Seat No.: _____ Enrolment No.______

GUJARAT TECHNOLOGICAL UNIVERSITY

M.E Sem-I Regular Examination January / February 2011

Subject code:710901N

Subject Name: Theory of Elasticity

Date: 31 /01 /2011 **Time: 02.30 pm – 05.00 pm**

Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **English version is Authentic**

Q.1 (a) Define state of stress at a point and prove that if stress components acting in three mutually perpendicular planes passing through a point are known then stress components on any plane passing through that point can be determined. **07**

(b) Prove that of the nine rectangular components of stress only six are independent due to equality of cross shears. **07**

Q.2 (a) The three principal stresses at point P are 4 MPa, 5 MPa and 6 MPa respectively. Determine the unit normal for the plane upon which the normal stress is 5 MPa and shearing stress is 0.5 MPa. **07**

(b) The state of stress at a point P in a Cartesian frame of reference (x, y, z) is given as **07**

$$\tau_{ij} = \begin{bmatrix} 10{,}000 & 10{,}000 & 10{,}000 \\ 10{,}000 & -5000 & 10{,}000 \\ 10{,}000 & 10{,}000 & -5000 \end{bmatrix} \text{ N/cm}^2$$

Determine the normal and shearing stresses on a plane that is equally inclined to all the three axes.

OR

(b) Determine the principal stresses and their directions for a given state of stress. **07**

$$\tau_{ij} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \text{ N/cm}^2$$

Where in usual notations the three invariants are as follows:

$$I_1 = \tau_{xx} + \tau_{yy} + \tau_{zz}$$

$$I_2 = \tau_{xx}\,\tau_{yy} + \tau_{yy}\,\tau_{zz} + \tau_{zz}\,\tau_{xx} - \tau_{xy}^2 - \tau_{yz}^2 - \tau_{zx}^2$$

$$I_3 = \begin{vmatrix} \tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \tau_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \tau_{zz} \end{vmatrix}$$

Q.3 (a) Draw and comment on the nature of the Mohr's circle diagram for the following cases where in the three principal stresses σ_1, σ_2 and σ_3 are given as: **07**

(i) $\sigma_1 \geq \sigma_2 \geq \sigma_3$ (ii) $\sigma_1 = \sigma_2 \geq \sigma_3$ (iii) $\sigma_1 = \sigma_2 = \sigma_3$

(b) Given the displacement field **07**

$$u_x = k(x^2 + 2z)\ u_y = k\,(4x + 2y^2 + z)\ ,\ u_z = 4kz^2$$

where $k = 0.001$ can be assumed as very small.

Determine:

(a) The engineering extensional strains at the point $P(2, 2, 3)$, along the directions $(0, 1/\sqrt{2}, 1/\sqrt{2})$ and $(1,0,0)$.

OR

Q.3 (a) The displacement components of an incompressible continuum are **07**

$u_x = (1-y^2)(a+bx+cy^2), \quad u_y\Big|_{y=\pm\sqrt{3}} = 0, \quad u_z = 0$

Where a, b and c are infinitesimal constants. Determine u_y if $D = 1 + \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z}$ in usual notations.

(b) The displacement field for a body which satisfies the compatibility condition is given by **07**

$u = (x^2+y)\vec{i} + (3+z)\vec{j} + (x^2+2y)\vec{k}$. Write down the displacement gradient matrix at point (2, 3, 1).

Q.4 (a) Show with the mathematical proof that there are only two elastic constants involved in relations between principal stresses and principal strains for an isotropic material. **07**

(b) Define modulus of rigidity, bulk modulus, Young's modulus and Poisson's ratio. Prove that an isotropic material having a Poisson's ratio as 0.5 is incompressible in nature. **07**

OR

Q. 4 (a) Prove that the displacement equation of equilibrium in x – direction is **07**

$(\lambda+\mu)\frac{\partial \Delta}{\partial x} + \mu\nabla^2 u_x = 0$

Where u_x the displacement in x direction, λ and μ is Lame's coefficient and Δ is a volumetric strain.

(b) A cubical element is subjected to the following state of stress. **07**

$\sigma_x = 100$ MPa, $\sigma_y = -20$ MPa, $\sigma_z = -40$ MPa

$\tau_{xy} = \tau_{yz} = \tau_{zx} = 0$

Assuming the material to be homogeneous and isotropic determine the principal shear strain and octahedral shear strain, if E = 2 x 105 MPa and ν = 0.25. Assume in usual notations

G = E/2(1+ ν)

Q.5 (a) Consider a thin disk with a hole with an inner radius 'a' and outer radius 'b' and traction free surface (i.e. $\sigma_r = 0$ at r = a, and r = b) subjected to a temperature distribution which varies with radius 'r' and is independent of the angular displacement 'θ'. Derive the expression for radial and angular stresses for a given case. **07**

(b) Determine the thermal stresses induced in three directions (r, θ, z) on a long **hollow** circular cylinder with inner radius 'a' and outer radius 'b' when the temperature is symmetrical about the axis and does not vary along its axis. **07**

OR

Q.5 (a) Explain the principle of superimposition and prove that the principle is valid for two different forces acting at two different points. **07**

(b) Derive the mathematical expression for Castigliano's first principle from the concepts of elastic strain energy. Illustrate the use of this principle for any one real time engineering application. **07**

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