S.E Etrx / Tele comm Applied maths IV 18/5/06

6 : 1sthalf-D98

Con.1809-06. **(REVISED COURSE)** **TV-8070**

(3 Hours) **[Total Marks : 100**

N.B. (1) Question No. **1** is **compulsory**.
(2) Answer any **four** questions from Question Nos. **2** to **7**.
(3) Answers to **individual** questions must be **grouped** and written **together**.

1. (a) State Cauchy's Integral formula and hence evaluate — 5

$$\oint_C \frac{(Z-1)(Z-2)}{(Z-3)(Z-4)}\,dZ$$

Where (i) C is $|Z| = 3{\cdot}5$
(ii) C is $|Z| = 4$.

(b) Find Eigen values of adj A and $A^2 - 2A + I$ 5

Where $A = \begin{bmatrix} 2 & 3 & 4 \\ 0 & 4 & 2 \\ 0 & 0 & 3 \end{bmatrix}$.

(c) Find the rate of change of $\phi = xyz$ in the direction of the normal to the surface $x^2 y + y^2 x + yz^2 = 3$ at (1, 1, 1). 5

(d) State Cauchy's Residue Theorem and hence evaluate $\oint_C \frac{Z^2+4}{(Z-2)(Z+3i)}\,dZ$ 5

Where C is —
(i) $|Z + 1| = 2$, (ii) $|Z - 2| = 2$.

2. (a) Find residue at the poles for the function $f(Z) = \frac{Z^2 - 2Z}{(Z+1)^2 (Z^2+4)}$. 5

(b) Find e^A and 4^A if $A = \begin{bmatrix} \frac{3}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{3}{2} \end{bmatrix}$. 5

(c) Prove that — 5

(i) $\nabla(r^n) = n\, r^{n-2}\, \bar{r}$

(ii) $\nabla\left(\frac{\bar{a}\cdot\bar{r}}{r^n}\right) = \frac{\bar{a}}{r^n} - \frac{n(\bar{a}\cdot\bar{r})\bar{r}}{r^{n+2}}$

(d) Evaluate —

$$\int_0^{3+i} Z^2\, dZ$$

(i) along the real axis upto 3 and then vertically to 3 + i. 5
(ii) along the curve $x = 3y^2$. 5

3. (a) Evaluate $\int_0^{2\pi} \frac{d\theta}{a + b\cos\theta}$; $a > b > 0$. 5

(b) Define Minimal Polynomial and derogatory matrix. Verify whether $A = \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$ is derogatory or not. 5

(c) Prove that $\overline{F} = 2xyz^2 i + \left[x^2 z^2 + z \operatorname{Cos}(yz) \right] j + \left[2x^2 yz + y \operatorname{Cos}(yz) \right] K$ is a conservative vector field. 5

Find $\int_{A(0,\,0,\,1)}^{B\left(1,\,\frac{\pi}{4},\,2\right)} \overline{F} \cdot d\overline{r}$.

(d) If $A = \begin{bmatrix} 5 & 3 \\ 1 & 3 \end{bmatrix}$, obtain A^n interms of A. Hence find A^4. 5

4. (a) Find Laurent's Series expansion for $f(Z) = \frac{2-Z^2}{Z(1-Z)(2-Z)}$ valid for the region $1 < |Z| < 2$. 5

(b) If A is a symmetric matrix of order 3 and the Eigen values are $\lambda_1 = 8$, $\lambda_2 = 2$, $\lambda_3 = 2$ and the corresponding eigen vectors are $X_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$ for λ_1, $X_2 = \begin{pmatrix} -\frac{1}{2} \\ 0 \\ 1 \end{pmatrix}$ for λ_2 and X_3 for λ_3 then, 5

Find (i) X_3 corresponding to $\lambda_3 = 2$
(ii) Find AX_1, AX_2
(iii) Find $A^{10}X_3$.

(c) Using Greens Theorem, Evaluate $\oint_C (3x^2 - 8y^2)\,dx + (4y - 6xy)\,dy$ where C is the closed curve bounded by $y = \sqrt{x}$ and $y = x^2$. 5

(d) Find the constants 'a' and 'b' so that the surface $ax^2 - byz = (a+2)x$ will be orthogonal to the surface $4x^2y + z^3 = 4$ at $(1, -1, 2)$. 5

5. (a) Evaluate $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+a^2)(x^2+b^2)}\,dx$; $a > 0$, $b > 0$. 6

(b) Reduce the given Quadractic form to cannonical form by Orthogonal Transformation and hence find rank, index, signature and value class. $Q = 3x^2 + 3y^2 + 3z^2 - 2yz + 2zx + 2xy$. 8

(c) Evaluate $\iint_S (\nabla \times \bar{F}) \cdot \hat{n}\, dS$ where S is the surface bounded by $x = 0$, $y = 0$, $x = a$, $y = b$ and $\bar{F} = (x^2 - y^2)i + 2xy\,j$. 6

6. (a). Define : Singular point, Essential singularity, Removable singularity and Residue of a function. 4

(b) Prove that, if λ is an Eigen value of A then $f(\lambda)$ is an Eigen value of $f(A)$. 4

(c) Using Cayley Hamilton Theorem, prove that — 6

$$A^{-1} = A^2 - 5A + 9I \text{ where } A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}.$$

(d) Evaluate $\iint_S \bar{F} \cdot \hat{n}\, ds$ where $\bar{F} = (x^2 - yz)\,i + (y^2 - zx)\,j + (z^2 - xy)\,K$ taken over the Parallelopiped $0 \le x \le a$, $0 \le y \le b$, $0 \le z \le c$. 6

7. (a) If $f(K) = \int_C \frac{4Z^2 + Z + 4}{Z - K}$ and C is $4x^2 + 9y^2 = 36$, Find (i) $f(1)$, (ii) $f(i)$, (iii) $f'(i)$, (iv) $f''(-i)$. 5

(b) Is the following matrix diagonable ? Explain : $A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$. 5

Find algebraic multiplicity and Geometric Multiplicity of A.

(c) If $A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ show that $A^n = A^{n-2} + A^2 - I$. 5

(d) Evaluate, using Divergence Theorem, $\iint_S \bar{F} \cdot \hat{n}\, dS$, where $\bar{F} = x^3 i + y^3 j + z^3 k$ and S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$. 5
