9208/A11 **OCTOBER 2009**

MATHEMATICS

Time : Three hours Maximum : 100 marks

PART A — ($6 \times 5 = 30$ marks)

Answer any SIX questions.

1. Find $\frac{dy}{dx}$ when $y = x^{x^{\cdots\cdots\infty}}$.

2. If $x = \sin t$ $y = \cos pt$ prove that $(1 - x^2)y'' - xy' + p^2 y = 0$.

3. Integrate $I = \int \frac{dx}{(2 \sin x + 3 \cos x)^2}$.

4. Evaluate $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}}\, dx$.

5. Find by vector method the angle between the diagonals of a cube.

6. Find the rank of the matrix by reducing it into normal form $\begin{pmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{pmatrix}$.

7. Prove that the lines joining the origin to the points of intersection of the line $x - y = 2$ and the curve $5x^2 + 12xy - 8y^2 + 8x - 4y + 12 = 0$ make equal angles with the axes.

8. Find the equations of the tangents to the circle $x^2 + y^2 = 25$ which pass through $(7, 1)$.

9. Find the mean, median and mode for the following data 18, 15, 18, 16, 17, 18, 15, 19, 17, 17.

10. Between the hours 2 pm and 4 pm the average number of phone calls per minute coming into the switch board of a company is 2.35. Find the probability that during one particular minute there will be atmost 2 phone calls.

PART B — ($4 \times 10 = 40$ marks)

Answer any FOUR questions.

11. Find the angle of intersection between the curves $x^2 - y^2 = a^2$ and $x^2 + y^2 = a^2\sqrt{2}$.

12. Evaluate $\int (3x + 2)\sqrt{x^2 + x + 1}\, dx$.

13. Show that the four points whose position vectors are $3\vec{i} + 2\vec{j} + 4\vec{k}$; $6\vec{i} + 3\vec{j} + 2\vec{k}$; $5\vec{i} + 7\vec{j} + 3\vec{k}$; $2\vec{i} + 2\vec{j} + 5\vec{k}$ are coplanar.

14. Solve the system of equations :

$x + 2y + 3z = 1$
$2x + 3y + 2z = 2$ by matrix method.
$3x + 3y + 4z = 1$.

15. Solve $y^2p - xyq = x(z - 2y)$.

16. Marks of 10 students in Maths and Statistics are given below :

Maths :	32	38	48	43	40	22	41	69	35	64
Statistics :	30	31	38	43	33	11	27	76	40	59

Obtain the correlation coefficient.

PART C — ($2 \times 15 = 30$ marks)

Answer any TWO questions.

17. (a) If $y = \sin^{-1} x$ prove that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - n^2 y_n = 0$

(b) Find $\int \frac{dx}{5 + 4\cos x}$.

18. (a) Find the value of h and f so that the equation $9x^2 + 2hxy + 4y^2 + 6x + 2fy - 3 = 0$ represents a pair of parallel lines and find the distance between them.

(b) Solve $(mz - ny)p + (nx - lz)q = ly - mx$.

19. (a) Calculate the standard deviation of the following data :

X :	2.0	2.5	3.0	3.5	4.0	4.5	5.0
f :	5	38	65	92	70	40	10

(b) Apply the principle of least squares to fit a straight line $y = a + bx$ to the following data :

x :	2	4	6	8	10	12	14
y :	10	14	15	16	15	17	18