N.B. : (1) Question No. **1** is **compulsory**.
(2) Attempt any **four** questions from question Nos. **2** to **7**.
(3) If in doubt make **suitable** assumption. **Justify** your assumption and proceed.
(4) **Figures** to the **right** indicate **full** marks.

1. (a) If the angle between the surfaces $x^2 + axz + byz = 2$ and $x^2z + xy + y + 1 = z$ at $(0, 1, 2)$ is $\cos^{-1}\frac{1}{\sqrt{3}}$ then find the constants a and b. 5

(b) Find the eigen values and eigen vectors of the orthogonal matrix :– 5

$$B = \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$$

(c) Evaluate $\int_c f(z)\,dz$ along the parabola $y = 2x^2$ from $z = 0$ to $z = 3 + 18i$ where $f(z) = x^2 - 2ixy$. 5

(d) Find unit normal vector to the unit sphere at point – 5

$$\left(\frac{a}{\sqrt{3}}, \frac{a}{\sqrt{3}}, \frac{a}{\sqrt{3}}\right)$$

2. (a) Find the directional derivative of $xy^2 + yz^3$ at the point $(2, -1, 1)$ along the tangent to the curve $x = a\sin t$, $y = a\cos t$, $z = at$ at $t = \frac{\pi}{4}$. 6

(b) Verify Cauchy's integral theorem for $f(z) = e^z$ along a circle $c : |z| = 1$. 6

(c) Reduce the Quadratic form – 8
$8x^2 + 7y^2 + 3z^2 + 12xy + 4xz - 8yz$ to sum of squares and find the corresponding Linear transformation also find the rank, index and signature.

3. (a) Using Caley–Hamilton theorem for – 6

$$A = \begin{bmatrix} -3 & -4 & -4 \\ 2 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

Find $A^{64} + 2A^{37} - 58I$.

(b) Prove that $\nabla^2 f(r) = f''(r) + \frac{2}{r}f'(r)$ and hence show that $\nabla^4 e^r = \left(1 + \frac{4}{r}\right)e^r$. 6

(c) Find all possible Laurent's expansion of the function :– 8

$$f(z) = \frac{7z-2}{z(z-2)(z+1)} \text{ about } z = -1.$$

4. (a) If $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 0 \\ 1/2 & 2 \end{bmatrix}$ then prove that both A and B are not diagonalizable but AB is not diagonalizable. 6

(b) Verify Green's theorem in plane for :– 6

$\oint_c (x^2 - 2xy)\,dx + (x^2y + 3)\,dy$ where c is the boundary of the region defined by $y^2 = 8x$ and $x = 2$.

[TURN OVER

N.B. : (1) Question No. **1** is **compulsory**.
(2) Attempt any **four** questions from question Nos. **2** to **7**.
(3) If in doubt make **suitable** assumption. **Justify** your assumption and proceed.
(4) **Figures** to the **right** indicate **full** marks.

1. (a) If the angle between the surfaces $x^2 + axz + byz = 2$ and $x^2z + xy + y + 1 = z$ at $(0, 1, 2)$ is $\cos^{-1}\frac{1}{\sqrt{3}}$ then find the constants a and b. **5**

(b) Find the eigen values and eigen vectors of the orthogonal matrix :– **5**

$$B = \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$$

(c) Evaluate $\int_c f(z)\,dz$ along the parabola $y = 2x^2$ from $z = 0$ to $z = 3 + 18i$ where $f(z) = x^2 - 2ixy$. **5**

(d) Find unit normal vector to the unit sphere at point – **5**

$$\left(\frac{a}{\sqrt{3}}, \frac{a}{\sqrt{3}}, \frac{a}{\sqrt{3}}\right)$$

2. (a) Find the directional derivative of $xy^2 + yz^3$ at the point $(2, -1, 1)$ along the tangent to the curve $x = a \sin t$, $y = a \cos t$, $z = at$ at $t = \frac{\pi}{4}$. **6**

(b) Verify Cauchy's integral theorem for $f(z) = e^z$ along a circle $c : |z| = 1$. **6**

(c) Reduce the Quadratic form – **8**
$8x^2 + 7y^2 + 3z^2 + 12xy + 4xz - 8yz$ to sum of squares and find the corresponding Linear transformation also find the rank, index and signature.

3. (a) Using Caley–Hamilton theorem for – **6**

$$A = \begin{bmatrix} -3 & -4 & -4 \\ 2 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

Find $A^{64} + 2A^{37} - 58I$.

(b) Prove that $\nabla^2 f(r) = f''(r) + \frac{2}{r}f'(r)$ and hence show that $\nabla^4 e^r = \left(1 + \frac{4}{r}\right)e^r$. **6**

(c) Find all possible Laurent's expansion of the function :– **8**

$$f(z) = \frac{7z-2}{z(z-2)(z+1)} \quad \text{about } z = -1.$$

4. (a) If $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 0 \\ 1/2 & 2 \end{bmatrix}$ then prove that both A and B are not diagonalizable but AB is not diagonalizable. **6**

(b) Verify Green's theorem in plane for :– **6**

$\oint_c (x^2 - 2xy)\,dx + (x^2y + 3)\,dy$ where c is the boundary of the region defined by $y^2 = 8x$ and $x = 2$.

(c) (i) Evaluate $\int_{-\infty}^{\infty} \frac{x^2}{(x^2 + a^2)(x^2 + b^2)} dx$, $a > 0$, $b > 0$

(ii) Evaluate $\int_{0}^{2\pi} \frac{d\theta}{\sqrt{2} - \cos\theta}$. 4

5. (a) Evaluate $\int_{c} \frac{\sin^6 z}{(z - \pi/6)^3} dz$ where c is $|z| = 1$. 6

(b) Define Minimal polynomial and derogatory matrix and Test whether the matrix 6

$$A = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix} \text{ is derogatory.}$$

(c) Verify Gauss-Divergence theorem for :– 8
$F = 4xi + 2y^2j + z^2k$ taken over the region of the cylinder bounded by $x^2 + y^2 = 4$, $z = 0$ and $z = 3$.

6. (a) If $A = \begin{bmatrix} -1 & 4 \\ 2 & 1 \end{bmatrix}$ then prove that $3 \tan A = A \tan 3$. 6

(b) Evaluate $\int_{c} (z - z^2)\, dz$ where c is the upper half of circle $|z - 2| = 3$. 6

(c) (i) Show that $\vec{F} = (ye^{xy} \cos z)i + (xe^{xy} \cos z)j + (-e^{xy} \sin z)k$ is irrotational and find the scalar potential ϕ such that $\vec{F} = \nabla\phi$. 6

(ii) Find div F where $\vec{F} = \frac{xi - yj}{x^2 + y^2}$. 2

7. (a) State and prove Cauchys-Residue theorem and hence – 6

Evaluate $\int_{c} \frac{1+z}{z(2-z)} dz$ where c is $|z| = 1$.

(b) Evaluate $\iint_{s} F \cdot nds$ where $F = (x + y^2)\, i - 2xj + 2yzk$ and s is the surface of the plane $2x + y + z = 6$ in the first octant. 6

(c) Show that the matrix 8

$$A = \begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$$

is diagonalizable, also find the diagonal form and diagonalizing matrix P.