S.E.(Electricals, Etrx, Bio-Medical, Instrument, Etrx&TC)

exam-mah-4-7 77 Sem-IV / Rev. App. Mathematics - IV 19/6/07.

Con.3342-07. (Library) **(REVISED COURSE)** **ND-1621**

(3 Hours) **[Total Marks : 100**

N.B.(1) Question No. **1** is **compulsory**.
(2) Solve any **four** questions from the remaining **six** questions.
(3) If in doubt, make **suitable** assumption, justify your assumption and proceed.
(4) Figures to the **right** indicate **full** marks.

1. (a) State and prove Cauchys-Integral formula. 5
(b) Integrate xz along the st. line from A (1, 1) to B (2, 4) in the complex plane. 5
(c) The matrix A is defined as $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -2 & 6 \\ 0 & 0 & -3 \end{bmatrix}$ 5

find the eigenvalues of $3A^3 + 5A^2 + 6A + I$.

(d) Find the directional derivative of $\phi = \dfrac{1}{(x^2+y^2+z^2)^{1/2}}$ at P (1, –1, 1) in the direction of 5

$\vec{a} = i + j + k$.

2. (a) Represent the function $f(z) = \dfrac{4z+3}{z(z-3)(z+2)}$ in laurents series : 6
   (i) within $|z| = 1$
   (ii) in the annular region between $|z| = 2$ and $|z| = 3$.
(b) Verify Cayley-Hamilton theorem for $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ and hence find A^{-1}. 6
(c) Show that the vector function $F = (x^2 - yz)\,i + (y^2 - zx)\,j + (z^2 - xy)\,k$ is irrotational. 8
Find scalar potential ϕ such that $F = \nabla\phi$.

3. (a) Find a matrix P which diagonalizes the matrix $A = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$, verify that $P^{-1}AP = D$ 6

where D is the diagonal matrix.

(b) Determine the residue at the poles for the function $f(z) = \dfrac{z^2}{(z^2+3z+2)^2}$. 6

(c) Verify Green's theorem for $\int_C (y - \sin x)\,dx + \cos x\,dy$ where C is the plane triangle 8

enclosed by the lines $y = 0$, $x = \dfrac{\pi}{2}$ and $y = \dfrac{2x}{\pi}$.

4. (a) Define minimal polynomial and Derogatory matrix. Test whether the matrix 6

$$A = \begin{bmatrix} 7 & 4 & -1 \\ 4 & 7 & -1 \\ -4 & -4 & 4 \end{bmatrix}$$ is derogatory ?

(b) Evaluate $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{z^2 + 3z + 2} dz$ 6

where C is (i) $|z| = 1$

(ii) $|z| < 2$

[TURN OVER

(c) Apply Stokes theorem to evaluate 8

$$\int_C [(x+2y)\,dx + (x-z)\,dy + (y-z)\,dz]$$

Where C is the boundry of the triangle with vertices (2, 0, 0), (0, 3, 0) and (0, 0, 6) oriented in the counter clockwise direction.

5. (a) Prove that (i) $\nabla\left\{\nabla \cdot \frac{\bar{r}}{r}\right\} = -\frac{2}{r^3}\bar{r}$ 6

(ii) $\nabla \times \left(\vec{a} \times \vec{r}\right) = 2\vec{a}$ where a is constant vector.

(b) If $A = \begin{bmatrix} \frac{3}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{3}{2} \end{bmatrix}$ then find e^A and A^{10} 6

(c) (i) Show by the method of residues that $\int_0^{\pi} \frac{a}{a^2 + \sin^2\theta}\, d\theta = \frac{\pi}{\sqrt{1+a^2}}$ 8

(ii) Evaluate $\int_0^{\infty} \frac{dx}{x^4 + 16}$

6. (a) Test whether the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ 2 & 2 & -1 \\ 1 & 2 & -1 \end{bmatrix}$ is diagonalizable. 6

(b) Evaluate $\int_c F\cdot dr$ where $F = (x^2+y^2)\,i - 2xyj$ and C is the rectangle in the xy-plane bounded by y = 0, x = a, y = b, x = 0 6

(c) Apply Divergence theorem, to evaluate $\iint_S F.ds$ where $F = 4xi - 2y^2j + z^2k$ and S is the region bounded by $y^2 = 4x$, x = 1, z = 0, z = 3. 8

7. (a) (i) If λ_1, λ_2 and λ_3 are the eigenvalues of the matrix $\begin{bmatrix} -2 & -9 & 5 \\ -5 & -10 & 7 \\ -9 & -21 & 14 \end{bmatrix}$ 6

then find the value of (i) $\lambda_1 + \lambda 2 + \lambda_3$

(ii) $\lambda_1, \lambda_2\, \lambda_3$

(ii) Prove that the characteristic roots of a Hermitian matrix are real.

(b) If $f(a) = \int_c \frac{4z^2 + z + 4}{z - a}\, dz$ and C is $4x^2 + 9y^2 = 36$ 6

then find (1) f(1) (2) f(i) (3) f'(i) (4) f''(i)

(c) Reduce the quadratic form $q = 2x^2 + y^2 - 3z^2 - 8yz - 4xz + 12xy$ to real canonical form and find its rank, index and signature. 8

xxxxxxxxxxxxxxx