SOLUTION & ANSWER FOR AIEEE-2009
VERSION – A

[PHYSICS, CHEMISTRY & MATHEMATICS]

PART A – PHYSICS

1. Statement 1 : For a charged particle moving from Point P to Point Q ---------

Ans : Statement – 1 is true, Statement – 2 is true; Statement – 2 is the correct explanation of Statement - 1

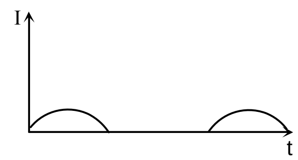
2. The above is a plot of binding energy per nucleon E_b, against the nuclear mass M ---------

Ans : (i) and (iv)

Sol: Heavy nuclide disintegrates to lighter ones by releasing energy. Lighter nuclei combine to form releasing energy.

3. A p – n junction (D) shown in the figure can act -----

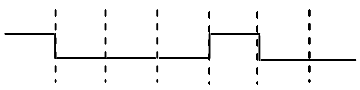
Ans :



Sol: Diode acts as a half wave rectifier without filter.

4. The logic circuit shown below has the input waveforms `A' ----------

Ans :



Sol: $\overline{\overline{A}+\overline{B}} = A.B$ = AND gate

5. If x, v and a denote the displacement, the velocity and the acceleration --------

Ans : $\frac{aT}{x}$

Sol: $\frac{aT}{x} = \frac{-A\omega^2 \sin\omega t.T}{A\sin\omega t} = -\omega^2 T$ = constant

6. In an optics experiment, with the position of the object ------

Ans : (2f, 2f)

Sol: $\frac{1}{u}+\frac{1}{v}=\frac{1}{f} \Rightarrow u = v = 2f$

7. A thin uniform rod of length `ℓ' and mass m is swinging freely ----

Ans : $\frac{1}{6}\frac{\ell^2\omega^2}{g}$

Sol: $E = \frac{1}{2}I\omega^2 = \frac{1}{2}\frac{m\ell^2}{3}\omega^2 = mgh$

$\Rightarrow h = \frac{\ell^2\omega^2}{6g}$

8. Let $P(r) = \frac{Q}{\pi R^4}r$ be the charge density distribution for a --------

Ans : $\frac{Qr_1^2}{4\pi\varepsilon_0 R^4}$

Sol: $q = \int_0^{r_1} 4\pi r^2 dr \frac{Q}{\pi R^4} r = \frac{Qr_1^4}{R^4}$

$E = \frac{Qr_1^4}{R^4}.\frac{1}{4\pi\varepsilon_0 r_1^2} = \frac{Qr_1^2}{4\pi\varepsilon_0 R^4}$

9. The transition from the state n = 4 to n = 3 in a ----------

Ans : $5 \rightarrow 4$

Sol: Transition $n_x \rightarrow n_{x-1}$ is minimum for larger x.

10. One kg of a diatomic gas is at a pressure of ---------

Ans : 5×10^4 J

Sol: PV = nRT

$$P\frac{m}{\rho} = nRT$$

$$V = \frac{5}{2}nRT = \frac{5}{2}\rho\frac{m}{V} = 5 \times 10^4\ J$$

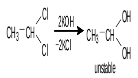
11. Statement 1 : The temperature dependence of resistance is usually --------------

Ans : Statement – 1 is false, Statement – 2 is true.

12. The magnitude of the magnetic field (B) due to the loop ---------

Ans : $\dfrac{\mu_0 I(b-a)}{24ab}$

Sol: $B = \dfrac{\mu_0 I}{2}\left(\dfrac{30}{360}\right)\left(\dfrac{1}{a}-\dfrac{1}{b}\right)$

= 

13. Due to the presence of the current I_1 at the ---------

Ans : The forces on AD and BC are zero.

Sol: $\bar{B}$ vector is parallel to each current element along AD and BC

$Id\bar{\ell} \times \bar{B} = 0$

14. A mixture of light, consisting of wavelength 590 nm and an unknown

Ans : 442. 5 nm

Sol: $3\beta_{kn\ own} = 4\ \beta_{unknown}$

$\therefore\ 3\lambda_{known} = 4\ \lambda_{unknown}$

$\lambda_{unknown} = \dfrac{3}{4}\lambda_{known} = 442.5$ nm

15. Two points P and Q are maintained at the potentials ----------

Ans : 2.24×10^{-16} J

Sol: $W = q\Delta V$

$= -1.6 \times 10^{-19} \times (-14) \times 100$

$= 2.24 \times 10^{-16}$ J

16. The surface of a metal is illuminated with the light -------------

Ans : 1.41 eV

Sol: $h\upsilon = \dfrac{1240}{400} = 3.1 eV$

$KE_{max} = 1.68$ eV

$\therefore\ \phi = 3.1 - 1.68 = 1.41$ eV

17. A particle has an initial velocity of ------

Ans : $7\sqrt{2}$ units

Sol: $\bar{v} = \bar{u} + \bar{a}t$

$= (3\hat{i} + 4\hat{j}) + 10(0.4\hat{i} + 0.3\hat{j})$

$|\bar{v}| = |(7\hat{i} + 7\hat{j})|$

18. A motor cycle starts from rest and accelerates ----------

Ans : 98 m

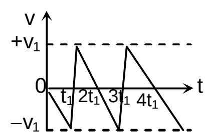
Sol: $f = f_0\left(\dfrac{330 - u_\ell}{330}\right) = 0.94\ f_0$

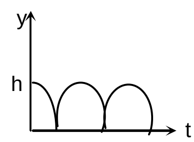
$330 - u_\ell = 0.94 \times 330$

$u_\ell = 19.8$ m/s

$\therefore\ S = \dfrac{1}{2}at^2 = 98$ m

19. Consider a rubber ball freely falling from a height ------------------

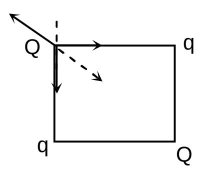
Ans : 



Sol: Velocity reverses instantaneously; downward direction of velocity needs to be treated as negative according to sign conservation.

20. A charge Q is placed at each of the opposite corners of a -------

Ans : $-2\sqrt{2}$

Sol: 

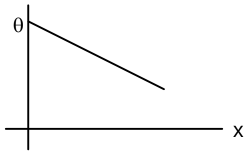
Magnitudes of forces

$$\frac{1}{K}\frac{Q^2}{(\sqrt{2}a)^2} = \frac{1}{K}\frac{Qq}{a^2}.2\frac{1}{\sqrt{2}}$$

$$\frac{Q}{q} = -2\sqrt{2}$$

21. A long metallic bar is carrying heat from one of its ends to the other ------

Ans :



Sol: $\Delta Q = -KA\dfrac{\Delta T}{\Delta X}$

ΔQ constant $\Rightarrow \dfrac{\Delta T}{\Delta X}$ constant.

22. A transparent solid cylindrical rod has a refractive ---------

Ans : $\sin^{-1}\left(\dfrac{1}{\sqrt{3}}\right)$

Sol: $\text{Sin }(90 - r) = \dfrac{1}{\mu} = \dfrac{\sqrt{3}}{2}$

$\text{Cos } r = \dfrac{\sqrt{3}}{2} \Rightarrow r = 30°$

$\dfrac{\sin i}{\sin r} = \mu = \dfrac{2}{\sqrt{3}}$

$\sin i = \dfrac{1}{\sqrt{3}}$

$I = \sin^{-1}\left(\dfrac{1}{\sqrt{3}}\right)$

23. Three sound waves of equal amplitudes have frequencies -------

Ans : 1

Sol: Resultant frequency of (υ – 1) and (υ + 1) with υ cannot produce any beat. Beat can produce either with υ and (υ + 1) or υ and (υ – 1) where the beat frequency is 1.

24. The height at which the acceleration due to gravity becomes ----------------

Ans : 2 R

Sol: $g = \dfrac{GM}{R^2}$

$\dfrac{g}{9} = \dfrac{GM}{(R+h)^2} \Rightarrow \dfrac{(R+h)^2}{R^2} = 9$

h = 2 R

25. Two wires are made of the same material and have the same volume ------

Ans : 9 F

Sol: $\dfrac{\frac{F}{A}}{\frac{(\Delta\ell)}{\ell}} = Y \Rightarrow F = YA\dfrac{(\Delta\ell)}{\ell}$

$V_1 = A\ell\ ;\ V_2 = 3A.\dfrac{\ell}{3} = V_1$

$\dfrac{\frac{F'}{3A}}{\frac{\Delta\ell}{\ell/3}} = Y \Rightarrow F' = Y.3A\dfrac{(\Delta\ell)}{\ell}.3 = 9\text{ F}$

26. In an experiment the angles are required to be measured using an -------------

Ans : One minute

Sol: $\dfrac{1.\text{m.s.d.}}{N} = \dfrac{1/2}{30} = \dfrac{1}{60}{}^\circ$

= 1'

27. An inductor of inductance L = 400 mH and resistors -----

Ans : $12\ e^{-5t}$ V

Sol: $I = \dfrac{L}{R} = 0.2$

$E = E_0 e^{-t/\tau} = 12\ e^{-5t}$

R_1 has no role

28. Assuming the gas to be ideal the work done ------

Ans : 400 R

Sol: $V_1 = \dfrac{nRT_1}{P}$

$V_2 = \dfrac{nRT_2}{P}$

$V_2 - V_1 = \dfrac{nR}{P}(T_2 - T_1)$

$P(V_2 - V_1) = nR(T_2 - T_1) = 400\text{ R}$

29. The work done on the gas in taking it from D to A is --------

Ans : –414 R

Sol: $V_A P_A = V_D P_D \Rightarrow \dfrac{V_A}{V_D} = \dfrac{P_D}{P_A} = \dfrac{1}{2}$

$W = nRT\ \text{In}\left(\dfrac{V_2}{V_1}\right) = 2R.300\ \text{In}\left(\dfrac{1}{2}\right)$

= –414 R

30. The net work done on the gas in the cycle -----

Ans : 276 R

Sol: $W_{AB} = 400\ R$; $W_{CD} = -400\ R$

$W_{DA} = -414\ R$;

$W_{BC} = nRT\ \ln \frac{V_C}{V_B}$

$= 2R.500.\ \ln 2$

$= 693\ R$

$W_{total} = (693 - 414)\ R$

$= 279\ R$

PART B – CHEMISTRY

31. Knowing that the Chemistry of lanthanoids .... is incorrect?

Ans: Ln(III) compounds are generally colourless.

Sol : Except La^{3+},Lu^{3+}, other lanthanoids exhibit colour in both solid state and in aqueous solution.

32. A liquid was mixed with ethanol and a drop of concentrated H_2SO_4 was added.........

Ans: CH_3COOH

Sol : Acetic acid forms ethyl acetate (ester) having fruity smell when heated with ethanol in presence of a drop of con.H_2SO_4.

$$CH_3 - COOH + C_2H_5OH \xrightleftharpoons{con.H_2SO_4} CH_3 - COOC_2H_5 + H_2O$$

33. Arrange the carbanions, $(CH_3)_3\bar{C}$, $\bar{C}Cl_3$, $(CH_3)_2\bar{C}H$, ...

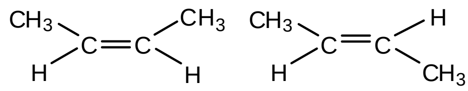
Ans: $\overset{(-)}{C}Cl_3 > C_6H_5C\overset{(-)}{H_2} > (CH_3)_2\bar{C}H > (CH_3)_3C^{(-)}$

Sol : $\overset{(-)}{C}Cl_3$ is stable due to the –I effect of three chlorine atoms. Further it is stabilised by the resonance due to the presence of d–orbitals on chlorine. $C_6H_5C\overset{(-)}{H_2}$ is stabilised by resonance. $(CH_3)_3C^{(-)}$ is the least stable ion due to the +I effect of three methyl groups.

34. The alkene that exhibits geometrical ....

Ans: 2–butene

Sol :



Cis-but-2-ene trans-but-2-ene

35. In which of the following .... the sequence is not strictly..........

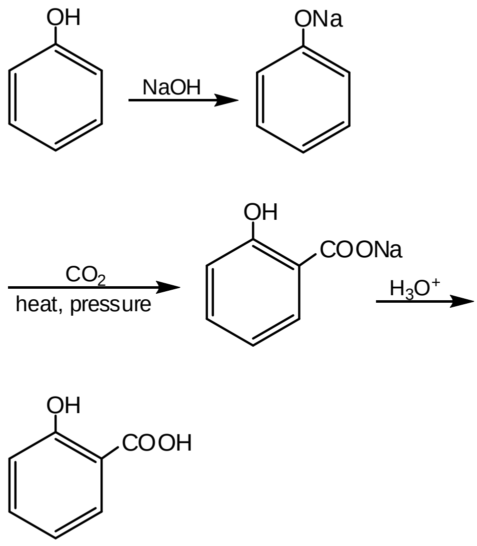
Ans: $NH_3 < PH_3 < AsH_3 < SbH_3$: increasing basic strength.

Sol : NH_3 is the most basic . Basic strength decreases from NH_3 to SbH_3.

36. The major product obtained .... of phenol with sodium hydroxide ....

Ans: Salicylic acid

Sol :



Salicyclic acid

It is Kolbe's reaction.

37. Which of the following .... is incorrect .... physisorptions?

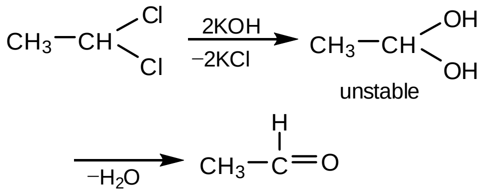
Ans: Enthalpy of adsorption ($\Delta H_{adsorption}$) is low and positive.

Sol : ΔH is negative for adsorption.

38. .... on heating with aqueous KOH, produces acetaldehyde?

Ans: CH_3CHCl_2

Sol :



39. In an atom, an electron is moving ....

Ans: 1.92×10^{-3} m

Sol : $\Delta x\, m\Delta v = \dfrac{h}{4\pi}$ $\quad \Delta v = \dfrac{0.005}{100} \times 600 = 0.03$

$$\Delta x = \frac{6.6 \times 10^{-34}}{4 \times 3.14 \times 9.1 \times 10^{-31} \times 0.03}$$

$= 1.92 \times 10^{-3}$ m

40. In a fuel cell methanol is used ....

Ans: 97%

Sol : Efficiency = $\dfrac{\Delta G}{\Delta H}$

$\Delta G = (-394.4 \text{ kJ} + 2 \times -237.2 \text{ kJ}) - (-166.2 \text{ kJ}) = -702.6$ kJ

$\Delta H = -726$ kJ

Efficiency = $\dfrac{-702.6}{-726} \times 100 = 97\%$

41. Two liquids X and Y form an ideal solution. ....

Ans: 400 and 600

Sol : $\dfrac{1}{4}X + \dfrac{3}{4}Y = 550$

$\dfrac{1}{5}X + \dfrac{4}{5}Y = 560$

Solving X = 400 mm Hg

Y = 600 mmHg

42. The half life period of a first order ....

Ans: 46.06 minutes

Sol : $\lambda = \dfrac{2.303}{t} \log \dfrac{N_0}{N_t}$

$\dfrac{0.693}{t_{1/2}} = \dfrac{2.303}{t} \log \dfrac{100}{1}$

$\dfrac{0.693}{6.93} = \dfrac{2.303}{t} \times 2$

t = 46.06 minutes

43. Given:

$E^0_{Fe^{3+}/Fe} = -0.036$ V, ...

Ans: 0.770 V

Sol :

	E°(V)	nE°(V)
$Fe^{3+} + 3e^- \rightarrow Fe$	–0.036	–0.108.... (1)
$Fe^{2+} + 2e^- \rightarrow Fe$	–0.439	–0.878.... (2)
(1) – (2) $\rightarrow$		
$Fe^{3+} + 1e^- \rightarrow Fe^{2+}$	0.77	0.77

44. On the basis of the following ....

Ans: –228.88 kJ

Sol : Given $H_2O_{(l)} \rightarrow H^+_{(aq)} + OH^-_{(aq)}$

$\Delta H = 57.32$ kJ

$\therefore 57.32 \text{ kJ} = \Delta f H^0_{OH^-_{(aq)}} - (-286.2 \text{ kJ})$

$\Delta f H^0_{OH^-_{(aq)}} = -228.88$ kJ

45. Copper crystallises in fcc with a ....

Ans: 127 pm

Sol : $\sqrt{2}\,a = 4r$

$r = \dfrac{\sqrt{2} \times 361 \text{ pm}}{4} = 127$ pm

46. .... has an optical isomer?

Ans: $[Co(en)_2(NH_3)_2]^{3+}$

Sol : Cis $[Co(en)_2(NH_3)_2]^{3+}$ is optically active.

47. Solid $Ba(NO_3)_2$ is gradually dissolved ....

Ans: 5.1×10^{-5} M

Sol : $K_{sp(BaCO_3)} = [Ba^{2+}]\,[CO_3^{2-}] = 5.1 \times 10^{-9}$

$\therefore [Ba^{2+}] = \dfrac{5.1 \times 10^{-9}}{10^{-4}}$

$= 5.1 \times 10^{-5}$ M

48. .... Xenon compounds is not fesible?

Ans: $XeO_3 + 6HF \rightarrow XeF_6 + 3H_2O$

Sol : XeO_3 reacts with aqueous alkali but not with acids.

49. Using MO theory .... shortest bond length?

Ans: O_2^{2+}

Sol : In O_2^{2+} the bond order is three, hence the shortest bond length.

50. In context with the transition elements, ... the following statements is incorrect?

Ans: In the highest oxidation states the transition metals show basic character and form cationic complexes.

Sol : In the highest oxidation state the transition metals show acidic character.

51. Calculate the wavelength ....

Ans: 0.40 nm

Sol : $\lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34}\ Js}{1.67 \times 10^{-27}\ kg \times 10^{3}\ ms^{-1}}$

$= 4 \times 10^{-10}$ m = 0.4 nm

52. A binary liquid solution is prepared by ....

Ans: The solution is non–ideal, showing +ve deviation from Raoult's Law.

Sol : n–heptane reduces the attraction between the ethanol molecules and there by mutually increases the vapourising tendency.

53. The number of stereoisomers possible ....

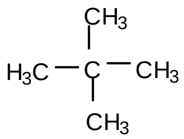
Ans: 4

Sol : The compound CH_3–CH =CH–CH(OH)–Me can show geometrical as well as optical isomerism. Hence the cis and trans isomers can have the (+) and (–) forms. So the total number of stereoisomers is 4.

54. The IUPAC name ....

Ans: 2, 2-dimethylpropane

Sol :



: 2, 2-dimethylpropane

(neopentane)

55. .... correct order of ionic radius is:

Ans: $Na^{+} > Li^{+} > Mg^{2+} > Be^{2+}$

Sol : The ionic radii in pm are

$Be^{2+} = 31$ $Mg^{2+} = 72$

$Li^{+} = 76$ $Na^{+} = 102$

56. The two functional groups present ....

Ans: $>C = O$ and –OH

Sol : Carbohydrates are polyhydroxy carbonyl compounds.

57. The bond dissociation energy of B – F in ....

Ans: Significant $p\pi - p\pi$ interaction between B and F in BF_3 whereas there is no possibility of such interaction between C and F in CF_4.

Sol : $p\pi - p\pi$ back bonding in BF_3 makes B – F bonds stronger whereas in CF_4 no such back bonding is possible, as there is no vacant p–orbital in carbon.

58. In Cannizzaro reaction .... the slowest step is:

Ans: the transfer of hydride to the carbonyl group

Sol : The rate determining step in Cannizzaro reaction is the transfer of hydride ion from the anion formed by the addition of $\overset{(-)}{OH}$ to the carbonyl group of the other aldehyde molecule.

59. .... represents linkage isomers?

Ans: $[Pd(PPh_3)_2(NCS)_2]$ and $[Pd(PPh_3)_2(SCN)_2]$

Sol : Linkage of the ambidentate ligand CNS is through two different sites in the complexes.

60. Buna – N synthetic rubber ....

Ans: $H_2C = CH - CN$ and
$H_2C = CH - CH = CH_2$

Sol : Buna – N is a copolymer of 1, 3–butadiene and acrylonitrile ($CH_2 = CH - CN$)

PART C – MATHEMATICS

61. Let a, b, c be such that $b(a + c) \neq 0$. ....

Ans: any odd integer

Sol: $\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} +$

$$\begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ (-1)^{n+2}a & (-1)^{n+1}b & (-1)^n c \end{vmatrix} = 0$$

$\Delta + (-1)^n \begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ a & -b & c \end{vmatrix}$

$\Delta + (-1)^n \Delta = 0 \Rightarrow$ n is any odd integer

62. If the mean deviation of the numbers 1, …

Ans: 10.1

Sol: The mean of the given numbers is the middle term 1 + 50d

$\therefore$ M.D = $\dfrac{2(d + 2d + ... + 50d)}{101}$

$= \dfrac{2}{101} d(1 + 2 + ... + 50)$

$= \dfrac{2}{101} d \dfrac{(50) \times (51)}{2} = \dfrac{2550}{101} d = 255$

d = 10.1

63. If the roots of the equation $bx^2 + cx + a = 0$

Ans: greater than –4ab

Sol: Roots of $bx^2 + cx + a = 0$ are imaginary

$\Rightarrow c^2 - 4ab < 0$

$\Rightarrow c^2 < 4ab$ — (1)

Now the extremum value of

$3b^2 x^2 + 6bcx + 2c^2$ is

$\dfrac{-\left((6bc)^2 - 4 \times 3b^2 \times 2c^2\right)}{4 \times 3b^2}$

$= \dfrac{-12b^2c^2}{12b^2} = -c^2 > -4ab$ by (1)

64. Let A and B denote the statements ….

Ans: both a and b are true

Sol: $\sum(\cos\alpha \cos\beta + \sin\alpha \sin\beta) = \dfrac{-3}{2}$

$\therefore \sum\cos^2\alpha + \sum\sin^2\alpha + 2\sum\cos\alpha \cos\beta + 2\sum\sin\alpha \sin\beta = 0$

i.e., $(\cos\alpha + \cos\beta + \cos\gamma)^2 + (\sin\alpha + \sin\beta + \sin\gamma)^2 = 0$

i.e., $\cos\alpha + \cos\beta + \cos\gamma = 0$ and $\sin\alpha + \sin\beta + \sin\gamma = 0$

65. The lines $p(p^2 + 1)x - y + q = 0$ ….

Ans: Exactly one value of p

Sol: Given two lines are parallel

slope $m_1 = p(p^2 + 1)$, $m_2 = -(p^2 + 1)$

Since $m_1 = m_2$ we get $p = -1$

$\therefore$ exactly one value of p

66. If A, B and C are three sets such that $A \cap B$

Ans: B = C

Sol: $A \cap B = A \cap C$ and $A \cup B = A \cup C \Rightarrow B = C$

67. If $\bar{u}, \bar{v}, \bar{w}$ are non-coplanar vectors ….

Ans: exactly one value of (p, q)

Sol: $3p^2[u\ v\ w] - pq[v\ w\ u] - 2q^2[w\ v\ u] = 0$

$\Rightarrow 3p^2[u\ v\ w] - pq[u\ v\ w] + 2q^2[u\ v\ w] = 0$

$\Rightarrow [u\ v\ w]\ (3p^2 - pq + 2q^2) = 0$

$3p^2 - pq + 2q^2 = 0$ ($\because [u\ v\ w] \neq 0$)

Since discriminant of the above equation < 0, the equation has real roots only when p = 0 & q = 0.

68. Let the line $\dfrac{x-2}{3} = \dfrac{y-1}{-5} = \dfrac{z+2}{2}$ lie in the …

Ans: (–6, 7)

Sol: The normal to the plane is perpendicular to the line

$\therefore 3 \times 1 + 3 \times -5 + 2x - \alpha = 0$

$\Rightarrow \alpha = -6$

(2, 1, –2) lies in the plane

$\therefore 2 + 3 \times 1 + 6 \times -2 + \beta = 0$

$\Rightarrow \beta = 7$

$\therefore (\alpha, \beta) = (-6, 7)$

69. From 6 different novels and 3 different …..

Ans: atleast 1000

Sol: 6 novels, 3 dictionaries

To select 4 novels and 1 dictionary. This can be done in ${}^{6}C_4 \times {}^{3}C_1$ ways = 15 × 3 = 45 ways

		D		

Dictionary is to be placed in the middle. 4 novels can be arranged in the 4 places in 4! = 24 ways

$\therefore$ Number of ways = 45 × 24 = 1080

70. $\int_0^{\pi} [\cot x] dx,$ where [.] denotes the greatest …

Ans: $-\frac{\pi}{2}$

Sol: $\int_0^{\pi} (\cot x) dx$

Put $x = \frac{\pi}{2} - t \Rightarrow \quad dx = -dt$

$$= -\int_{\pi/2}^{-\pi/2} \cot\left[\pi/2 - t\right] dt$$

$$= -\int_{-\pi/2}^{\pi/2} [\tan x] dx$$

Now, consider the greatest integer function

$$\int_{-n}^{n} [x] dx = -n$$

$$\therefore \int_{-\pi/2}^{\pi/2} [\tan x] dx = \frac{-\pi}{2}$$

71. For real x, let $f(x) = x^3 + 5x + 1$, then….

Ans: f is one-one and onto R

Sol: $f'(x) = 3x^2 + 5 > 0 \ \forall\ x \Rightarrow f(x)$ is increasing

Thus f is one-one

Also $f'(x) \neq 0$ for any real x. Thus f attains neither a maximum nor a minimum at any real points. That is f is an ever increasing function . Hence it is one-one and onto

72. In a binomial distribution $B\left(n, p = \frac{1}{4}\right)$, if ….

Ans: $\frac{1}{\log_{10} 4 - \log_{10} 3}$

Sol: $p = \frac{1}{4} \ \therefore q = \frac{3}{4}$

$P(X = x) = {}^{n}C_x \ p^x q^{n-x}$

Given that $1 - P(x = 0) \geq \frac{9}{10}$

$\therefore P(X = 0) \leq 1 - \frac{9}{10} = \frac{1}{10}$

i.e., ${}^{n}C_0 P^0 . q^{n-0} \leq \frac{1}{10}$

i.e., $\left(\frac{3}{4}\right)^n \leq \frac{1}{10}$

i.e., $n \log_{10} \frac{3}{4} \leq -1$

$$\therefore n \leq \frac{-1}{\log_{10} \frac{3}{4}}$$

$$= \frac{1}{\log_{10} 4 - \log_{10} 3}$$

73. If P and Q are the points of intersection of ….

Ans: all except one value of p

Sol: Equation of circle passing through the intersection of the given circles is

i.e., $x^2 + y^2 + 3x + 7y + 2p - 5$
$+ \lambda(x + 5y + 2p - 5 + p^2) = 0$

Given that it passes through (1, 1)

i.e., $(7 + 2p) + \lambda(1 + p)^2 = 0$

$\Rightarrow \lambda = \frac{-(7 + 2p)}{(1 + p)^2}$

i.e., λ does not exist at p = –1

$\therefore$ all except one value of p

74. The projection of a vector on the three …….

Ans: $\left(\frac{6}{7}, \frac{-3}{7}, \frac{2}{7}\right)$

Sol: The given vector is 6i − 3j + 2k. Its direction cosines are $\left(\frac{6}{\sqrt{6^2+3^2+2^2}}, \frac{-3}{\sqrt{6^2+3^2+2^2}}, \frac{2}{\sqrt{6^2+3^2+2^2}}\right)$

$= \left(\frac{6}{7}, \frac{-3}{7}, \frac{2}{7}\right)$

75. If $\left|Z - \frac{4}{z}\right| = 2$, then the maximum value …..

Ans: $\sqrt{5}$ + 1

Sol: $\left|z + \frac{4}{z}\right| = 2 \Rightarrow |z| - \left|\frac{4}{2}\right| \le 2$

$\Rightarrow |z|^2 - 2|z| - 4 \le 0$

$\Rightarrow |z| \in \left[0, 1 + \sqrt{5}\right]$

$\therefore$ Maximum value of |z| is 1 + $\sqrt{5}$

76. Three distinct points A, B and C are ….

Ans: $\left(\frac{5}{4}, 0\right)$

Sol: Let (x, y) be the any point

Then $\frac{\sqrt{(x-1)^2 + y^2}}{\sqrt{(x+1)^2 + y^2}} = \frac{1}{3}$

$\Rightarrow 9((x-1)^2 + y^2) = (x+1)^2 + y^2$

$\Rightarrow 8x^2 + 8y^2 - 20x + 8 = 0$

$\Rightarrow x^2 + y^2 - \frac{5}{2}x + 1 = 0$

Circumcentre $\left(\frac{5}{4}, 0\right)$

77. The remainder left out when $8^{2n} - (62)^{2n+1}$ …

Ans: 2

Sol: $8^{2n} - (62)^{2n+1} = (9-1)^{2n} - (63-1)^{2n+1}$

$= [9^{2n} - {}^{2n}C_1 \cdot 9^{2n-1} + \ldots + (-1)^{2n}] -$

$[63^{2n+1} - {}^{(2n+1)}C_1\, 63^{2n} + \ldots + (-1)^{2n+1}]$

$= [M(9) + 1] - [M(9) - 1]$

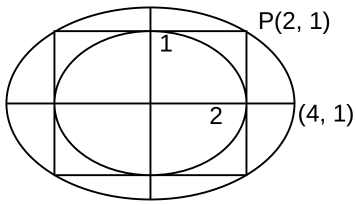
$= M(9) + 2$

$\therefore$ The remainder when $8^{2n} - (62)^{2n+1}$ is divided by 9 is 2

78. The ellipse $x^2 + 4y^2 = 4$ is inscribed …

Ans: $x^2 + 12y^2 = 16$

Sol:



Equation of required ellipse is $\frac{x^2}{4^2} + \frac{y^2}{b^2} = 1$

It passes through (2, 1) $\Rightarrow$ b = $\frac{3}{4}$

$\therefore$ Equation is $x^2 + 12y^2 = 16$

79. The sum to infinity of the series …..

Ans: 3

Sol: Put S = $\frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \ldots$

$\frac{1}{3}S = \frac{2}{3^2} + \frac{6}{3^3} + \frac{10}{3^4} + \ldots$

$\frac{2}{3}S = \frac{2}{3} + \frac{4}{3^2} + \frac{4}{3^3} + \ldots$

$= \frac{2}{3} + \frac{4}{9} \cdot \frac{1}{1 - \frac{1}{3}}$

$= \frac{2}{3} + \frac{2}{3} = \frac{4}{3}$

S = 2

$\therefore$ required sum = 1 + 2 = 3

80. The differential equation which represents the …

Ans: $yy'' = (y')^2$

Sol: $y = c_1 e^{c_2 x} \Rightarrow \log y = \log c_1 + c_2 x$

$\therefore \frac{1}{y} . y' = c_2$

Again differentiating

$\frac{y.y'' - (y').(y')}{(y)^2} = 0$

i.e., $yy'' = (y')^2$

81. One ticket is selected at random from 50 ….

Ans: $\frac{1}{14}$

Sol: Product is zero. Total number of selections are 00, 01, 02, …, 10, 20, 30, 40. There are 14 cases out of which only 08 (sum of digits should be 8) is the favorable case.

$\therefore$ Required probability = $\frac{1}{14}$

82. Let y be an implicit function of x defined …

Ans: – 1

Sol: Put $x^x = t$.

Then $t^2 - 2t\cot y + \cot^2 y - \operatorname{cosec}^2 y = 0$

$\Rightarrow (t - \cot y)^2 = \operatorname{cosec}^2 y$

$\Rightarrow t - \cot y = \pm \operatorname{cosec} y$

$\Rightarrow t = \cot y \pm \operatorname{cosec} y$

$\Rightarrow x^x = \cot y \pm \operatorname{cosec} y$

Differentiating, $x^x(1 + \log x)$

$= (-\operatorname{cosec}^2 y \pm \operatorname{cosec} y \cot y)\, y'$

$= -\operatorname{cosec} y(\operatorname{cosec} y \pm \cot y)\, y'$

$= -\operatorname{cosec} y \; x^x\, y'$

$$\Rightarrow \quad y' = \frac{-(1+\log x)}{\operatorname{cosec} y}$$

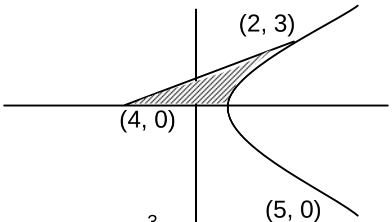
When x = 1, $1 - 2\cot y - 1 = 0 \Rightarrow y = \frac{\pi}{2}$

$$\therefore y'(1) = \frac{-(1+\log 1)}{\operatorname{cosec}\frac{\pi}{2}} = -1$$

83. The area of the region bounded by the ….

Ans: 9

Sol:



Required area = $\int_0^3 (x_1 - x_2)\,dy = 9$

84. Given $P(x) = x^4 + ax^3 + bx^2 + cx + d$ such ….

Ans: P(–1) is not minimum and P(1) is the maximum of P

Sol: $P(x) = x^4 + ax^3 + bx^2 + cx + d$

$P'(0) = 0 \Rightarrow (4x^3 + 3ax^2 + 2bx + c)$ at x = 0 is zero $\Rightarrow$ c = 0.

Now, $P'(x) = 4x^3 + 3ax^2 + 2bx$

$= x(4x^2 + 3ax + 2b)$.

Since x = 0 is the only solution, $4x^2 + 3ax + 2b > 0$ for all $x \in R$.

$\therefore P'(-1) < 0$ and $P'(1) > 0$. $\Rightarrow$ derivative changes sign from –ve to +ve and x = 0 is a point of minimum.

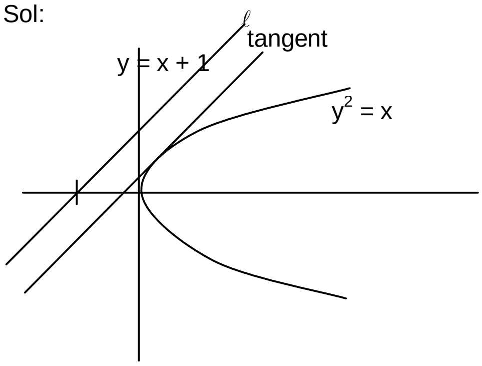
Given $P(-1) < P(1) \Rightarrow P(1)$ is maximum and P(0) is minimum in the interval [– 1, 1].

Thus in [–1, 1], p(–1) is not minimum and p(1) is the maximum

85. The shortest distance between the line …

Ans: $\frac{3\sqrt{2}}{8}$

Sol:



Consider $x = y^2$. Slope of the curve = $\frac{1}{2y}$

Shortest distance = perpendicular distance $\Rightarrow$ slope of tangent to the curve should be same as slope of line.

$\therefore \frac{1}{2y} = 1 \Rightarrow y = \frac{1}{2}$

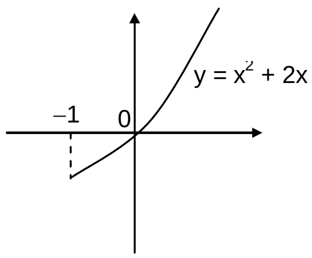
$\Rightarrow x = \frac{1}{4}$

Distance of $\left(\frac{1}{4}, \frac{1}{2}\right)$ from line x + 1 = y is $\frac{3\sqrt{2}}{8}$

86. Let $f(x) = (x + 1)^2 - 1, x \geq -1$

Ans: Statement -1 is true, Statement – 2 is true Statement – 2 is a correct explanation for Statement - 1

Sol:



$f(x) = (x + 1)^2 - 1, x \geq -1$

$f(x) = x^2 + 2x, x \geq -1$

Obviously f is a bijection.(Refer graph)

$\therefore$ Inverse of f exists

$f^{-1}(x) = \sqrt{x+1} - 1$

Let $f(x) = f^{-1}(x)$

$x^2 + 2x = \sqrt{x+1} - 1$

$\therefore x^2 + 2x + 1 = \sqrt{x+1}$

$\Rightarrow (x + 1)^2 = \sqrt{x+1}$

Possible only for x = 0 and x = –1

Statement 1 true and as shown above follows from statement 2

87. Let f(x) = x|x| and g(x) = sinx

Ans: Statement -1 is true, Statement 2 is false

Sol: $f(x) = \begin{cases} -x^2 & x < 0 \\ x^2 & x \geq 0 \end{cases}$ g(x) = sinx

$\therefore$ $(g \circ f)(x) = \begin{cases} -\sin x^2 & x < 0 \\ x^2 & x \geq 0 \end{cases}$

Let $h(x) = (g \circ f)x$

$\therefore$ $h'(x) = \begin{cases} -2x.\cos(x^2) & x < 0 \\ +2x\cos(x^2) & x \geq 0 \end{cases}$

at x = 0, h'(x) = 0 is differentiable

Again

$$h''(x) = \begin{cases} 2x^2\sin x^2 - 2\cos(x^2) & x < 0 \\ -2x^2\sin x^2 + 2\cos(x^2) & x \geq 0 \end{cases}$$

$\therefore$ $h''(0) = \begin{cases} -2 & x < 0 \\ 2 & x > 0 \end{cases}$

i.e., h"(x) is not differentiable

88. Statement – 1: The variance of the first n …

Ans: Statement - 1 is true, Statement - 2 is false

Sol: Statement 2 is true (Direct result)

$x_i = 2, 4, 6, \ldots 2n$

$$\sigma^2 = \frac{1}{n}\sum x_i^2 - \left(\frac{\sum x}{n}\right)^2$$

$$= \frac{2^2}{n}\sum n^2 - \frac{2^2}{n^2}\left(\sum n\right)^2$$

$$= \frac{2^2}{n}\left(\frac{n(n+1)(2n+1)}{6} - \frac{1}{n}\left(\frac{n(n+1)}{2}\right)^2\right)$$

$$= \frac{2^2}{n}.\frac{n(n+1)}{2}\left(\frac{(2n+1)}{3} - \frac{(n+1)}{2}\right)$$

$$= 2(n+1)\left(\frac{4n+2-3n-3}{6}\right)$$

$$= \frac{n^2-1}{3}$$

$\therefore$ Statement (1) is false

89. Statement – 1 : ~(p ↔ ~q) is equivalent to …

Ans: Statement -1 is true and Statement - 2 is false

Sol: Truth table for ~ (p ↔ ~q)

p	q	~q	p ↔ ~q	~(p ↔ ~ q)	p ↔ q
T	T	F	F	T	T
T	F	T	T	F	F
F	T	F	T	F	F
F	F	T	F	T	T

$\therefore$ Statement I is true and II is false

90. Let A be a 2 × 2 matrix

Ans: Statement - 1 is true, Statement - 2 is true; Statement - 2 is not a correct explanation for Statement - 1

Sol: For a 2 × 2 matrix indeed |adj A| = |A|

Since $|\text{adj } A| = |A|^{n-1}$, where n is the order of the matrix

Statement 2 is true

In general, for a n^{th} order matrix, adj (adj A) ≠ A.

But for a 2 × 2 matrix adj (adj A) = A

Statement 1 true

But Statement 1 does not follow from Statement 2