Con. 5943-09. (REVISED COURSE) SP-7859

S.E.(EXTC) Sem IV (R) 6/01/10

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(3 Hours) [Total Marks : 100

Applied Mathematics IV

N.B. (1) Question No.1 is **compulsory**.
(2) Attempt any **four** questions out of the remaining **six** questions. 2.30 to 5.30 p
(3) **Figures** to the **right** indicate **full** marks.

1. (a) Prove that the eigen values of an orthogonal matrix are +1 or –1. **5**

(b) Show that a harmonic function 'u' satisfies the differential equation $\frac{\partial^2 u}{\partial z\,\partial \bar{z}} = 0$. **5**

(c) $\bar{F} = (x + 2y + az)\,\bar{i} + (bx - 3y - z)\,\bar{j} + (4x + cy + 2z)\,\bar{k}$ if $\bar{F}$ is irrotational then find **5**
(i) the constants a, b, c (ii) a scalar function ϕ such that $\bar{F} = \nabla Q$.

(d) Prove that $J_n(x)$ is an even function if n is an even number and is an odd function if n is an odd number. **5**

2. (a) If $f(z) = z^n$ then show that f(z) is an analytic function and hence find f' (z). **6**

(b) Show that the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ satisfies Cayley-Hamilton Theorem and hence **7**

find A^{-1} if exists.

(c) Expand f(x) = 1 in 0 < x < 1 in a series as $1 = \sum_{i=1}^{\infty} \frac{2}{\lambda_i J_1(\lambda_i)} J_0(\lambda_i x)$ **7**

3. (a) Find the analytic function whose real part is $\frac{\sin(2x)}{\cosh(2y) + \cos(2x)}$ **6**

(b) Find eigen values and eigen vectors of A^3 where $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$. Is A derogatory ? **7**

(c) Evaluate $\int_C \bar{F} \times d\bar{r}$ where $\bar{F} = (2xy + z^2)\,\bar{i} + x^2\bar{j} + 3xz^2\bar{k}$ along the curve $x = t$, $y = t^2$, **7**
$z = t^3$ from (0, 0, 0) to (1, 1, 1).

4. (a) Expand $f(z) = \frac{1}{z(z+1)(z-2)}$ in Laurent's series when — **6**

(i) $|z| < 1$ (ii) $1 < |z| < 2$ (iii) $|z| > 2$

(b) Evaluate $\int_0^{\pi} \frac{d\theta}{3 + 2\cos\theta}$ **7**

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(c) Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ 7

5. (a) Evaluate $\int_C \frac{z^2}{z^4 - 1} dz$ where C is the circle — 6

(i) $|z| = 3/4$ (ii) $|z - 1| = 1$ (iii) $|z + i| = \frac{1}{2}$

(b) Evaluate $\int_0^{Hi} \left(x^2 + iy \right) dz$ along the path (i) $y = x$ (ii) $y = x^2$. Is the line integral independent of the path. 7

(c) Verify Stoke's Theorem for $\bar{F} = y\bar{i} + z\bar{j} + x\bar{k}$ over the surface $x^2 + y^2 = 1 - z, (z > 0)$ 7

6. (a) Prove that $J_{-3/2}(x) = -\sqrt{\frac{2}{\pi x}} \left[\frac{\cos(x)}{x} + \sin(x) \right]$ 6

(b) Find the bilinear transformation which maps the points 2, i, –2 on to the points 1, i, –1. Is this transformation parabolic ? 7

(c) Reduce the following quadratic form $Q = 2x^2 + y^2 - 3z^2 - 8yz - 4xz$ to normal form through congruent transformations. Also find it's rank and signature. 7

7. (a) Find 4^A if $A = \begin{bmatrix} 3/2 & 1/2 \\ 1/2 & 3/2 \end{bmatrix}$ 6

(b) Find the residues at singular points of $f(z) = \frac{z}{(z-1)^2 (z^2 - 1)}$ and hence evaluate $\int_C f(z)dz$ where C is $|z| = 2$. 7

(c) Using Green's Theorem evaluate $\int_C \left(e^{x^2} - xy \right) dx - \left(y^2 - x \right) dy$ where C is the circle $x^2 + y^2 = 1$. 7