SE CSEM IV / Comp / App. Maths-IV / Dec 09

Con. 5944-09. **(REVISED COURSE)** **SP-7802**

(3 Hours) **[Total Marks : 100**

N.B. (1) Question No. **1** is **compulsory**.
(2) Answer any **four** out of remaining **six** questions.
(3) Assume any **suitable** data whenever required and justify the **same**.

1. **a)** State and prove Cauchy's integral formula. *(5)*

b) Show that following matrices have the same characteristic equation. *(5)*

$$\begin{pmatrix} a & b & c \\ b & c & a \\ c & a & b \end{pmatrix}, \begin{pmatrix} b & c & a \\ c & a & b \\ a & b & c \end{pmatrix}, \begin{pmatrix} c & a & b \\ a & b & c \\ b & c & a \end{pmatrix}$$

c) Find all the basic feasible solution of the equations. *(5)*

$$2x_1 + 6x_2 + 2x_3 + x_4 = 3$$
$$6x_1 + 4x_2 + 4x_3 + 6x_4 = 2$$

d) Find the analytic function if $f(z) = u + iv$ if *(5)*

$$3u + 2v = y^2 - x^2 + 16xy$$

2. **a)** Show that the matrix $A = \begin{pmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{pmatrix}$ is a diagonisable. *(6)*

Find the diagonal matrix and transforming matrix.

b) Use Simplex method to Maximize $z = 3x_1 + 5x_2$ subject to the constraints *(6)*

$$3x_1 + 2x_2 \le 18$$
$$x_1 \le 4$$
$$x_2 \le 6$$
$$x_1, x_2 \ge 0$$

c) Evaluate (i) $\int_0^{2\pi} \frac{d\theta}{5 + 3\sin\theta}$ (ii) $\int_0^{\infty} \frac{dx}{x^2 + 1}$ *(8)*

3. **a)** Show that the $A = \begin{pmatrix} 2 & 3 & 4 \\ 0 & 2 & -1 \\ 0 & 0 & 1 \end{pmatrix}$ is derogatory and *(6)*

find its minimal polynomial.

b) Use Big M method to Maximize $z = 3x_1 - x_2$ subject to the constraints *(6)*

$$2x_1 + x_2 \ge 2$$
$$x_1 + 3x_2 \le 3$$
$$x_2 \le 4$$
$$x_1, x_2 \ge 0$$

c) Find the bilinear Transformation which maps the points $z = 1, i, -1$ onto the points $\omega = i, 0, -i$. Hence find the fixed points of the transformation and the image of $|z| < 1$. *(8)*

4. **a)** If $A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ Find A^{50} *(6)*

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b) Find Laurent's series which represents the function $f(z) = \frac{2}{(z-1)(z-2)}$ when *(6)*

(i) $|z| < 1$ (ii) $1 < |z| < 2$ (iii) $|z| > 2$

c) Using the Kuhn Tucker condition solve the NLpp *(8)*

Maximize $z = 7x_1^2 + 5x_2^2 + 6x_1$ subject to

$$x_1 + 2x_2 \le 10$$
$$x_1 - 3x_2 \le 9$$
$$x_1, x_2 \ge 0$$

5. a) If $f(z) = u + iv$ is analytic in R show that *(6)*

$$\left[\frac{\partial}{\partial x}|f(z)|\right]^2 + \left[\frac{\partial}{\partial y}|f(z)|\right]^2 = |f'(z)|^2$$

b) If $A = \begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$ *(6)*

find the characteristic roots and characteristic vectors of $A^3 + I$

c) Using the method of Lagrange's multipliers solve the NLpp *(8)*

optimize $z = x_1^2 + x_2^2 + x_3^2 - 10x_1 - 6x_2 - 4x_3$

subject to $x_1 + x_2 + x_3 = 7$

$x_i \ge 0$

6. a) Apply the principle of duality to solve Lpp *(6)*

Maximize $z = 3x_1 + 4x_2$

subject to the constraints $x_1 - x_2 \le 1$

$$x_1 + x_2 \ge 4$$
$$x_1 - 3x_2 \le 3$$
$$x_1, x_2 \ge 0$$

b) Using residue theorem evaluate $\int_C \frac{e^z}{(z^2 + \pi^2)^2} dz$ where C is $|z| = 4$. *(6)*

c) Use dual simplex method to solve the Lpp *(8)*

Maximize $z = -2x_1 - 2x_2 - 4x_3$

subject to the constraints $2x_1 + 3x_2 + 5x_3 \ge 2$

$$3x_1 + x_2 + 7x_3 \le 3$$
$$x_1 + 4x_2 + 6x_3 \le 5$$
$$x_1, x_2, x_3 \ge 0$$