University of Hyderabad,

Entrance Examination, 2007 8

M.Sc. (Statistics - OR)

Hall Ticket No.	

Time: 2 hours

Max. Marks: 75

Part A: 25

Part B: 50

Instructions

1. The OMR sheet contains space for answers to 100 questions. Answer Part A in 1 to 25 and Part B in 26 to 50. Ignore the remaining spaces.
2. Fill in your hall ticket number in the space provided in both the OMR sheet and on this page.
3. Calculators are not allowed.
4. Each correct answer in Part A carries **1 mark** and each wrong answer carries 0.33 $\left(\frac{1}{4}\right)$ mark. So do not gamble.
5. Each correct answer in Part B carries **2 marks** and each wrong answer carries 0.66 $\left(\frac{1}{2}\right)$ **mark.**
6. There will be no penalty if a question is unanswered.
7. Answers are to be given on the OMR sheet provided.
8. The appropriate answer should be coloured in by either a blue or a black ball point/sketch pen ~~or a black sketch pen~~. DO NOT USE A PENCIL.

J-2

SECTION-A

1. Seventeen teams take part in the Foot Ball Championship of a country. In how many ways can the gold, silver and bronze medals be distributed among the teams?

   (A) 17^3 (B) $\frac{17!}{3!\ 4!}$ (C) $\frac{17!}{14!}$ (D) 14!

2. How many subsets does a set with n elements have? (The empty set is considered as a subset of every set)

   (A) 2^n (B) n! (C) 2n (D) n(n-1)

3. Let N(A) be the number of elements in a set A. Then $N(A \cup B)$ is

   (A) N(A) + N(B) (B) $N(A) + N(B) - N(A \cap B)$
   (C) $N(A) - N(A \cap B)$ (D) $N(A) + N(B) + N(A \cap B)$

4. The probabilities of events A,B and $A \cap B$ are known. What is the probability of the event $\bar{A} \cup \bar{B}$?

   (A) $P(A) - P(A \cap B)$ (B) $P(B) - P(A \cap B)$
   (C) $P(\bar{A}) + P(\bar{B})$ (D) $1 - P(A \cap B)$

5. If X is a random variable with normal distribution with mean 1 and variance 2 denoted by N(1,2), what is the distribution of 2X?

   (A) N(2,2) (B) N(2,8) (C) N(2,4) (D) N(2,2)

6. Let $X_1,\ldots,X_n$ be a random sample from a normal distribution with mean θ and variance 25. Which one of the following hypotheses is simple.

   (A) $H_0:\theta \le 17$ (B) $H_0:\theta \ge 17$ (C) $H_0:\theta = 17$ (D) none of these

7. Suppose we subdivide the population into at least two subgroups (such as by marital status) and then draw a random sample from each of the groups. This type of sampling scheme is called

   (A) aggregate sampling (B) cluster sampling
   (C) stratified sampling (D) none of these

8. 10 books numbered 1,2,...,10 are to be arranged in a row; the probability that book number 7 has books number 3 and 4 on either side is

   (A) 8!/10! (B) 2(8!/10!) (C) 9!/10! (D) 7!/10!

J-2

9. The random variable X has the following probability distribution $P(X = 1) = P(X = -1) = \frac{3}{8}$, $P(X = 2) = P(X = -2) = \frac{1}{8}$

(A) The median is positive
(B) The mean is positive
(C) The median is zero and the mean is negative
(D) The mean and median are both zero

10. Let $P(X = n) = \left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^{n-1}$, n = 1,2,..., then $P(X \geq 10)$ is

(A) $\left(\frac{2}{3}\right)$ (B) $\left(\frac{2}{3}\right)^{9}$ (C) $\left(\frac{2}{3}\right)^{10}$ (D) $\left(\frac{2}{3}\right)^{11}$

11. A and B are independent events, then

(A) $P(A|B) = P(A|B^c)$
(B) $P(A|B) = P(A^c|B)$
(C) $P(A^c|B^c) = P(A|B)$
(D) $P(A) = P(B)$

12. The number of arrangements of letters in the word EXCESS is

(A) 720 (B) 360 (C) 180 (D) 90

13. $\lim_{n \to \infty} \frac{n!}{n^n}$ is

(A) 1 (B) 0 (C) ∞ (D) e

14. The covariance between two random variables X and Y is c_{xy}, then covariance between aX and bY is

(A) ab c_{xy} (B) $|ab|c_{xy}$ (C) $\frac{|a|}{|b|} c_{xy}$ (D) $\frac{|b|}{|a|} c_{xy}$

15. T_1 and T_2 are two unbiased estimators for a function $g(\theta)$ of the parameter θ, based on a sample of size n, we will prefer T_1 if

(A) $V(T_1) > V(T_2)$
(B) $ET_1^2 < ET_2^2$
(C) $cov(T_1,T_2) > 0$
(D) $E|T_1| < E|T_2|$

16. Let events A and B be mutually exclusive subsets of S. Which of the following statements is true concerning A and B?

(A) $(\bar{A} \cap \bar{B}) = \bar{\bar{B}}$
(B) $(\bar{A} \cap \bar{B}) = (A \cup B)$
(C) $(\bar{A} \cap B) = \bar{A}$
(D) $(\bar{A} \cup \bar{B}) = S$

17. Which of the following statements is always true for the normal distribution?

(A) $P(X \geq 6) = 1-P(X < 5)$
(B) $P(X \geq 6) = P(X > 6)$
(C) $P(X \geq 2) = 1-P(X \leq 1)$
(D) $P(X > 2) = P(X \geq 3)$

J-2

18. Let $C = \{(x,y);\ x,y \in \mathbb{R} \ni x^2-2x+y^2-4y > 4\}$, C is

(A) the boundary of a circle (B) the interior of a circle

(C) is a closed disc (D) the exterior of a circle

19. The function $f(x) = 5^x-4^x-3^x+2^x$, then for $f(x) = 0$

(A) $x = 0$ and $x = 1$ are solutions (B) $x = 0$ is the only solution

(C) $x = 1$ and $x = 2$ are solutions (D) has no solution

20. The random variable X has pdf $f_X(x) = |x|$, $-1<x<1$. Then

(A) $E(X) > 0$ (B) $E(X) < 0$ (C) $EX = 0$ (D) $P(X>0) > \frac{1}{2}$

21. The value of $\int_{-1}^{2} \frac{x}{|x| + 1}\, dx$

(A) does not exist (B) is 0 (C) is < 0 (D) is > 0

Answer Questions 22 and 23 based on the joint probability distribution of X and Y given below.

X \ Y	-1	0	1
0	0	1/4	1/4
1	1/6	1/6	1/6

22. $P(X = 0)$ is

(A) 1/6 (B) 1/3 (C) 5/12 (D) 1/2

23. $P(X = 1|Y = 0)$ is

(A) 1/5 (B) 2/5 (C) 3/5 (D) 4/5

24. $X \sim N(0,1)$ $0<a_1<a_2$ suppose $P(X \le a_1) = \alpha_1$; $P(X \le a_2) = \alpha_2$ then $P(-a_2 < X < -a_1)$ is

(A) $\alpha_2-\alpha_1$ (B) $\alpha_1-\alpha_2$ (C) $\alpha_1+\alpha_2$ (D) $\frac{\alpha_1+\alpha_2}{2}$

25. The moment generating functions of two independent random variables X_1 and X_2 are $M_1(t)$ and $M_2(t)$ respectively. The MGF of X_1+2X_2 is

(A) $M_1(t) + 2M_2(t)$ (B) $M_1(t) + M_2(2t)$

(C) $2M_1(t)M_2(t)$ (D) $M_1(t)M_2(2t)$

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J-2

SECTION-B

26. X is a random variable with Poisson distribution, $P(X=1) = 2P(X=0)$, then $P(X=2)$ is

(A) equal to $P(X=1)$ (B) twice $P(X=1)$

(C) equal to $P(X=0)$ (D) 4 times $P(X=0)$

27. From n distinct objections, the number of subsets of size 3 is twice the number of subsets of size 2. Therefore

(A) n = 10 (B) n = 12 (C) n = 8

(D) information given is not sufficient to determine n.

28. A group of 7 friends, 3 girls and 4 boys go to watch a film, they have tickets to seat numbers 7 to 13 (all in a row), they decide that only boys will sit on seats 7 and 13. In how many ways can these 7 friends be seated with the given condition?

(A) 720 (B) 5047 (C) 24 (D) 1440

29. $P(A^c \cup B^c) = 0.7$, $P(A) = 0.4$, $P(B) = 0.5$, then $P(A \cup B)$

(A) is 0.9 (B) cannot be obtained from the information given

(C) is 0.6 (D) is 1

30. $a_1-a_2 = -4$, $a_1-a_3 = 5$, $a_2-a_3 = 9$, with this information on a_1, a_2 and a_3

(A) both mean and variance can be obtained

(B) variance can be obtained but not the mean

(C) mean can be obtained but not variance

(D) neither mean nor variance can be obtained

31. There are 10 slips numbered 1,...,10 in a bag, two slips are drawn, the set of all possible outcomes S is

(A) $\{1,...,10\}$ (B) $\{(i,j);\ i,j \in \{1,...,10\}\}$

(C) $\{(i,j);\ i \neq j \in \{1,...,10\}\}$ (D) $\{\{i,j\};\ i \neq j \in \{1,...,10\}\}$

32. $f_X(x) = ce^{-|x|}$ $-\infty<x<\infty$, what should c be so that f_X is a probability density function?

(A) 1 (B) 1/2 (C) 1/4 (D) 2

J-2

33. Two distinct numbers are selected from {1,...,10}, the probability that the larger of the two is more than 50 is

(A) at most 1/4

(B) more than 1/4 but not more than 2/3

(C) more than 1/2 but not more than 2/3

(D) more than 2/3

34. The first and second raw moments of a random variable X are 12 and 100 respectively. Which of the following is correct.

(A) This can never happen.

(B) This can happen if X has binomial distribution

(C) This can happen if $X \sim N(\mu, \sigma^2)$ for some choices of (μ, σ^2)

(D) This is true for $X \sim P(12)$

35. X and Y are two random variables taking values 1,2,3,4 with the following distributions $P(X = i) = \frac{1}{4}$ i = 1,2,3,4; P(Y = 1) = 5/16, P(Y = 2) = 1/32, P(Y = 3) = 11/32, P(Y = 4) = 5/16 then

(A) EX = EY (B) EX > EY (C) P(X>1) < P(Y>1) (D) EX < EY

36. The correlation coefficient between X and Y denoted by ρ_{XY} is 0, which of the following statements is always true

(A) $\rho_{-X,-Y} > 0$ (B) $\rho_{X^2,Y^2} = 0$ (C) $\rho_{-X,-Y} = 0$ (D) $\rho_{-X,-Y} < 0$

37. The probability that among k randomly selected digits, the digits 0 and 1 are not there.

(A) $9^k/10^k$ (B) $8^k/10^k$ (C) $2^k/10^k$ (D) $1/10^k$

38. In every scanning cycle, a radar tracking a space object detects the object with constant probability p. What is the probability of detecting the object in n cycles?

(A) p^n (B) $1-p^n$ (C) $(1-p)^n$ (D) $1 - (1-p)^n$

39. An unbiased coin is tossed until a head is obtained. If N denotes the number of tosses required, what is P(N > 1)?

(A) 1/2 (B) 1 (C) 1/4 (D) 1/8

40. Let X be a random variable such that $P(X = i) = \frac{1}{2n+1}$ for i = -n, -n+1, ...,-1,0,1,...n. What is V(X)?

(A) $\frac{n(n+1)}{3}$ (B) $\frac{n^2(n+1)}{3}$ (C) $\frac{2^n(n+1)}{3}$ (D) $\frac{n^2(n+1)^2}{3}$

41. Let X_1 and X_2 be independent identically distributed random variables with variance one, what is the covariance between X_1+X_2 and X_1-X_2?

(A) 2 (B) -2 (C) 1 (D) 0

42. For a random variable X with $EX = 3$ and $EX^2 = 13$. $P(-2<X<8)$ is

(A) $\geq \frac{13}{25}$ (B) $\geq \frac{21}{25}$ (C) $\frac{7}{15}$ (D) $\frac{1}{2}$

43. $X_1,\ldots,X_n$ is a random sample from a distribution with probability density function,

$$f(x,\theta) = \theta x^{\theta-1}, \quad 0<x<1 \quad \text{for } \theta > 0$$
$$= 0, \quad \text{o.w}$$

(A) $\sum_{i=1}^{n} X_i$ is a sufficient statistic for θ

(B) $\prod_{i=1}^{n} X_i$ is a sufficient statistic for θ

(C) $\frac{1}{n}\sum_{i=1}^{n} X_i$ is sufficient statistic for θ

(D) none of the above is correct

44. Let $P(A) = 0.2$ and $P(B|A^c) = 0.7$, then

(A) $P(A \cup B) = 0.76$

(B) $P(A \cup B) = 0.24$

(C) $P(A \cup B) = 0.56$

(D) $P(A \cup B)$ cannot be determined with the information given

45. X_1 and X_2 independent Poisson random variables with parameter λ, an unbiased estimator for λ^2 is

(A) $\frac{X_1^2+X_2^2}{2}$ (B) $\frac{X_1+X_2}{2}$ (C) $\max(X_1,X_2)$ (D) $\frac{X_1^2+X_2^2-(X_1+X_2)}{2}$

46. $\{x: |x-2| > 2\} \cap \{x: |x-3| < 2\}$ is

(A) an open interval

(B) a closed set

(C) a non-empty finite set

(D) an open set which is not an interval

47. Consider the matrix $C = \begin{pmatrix} 3 & 5 & 2 \\ 1 & 0 & 2 \\ 5 & 10 & a \end{pmatrix}$. What should a be so that the rows of matrix are not linearly independent?

(A) 0 (B) 1 (C) 2 (D) 3

48. The value of $\binom{2n}{2} + \binom{2n}{n} + \ldots + \binom{2n}{2n}$ is

(A) 2^n (B) 2^{2n} (C) 2^{2n-1} (D) $2^{2n-1} - 1$

49. X and Y are random variables satisfying $\log Y = X \sim N(0,1)$, EY is

(A) e^2 (B) e (C) $e^{1/2}$ (D) none of these is correct

50. The set of vectors (1,0,0); (2,3,0); (4,5,6), (7,8,9) in $\mathbb{R}^3$ are

(A) A basis for $\mathbb{R}^3$.

(B) Linearly dependent but not spanning $\mathbb{R}^3$.

(C) Linearly dependent and spanning $\mathbb{R}^3$.

(D) Linearly independent.

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J-2