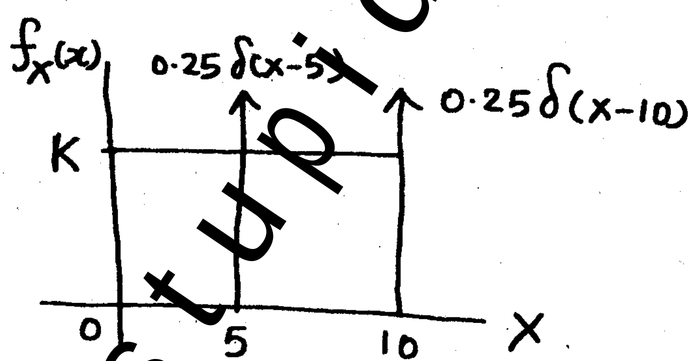
(b) In the photoelctric detector, Let Y be the number of photoelectrons produced in time τ, depends on the (normalized) incident energy X. If X were constant, say X = x, Y would be a poisson random variable with parameter x but as real light sources except for gain stabilized lasers – do not emit constant energy signals, X must be treated as a random variable. In certain situations the pdf of X is accurately modeled by— 10

$$f_X(x) = \begin{cases} \dfrac{1}{\mu_x} \exp\left(\dfrac{-x}{\mu_x}\right), & x \geq 0 \\ 0, & x < 0 \end{cases}$$

where $\mu_x = E[x]$

Compute E[Y].

7. (a) The pdf of a random variable is shown— 10



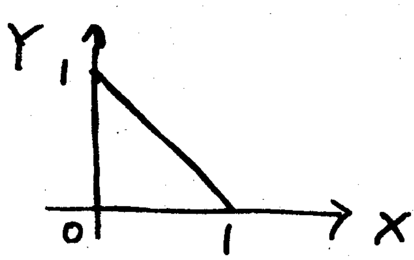
(i) Find the value of constant K.

(ii) Compute $P[X \leq 5]$ and $P[5 \leq X < 10]$.

(b) The joint probability density function of (x, y) is given by— 10

$$f_{XY}(x, y) = C(1 - x - y)$$

for values of x and y for which (x, y) lies within the triangle as shown—



outside the triangle $f_{XY}(x, y) = 0$.

Find : (i) C

(ii) $f_X(x)$

(iii) $f_Y(y)$.