SECTION - A

10 × 2 = 20

VERY SHORT ANSWER TYPE QUESTIONS

Attempt **ALL** questions. Each question carries **2 marks**.

1. Find the equation of the circle having the line joining $(1, 2)$, $(4, 5)$ as a diameter.

2. Find the equation of the sphere that passes through the point $(4, 3, -1)$ and having its centre $(3, 8, 1)$.

3. Find the equation of the tangent to the parabola $y^2 = 16x$ inclined at $60°$ to the x-axis.

4. Find the product of lengths of the perpendiculars from any point on the hyperbola $x^2/16 - y^2/9 = 1$ to its assymptotes.

5. Find the nth derivative of $\log(4x^2 - 9)$.

6. Evaluate $\int \dfrac{1}{5 - 2x^2 + 4x}\, dx$.

7. Evaluate $\int \dfrac{1}{1 + \sin 2x}\, dx$.

8. Evaluate $\displaystyle\int_0^{\pi/2} \dfrac{\cos^{5/2} x}{\sin^{5/2} x + \cos^{5/2} x}\, dx$.

9. Find the area cut off between $x = 0$ and $x = 4 - y^2$.

10. Find the order of the differential equation obtained by eliminating the arbitrary constants b and c from $xy = ce^x + be^{-x} + x^2$.

SECTION - B

5 × 4 = 20

SHORT ANSWER TYPE QUESTIONS

Attempt **any 5** questions. Each question carries **4 marks**.

11. Show that the area of the triangle formed by the two tangents from (x_1, y_1) to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ and the chord of contact is $\dfrac{r(S_{11})^{3/2}}{r^2 + S_{11}}$ where r is the radius of the circle.

12. Show that the common tangents to the circle $2x^2 + 2y^2 = a^2$ and the parabola $y^2 = 4ax$ intersect at the focus of the parabola $y^2 = -4ax$.

13. Show that the locus of poles of chords of the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ which subtend a right angle at the centre of the ellipse is $\dfrac{x^2}{a^4} + \dfrac{y^2}{b^4} = \dfrac{1}{a^2} + \dfrac{1}{b^2}$.

14. If PSP', QSQ are two perpendicular focal chords of a conic then show that $\dfrac{1}{PP'} + \dfrac{1}{QQ'}$ is a constant.

15. Evaluate $\displaystyle\int x\, Sin^{-1}\, x\, dx$.

16. Solve $(y^2 - 2xy)\, dx + (2xy - x^2)\, dy = 0$.

17. Solve $\dfrac{dy}{dx} + \dfrac{y}{x} = \dfrac{y^2}{x^2}$.

SECTION - C   5 × 7 = 35

LONG ANSWER TYPE QUESTIONS

Attempt **any** 5 questions. Each question carries 7 marks.

18. Show that the circles $x^2 + y^2 - 4x - 6y - 12 = 0$, $x^2 + y^2 + 6x + 18y + 26 = 0$ touch each other. Find the point of contact and the equation of the tangent at the point of contact.

19. Find the equation of the circle cutting orthogonally the circles $x^2 + y^2 - 8x - 2y + 16 = 0$, $x^2 + y^2 - 4x - 4y - 1 = 0$ and passing through the point $(1, 1)$.

20. Find the equation of the parabola whose focus is $(3, 5)$ and vertex is $(1, 3)$.

21. If $y = Tan^{-1}\, x$ then show that $(1 + x^2)\, y_{n+2} + (2n + 2)\, x\, y_{n+1} + n(n+1)\, y_n = 0$.

22. Evaluate $\displaystyle\int \dfrac{1}{3\cos x + 4\sin x + 6}\, dx$.

23. Evaluate $\displaystyle\int_{0}^{\pi/2} \dfrac{x\, dx}{\sin x + \cos x}$.

24. Using Simpson's rule to approximate the integral $\displaystyle\int_{1}^{5} \dfrac{dx}{1 + x}$ with $n = 4$.