

Board question paper (Maths) : March 2013

Note:

- i. All questions are compulsory
- ii. Figures to the right indicate full marks.
- iii. Solution of L.P.P. should be written on graph paper only.
- iv. Answers to both the sections should be written in the same answer book.
- vi. Answer to every new question must be written on a new page.

SECTION – II

Q.4. (A) Select and write the correct answer from the given alternatives in each of the following:

(6) [12]

i. Function $f(x) = x^2 - 3x + 4$ has minimum value at $x =$ _____

(A) 0 (B) $-\frac{3}{2}$

(C) 1 (D) $\frac{3}{2}$

ii. $\int \frac{1}{x} \cdot \log x \, dx =$ _____

(A) $\log(\log x) + c$ (B) $\frac{1}{2}(\log x)^2 + c$

(C) $2 \log x + c$ (D) $\log x + c$

iii. Order and degree of the differential equation

$$\left[1 + \left(\frac{dy}{dx} \right)^3 \right]^{\frac{7}{3}} = 7 \frac{d^2y}{dx^2} \text{ are respectively -}$$

(A) 2, 3 (B) 3, 2

(C) 7, 2 (D) 3, 7

(B) Attempt any THREE of the following:

(6)

i. If $x = at^2$, $y = 2at$, then find $\frac{dy}{dx}$.

ii. Find the approximate value of $\sqrt{8.95}$.

iii. Find the area of the region bounded by the parabola $y^2 = 16x$ and the line $x = 3$.

iv. For the bivariate data $r = 0.3$, $\text{cov}(X, Y) = 18$, $\sigma_x = 3$, find σ_y .

v. A triangle bounded by the lines $y = 0$, $y = x$ and $x = 4$ is revolved about the X-axis. Find the volume of the solid of revolution.

Q.5. (A) Attempt any TWO of the following:

(6) [14]

- i. A function $f(x)$ is defined as
 $f(x) = x + a$, $x < 0$
 $= x$, $0 \leq x < 1$
 $= b - x$, $x \geq 1$
 is continuous in its domain. Find $a + b$.
- ii. If $x = a \left(t - \frac{1}{t} \right)$, $y = a \left(t + \frac{1}{t} \right)$, then show that $\frac{dy}{dx} = \frac{x}{y}$
- iii. Evaluate : $\int \frac{1}{3 + 5 \cos x} dx$

(B) Attempt any TWO of the following:

(8)

- i. An insurance agent insures lives of 5 men, all of the same age and in good health. The probability that a man of this age will survive the next 30 years is known to be $\frac{2}{3}$. Find the probability that in the next 30 years at most 3 men will survive.
- ii. The surface area of a spherical balloon is increasing at the rate of $2 \text{ cm}^2/\text{sec}$. At what rate is the volume of the balloon is increasing when the radius of the balloon is 6 cm?
- iii. The slope of the tangent to the curve at any point is equal to $y + 2x$. Find the equation of the curve passing through the origin.

Q.6. (A) Attempt any TWO of the following:

(6) [14]

- i. If u and v are two functions of x , then prove that

$$\int uv \, dx = u \int v \, dx - \int \left[\frac{du}{dx} \int v \, dx \right] dx$$
- ii. The time (in minutes) for a lab assistant to prepare the equipment for a certain experiment is a random variable X taking values between 25 and 35 minutes with p. d. f.
 $f(x) = \frac{1}{10}$, $25 \leq x \leq 35$
 $= 0$, otherwise.
 What is the probability that preparation time exceeds 33 minutes? Also find the c. d. f. of X .
- iii. The probability that a certain kind of component will survive a check test is 0.6. Find the probability that exactly 2 of the next 4 tested components survive.

(B) Attempt any TWO of the following:

(8)

- i. If $ax^2 + 2hxy + by^2 = 0$, show that $\frac{d^2y}{dx^2} = 0$.
- ii. Find the area of the region common to the circle $x^2 + y^2 = 9$ and the parabola $y^2 = 8x$.
- iii. For 10 pairs of observations on two variables X and Y , the following data are available:
 $\sum(x-2) = 30$, $\sum(y-5) = 40$, $\sum(x-2)^2 = 900$,
 $\sum(y-5)^2 = 800$, $\sum(x-2)(y-5) = 480$.
 Find the correlation coefficient between X and Y .