T.E / VL / Ex+C / Rev / May'07 / PRP /

540-E 1stHf07

Con. 3358-07. **(REVISED COURSE)** **ND-2002**

(3 Hours) [Total Marks : 100

N.B. Answer any **five** questions.

1. (a) State and explain with example :—
   (i) Conditional probability 2
   (ii) Baye's Theorem. 4

   (b) If two events A and B are independant, show that —
   (i) $\overline{A}$ and $\overline{B}$ are independant 2
   (ii) A and $\overline{B}$ are independant. 2

   (c) For a certain binary communication channel, the probability that a transmitted '0' is received as a '0' is 0·95 while the probability that a transmitted '1' is received as '1' is 0·90. If the probability of transmitting a '0' is 0·4, find the probability that —
   (i) a '1' is received 3
   (ii) a '1' was transmitted given that '1' was received 4
   (iii) the error has occured. 3

2. (a) Define probability distribution function of random variable. State important properties of it and prove any one property. 6

   (b) If X is poisson distributed random variable, find moment generating function (M.G.F.) and characteristic function. 6

   (c) A continuous random variable has the probability density function defined by — 8

   $$f_x(x) = k \cdot x \cdot e^{-\lambda x} \quad ; \quad x \geq 0, \ \lambda > 0$$
   $$= 0 \quad ; \quad \text{else.}$$

   Determine the constant K and find mean and variance.

3. (a) If X is a continuous random variable and Y = aX + b then, prove that — 6

   $$fy(y) = \frac{1}{|a|} f_X\left(\frac{Y-b}{a}\right).$$

   (b) If a random variable X has uniform distribution in (–2, 2), find the probability density function fy(y) of Y = 3X + 2. 4

   (c) The joint probability density function of two dimensional random variable (x, y) is given by —

   $$f_{xy}(x, y) = 4xy\ e^{-(x^2 + y^2)} \ ; \ x \geq 0, \ y \geq 0$$

   (i) Find the marginal density functions of x and y. 4
   (ii) Find the conditional density function of Y given that X = x and the conditional density function of X given that Y = y. 4
   (iii) Check for independance of X and Y. 2

4. (a) Prove that if two random variables are independant, then density function of their sum is given by convolution of their density functions. 8

   (b) If X and Y are two independant exponential random variables with probability density functions given by 6

   $$f_x(x) = 2 \cdot e^{-2x} \quad ; \quad x \geq 0$$
   $$= 0 \quad ; \quad x < 0 \ \text{and}$$
   $$f_y(y) = 3 \cdot e^{-3y} \quad ; \quad y \geq 0$$
   $$= 0 \quad ; \quad y < 0$$

   Find the probability density function of z = x + y.

   (c) The joint probability density function of (x, y) is $f_{xy}(x, y) = 8 \cdot e^{-(2x + 4y)}$; x, y ≥ 0. 6
   If U = X / Y and V = Y, find the joint probability density function of (U, V) and hence find the probabilty density function of U.

5. (a) If X and Y are two random variables with standard deviations 6_x and 6_y and if C_{xy} is the covariance between them, then prove
   (i) $C_{xy}(x, y) = R_{xy}(x, y) - E[X] \cdot E[Y]$ 4
   (ii) $| C_{xy} | \leq 6x \cdot 6y$. 4

   Also deduce that 2

   $-1 \leq \rho \leq 1$.

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(b) If $X = \cos\theta$ and $Y = \sin\theta$ where θ is uniformly distributed over $(0, 2\pi)$. Prove that —
   (i) X and Y are uncorrelated. 5
   (ii) X and Y are not independant. 5

6. (a) Explain in brief — 12
   (i) WSS process
   (ii) Poisson process
   (iii) Queueing system.

   (b) A random process is given by 8

$$X(t) = \text{Sin}(\omega t + y)$$

where Y is uniformly distributed in $(0, 2\pi)$. Verify whether X(t) is a wide-sense stationary process.

7. (a) Explain power spectral density function. State its important properties and prove any one property. 8

   (b) For a random process having 6

$R_{xx}(\tau) = a \cdot e^{-b|\tau|}$ find the spectral density function, where a and b are constants.

   (c) The power spectral density of a WSS process is given by — 6

$$S_{xx}(w) = \frac{b}{a}(a - |w|) \; ; \; |w| \le a$$
$$= 0 \; ; \; |w| > a$$

Find the auto correlation of the function.

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