## Syllogism

## Introduction

Syllogism is originally a word given by the Greeks. Which means 'inference' or 'deduction'.

## Definitions of Some Important Terms

The terms defined below are used in the well defined method for solving the problems on syllogism.

## Proposition

A proposition is a sentence that makes a statement and gives a relation between two terms. It consists of three parts (a) the subject; (b) the predicate; (c) the relation between the subject and the predicate.

Some examples of propositions are being given below:
(i) All coasts are beaches.
(ii) No students are honest.
(iii) Some documents are secret
(iv) Some cloths are not cotton.

## Subject and Predicate

A subject is that part of the proposition about which something is being said. A predicate, on the other hand, is that term of the proposition which is stated about or related to the subject.

Thus, for example, in the four propositions mentioned above, 'coasts', 'students', 'documents' and 'cloths' are subjects while 'beaches', 'honest', 'secret' and 'cotton' are predicates.

## Categorical Propositions

A categorical proposition makes a direct assertion. It has no conditions attached with it. For example, "All S are P", "No S are P", "Some S are P" etc are categorical propositions, but "If S , then P " is not a categorical proposition.

## Types of Categorical Propositions

(a) Universal Proposition: Universal propositions either fully include the subject or fully exclude it. Examples are,

All coasts are beaches.
No Students are honest.
Universal propositions are further classified as
(i) Universal Positive Proposition: A proposition of the form "All S are P", for example, "All coasts are beaches", is called a universal positive proposition. And it is usually denoted by a letter "A".
(ii) Universal Negative Proposition: A proposition of the form "No S are P", for example, "No students are honest", is called a universal negative proposition. And it is usually denoted by a letter "E".
(b) Particular Proposition: Particular proposition either only partly include or only partly exclude the subject while making a statement. Examples are,

Some documents are secret.
Some cloths are not cotton.
Particular propositions are also further classified as
(i) Particular Positive Proposition: A proposition of the form "Some S are P", for example, "Some documents are secret", is called a particular positive proposition and it is denoted by the letter "I".
(ii) Particular Negative Proposition: A proposition of the form "Some S are not P" for example, "Some cloths are not cotton", is called a particular negative proposition. And is usually denoted by the letter " 0 ".

## Important Note

The definition of the $\mathbf{A}, \mathbf{E}, \mathbf{I}, \mathbf{O}$ propositions are very, very important and the student must have the ability to immediately recognise these types. With this need in mind we are listing these four types of propositions in the following table:

The Four Types of Propositions

| Type of <br> Proposition | Universal | Particular |
| :--- | :--- | :--- |
| Positive | All S are P P <br> All <br> Example: <br> All coasts are <br> beaches. | I Format: <br> Some S are P. <br> Example: <br> Nome documents <br> are secret. |
|  | E Format: <br> No S are P <br> Example: <br> No student are <br> honest. | O Format: <br> Some S are not P. <br> Example: <br> Some cloths are |

## Mediate and Immediate Inference

Mediate Inference: Syllogism is actually a problem of mediate inference. In mediate inference conclusion is drawn from two given statements. For example, if two statements are given: "All tips are balls" and "All balls are pencils", then a conclusion could be drawn that "All tips are pencils". This is a case of syllogism or mediate inference because conclusion is drawn from two propositions. We will learn how to draw conclusion (mediate inference) from the two given propositions later on in this chapter.

Immediate Inference: In immediate inference conclusion is drawn from only one given proposition. For example, let a given statement be "All coasts are beaches". Then, based on this statement, a conclusion could be drawn that "Some beaches are coasts". This is a case of immediate inference.

## Important Cases of Immediate Inference

In order to be able to solve syllogism problems completely and speedily we need to have a thorough idea of immediate inference. There are many aspects or methods of immediate inference. These include conversion, obversion, contraposition etc. We shall not study the less important of these methods. We shall see only two important cases of immediate inference.

## I. Implications (of a given proposition):

Let us see examples given below:
(a) Suppose we are given a proposition "All coasts are beaches". then this proposition naturally implies that the conclusion "Some coasts are beaches" must be true. It is very easy to comprehend because if "all" are beaches, then "some" ("some" is only a part of "all") must be beaches.
(b) Take the statement "No students are honest". If this statement is true the conclusion "Some students are not honest", too, must be true.
Thus, the above examples state that
(i) if a given proposition is of $\boldsymbol{A}$-type, then it also implies that the I-type conclusion must be true.
(ii) an $\mathbf{E}$-type proposition also implies an $\mathbf{O}$-type conclusion. Always remember the above two implications as thumb rules.

## II. Conversion

The second impportant method of immediate inference is conversion. Let us see as to how to convert a given proposition. The following rules are to be employed in order to convert a given proposition:
Step I: The subject becomes the predicate and the predicate becomes the subject.
Step II: The type of the given proposition is changed according to the pattern given in the table.

| Table: Rules of Conversion |  |
| :--- | :--- |
| A Statement of <br> the type | When converted becomes <br> a statement of the type |
| A format: <br> All S are P. <br> Example: <br> All coasts are beaehes. | I Format: <br> Some P are S. <br> Example: <br> Some beaches are coasts. |
| E Format: <br> No S are P. <br> Example: <br> No students are honest. | E Format: <br> No P are S. <br> Example: <br> No honest are students. |
| I Format: <br> Some S are P. <br> Example: <br> Some documents are secret. | I Format: <br> Some P are S. <br> Example: <br> Some secret are documents. |
| O Format: <br> Some S are not P. <br> Example: <br> Some cloths are not cotton. | O-Type statements <br> cannot be converted. |

## Analytical Method to Solve Syllogism

There can be two methods of solving syllogism. the analytical method and the method of Venn-diagrams.

To solve syllogism, I take analytical method instead of Venn-diagram method delebrately because I consider, analytical method is easier to comprehend and the candidate can solve questions on syllogism more quickly
by analytical method than venn-diagram method. Though, I will be discussing about Venn-Diagram also in the latter section of this book.

The analytical method for solving syllogism completely consists of the following easy steps:
I. Draw mediate inferences
II. Draw immediate inferences (implication or conversion)
III. Check for complementary pairs

## I. Draw Mediate Inferences

There are two extremely simple steps to draw mediate inferences:

Step I: Properly align the given sentences.
Step II: Use the given table to draw the conclusion.

## Step I: Aligning the given sentences

The first step is to properly align the two sentences. Before going into the details of 'aligning' we must remember that "The two given propositions must always have one common term, otherwise no conclusion could be drawn"

Now, by a properly aligned pair of propositions we mean that the two propositions should be written in such a way that the common term is the predicate of the first proposition and the subject of the second.

Here, there can be two cases, either the statements are already aligned or are not already aligend. If the statements are not already aligned, then they can be aligned by
(i) changing the order of the sentences and/or
(ii) convert one of the sentences.

Now, see the examples given below that will better illustrate the concept:
Align all the pair of statements in the examples given below:
Ex. 1: I. All boys are goodlooking. II. Some boys are Indian.

Discussion:
Here the term 'boys' is common to both the given statements. And the 'subject of the first statement is the subject of the second statement'. Hence the statements are not properly aligned.

We can align this by
(a) Converting the first statement

All boys are goodlooking $\rightarrow$ conversion $\rightarrow$ Some goodlooking are boys. Hence the aligned pair is
[Some goodlooking are boys.
Some boys are Indian.]
(b) Converting the second statement and changing the order of the sentences
The aligned pair will be
[Some Indians are boys. (converted form of second statement)
All boys are goodlooking.]
Ex. 2: I. Some girls are cute.
II. Some Americans are cute.

## Discussion:

Here the term 'cute' is common to both the given statements. And the predicate of the first statement is the predicate of the second statement. Hence the statement are not properly aligned.

We can align the pair by
(a) converting the first statement and changing the order of the statements. The aligned pair will be: [Some Americans are cute.
Some cute are girls (converted form of first statement).]
(b) converting the second statement, the aligned pair will be:
[Some girls are cute.
Some cute are Americans (converted form of second statement).]
Ex. 3: I. No table is chair.
II. Some doors are tables.

## Discussion:

Here the term 'table' is common to both the given statements. And the subject of the first statement is predicate of the second statement. Hence the statements are not properly aligned. We can align this pair by changing the order of the statements. The aligned pair will be:
[Some doors are tables. No table is a chair.]
Ex. 4: I. Some books are pants.
II. No pants are worthy.

## Discussion:

Here the term 'pants' is common to both the statements. And the sentences are already aligned, since predicate of the first statement is the subject of the second statement.

## Rule of IEA

We have seen that in order to align a sentence, conversion is necessary when the common term is either a subject in both the sentences or a predicate in both the sentences (see Ex. 1 and Ex. 2 in the previous section). In such cases we have to convert one of the sentences. A question may arise here as to which of the two statements to choose for conversion. For this remember the rule of IEA. That is, given a pair of to-be-aligned sentences, the priority should be given, while converting to I-type statement, to E-type statement and then to A-type statement, in that order. Hence, if in the given pair one sentence is of type I and the other of type A, then the sentence of type I should be converted.

## Step II: Use the table to draw conclusions

After step I (which involved aligning the two sentences) has been completed, we are left with the easy and selfexplanatory table which can be used to draw conclusions.

Table
To draw conclusions from a pair of aligned statements

| If the firsts <br> statement <br> is of the type | and the second <br> statement is <br> of the type | then the <br> conclusion <br> will be |
| :---: | :---: | :---: |
| A | $\mathbf{A}$ | $\mathbf{A}$ |
| A | $\mathbf{E}$ | $\mathbf{E}$ |
| A | 1 | - |
| A | O | - |
| E | $\mathbf{A}$ | $\mathbf{O}^{*}$ |
| E | E | - |
| E | I | $\mathbf{O}^{*}$ |
| E | O | - |
| I | $\mathbf{A}$ | I |
| I | I | $\mathbf{O}$ |
| I | O | - |
| I | A or E or I or O | - |
| O |  | - |

Note : The reader should note that
(i) There are only 6 cases where a conclusion can be drawn. These cases are highlighted in the table by bold letters and can be memorised in short form as:

| A | + | A | = |  |
| :---: | :---: | :---: | :---: | :---: |
| A | + | E | = | E |
| E | + | A | = | - |
| E | + | I | = | 0* |
| I | + | A | = | I |
| I | + | E |  | 0 |

(ii) The ' $\because$ ' sign in the third column of the above table means that no definite conclusion can be drawn.
(iii) Above table gives correct results if and only if the two sentences have been properly aligned.
(iv) Format of the conclusion (very important): The conclusion or the inference is itself a proposition whose subject is the subject of the first statement and whose predicate is the predicate of the second statement. The common term disappears.
(v) The meaning of $\mathbf{O}^{*}$ : In the third column of above table we have written $\mathrm{O}^{*}$ in place of O at two places. By O* we mean that the conclusion or inference is of type O but its format is exactly opposite the format mentioned in (iv). In this case the subject of the inference is the predicate of the second sentence and the predicate of the inference is the subject of the first sentence.

## An illustrative example

Draw inferences for the following pairs of statements:
(i) All books are hooks. All hooks are crooks.
(ii) All tables are chairs. All tables are glasses.
(iii) Some posters are goodlooking. All posters are expensive.
(iv) Some pencils are torches. No books are pencils.
(v) No bandit is kindhearted. All bandits are blackmailers.
(vi) Some roses are red. Some roses are good.

## Solution:

(i) Step I: The sentences are already aligned.

Step II: By the table, we see that A + A = A. Hence the inference will be of type A. Its subject will be the subject of the first sentence, ie books, and the predicate will be the predicate of the second sentence, ie crooks. Hence, inference: All books are crooks.
(ii) Step I: The common term 'tables' is a subject in both the sentences. Hence we will have to convert in order to align. Since both the sentences are of the same type (A), we may convert any of them. We choose to convert the first. Consequently, the aligned pair of sentences is:

Some chairs are tables. (Converted form of all tables are chairs.)
All tables are glasses.
Step II: From the table, I + A $=$ I. Hence the conclusion will be of type I, its subject being the subject of the first sentence (after aligning has taken place), ie 'chairs', and its predicate being the predicate of the second sentence, ie 'glasses'. Hence, Inference: Some chairs are glasses.
(iii) Step I: Again the common term 'posters' is a subject in both the sentences. By the rule of IEA we convert the I-type statement which is the first statement. Consequently, the aligned pair of sentences is:

Some goodlooking are posters (converted).
All posters are expensive.
Step II: I + A $=$ I, hence inference is:
Some goodlooking are expensive.
(iv) Step I: The common term 'pencils' is a subject in one sentence and a predicate in the other. Hence, changing the order of the statements is sufficient to align the two sentences. Consequently, the aligned pair will be:

No books are pencils.
Some pencils are torches.
Step II: $\mathrm{E}+\mathrm{I}=\mathrm{O}^{*}$. As we know, $\mathrm{O}^{*}$ means that the conclusion is of type $O$ but the subject of the conclusion is the predicate of the second sentence and the predicate of the conclusion is the subject of the first sentence. Hence, inference is:

Some torches are not books.
(v) Step I: By the rule of IEA, we convert the E-type statement. The aligned pair is:
No kindhearted is a bandit.
All bandits are blackmailers. Step II: $E+A=O^{*}$. Hence, inference is:

Some blackmailers are not kindhearted.
(vi) Step I: We convert the first sentence and obtain the following pair of aligned sentences:

Some red are roses.
Some roses are good.
Step II: I + I = _. This means that there can be no definite inference.

## II. Draw Immediate Inferences (implication or conversion)

We say this step as Step III. $\square$ Step III:Check for any immediate inferences (implication or conversion)
Consider the following. Here two statements are given, followed by two conclusions.
(i) Statements: All books are chairs.

All chairs are red.
Conclusions: (i) All books are red.
(ii) Some red (objects) are books.

Here we have (the sentences are already aligned) $\mathrm{A}+\mathrm{A}$ $=$ A. Hence the conclusion should be: All books are red. But if we convert this conclusion, we obtain: Some red (objects) are books. Hence, both the conclusions given above should be taken as true.
(i) Statements: Some buses are trucks. Some trucks are cars.
Conclusions: (i) Some trucks are buses.
(ii) Some cars are trucks.

Here we have (the sentences are already aligned) I + I $=$ ',' ie, no conclusion. But if we convert the first statement 'Some buses are trucks', we get 'Some trucks are buses'. Similarly, on converting the second statement, we get 'Some cars are trucks'.

Hence we observe that although there is no conclusion or mediate inference using steps I and II, still on converting the given statements themselves we find that both the given conclusions are true.
(iii) Statements: All buses are trucks.

All trucks are cars.

Conclusions: (i) Some buses are trucks.
(ii) Some cars are buses.

Here we have, $A+A=A$. Hence the conclusion should be: "All buses are cars." But this answer choice is not given. But if we convert this statement we get "Some cars are buses" which is the second given conclusion. Also, an immediate implication of "All buses are trucks" is "Some buses are trucks". Hence, here again, both the conclusions are correct.

Hence the above three examples show that while judging the given conclusions, we should not only take the conclusion (mediate inference) drawn from the table (if any) as correct, but the immediate inferences (immediate implications and/or conversions) of the given statements as well as of the conclusion drawn from the table, should also be treated as correct inferences.

## III. Check for Complementary Pair

We can say this step as Step IV.
Two statements make a complementary pair if
(a) both of them have the same subject and the same predicate.
(b) they fall into any of these categories:
(i) I and O type pair
(ii) E and I type pair
(iii) A and O type pair

Think over the following:

## Ex. 1: Conclusions:

(i) Some students are Indians.
(ii) Some students are not Indians.

It is easy to understand that one of these conclusions must be true. This is because when 'Some students are Indians’ is false the other conclusion 'Some students are not Indians' is automatically true. We call such a pair of sentences as complementary pair. Thus a pair of sentences is called complementary pair if it is so that when one is false other is true. Hence, in a complementary pair, at least one of the two statements is always true. This is a typical case where the choice "either (i) or (ii) follows" is true. Remember that this answer choice follows even without looking at the statements.

We give below some more examples of complementary pairs of statements:
Ex. 2: No student is a table. Some students are tables.
Ex. 3: All beautiful are kind. Some beautiful are not kind.
More generally, we can classify complementary pairs by the type of proposition. You may notice that in Ex 2, E and I type propositions made complementary pairs; in Ex 3, A and O type propositions made complementary pairs while in the earlier example $I$ and $O$ type propositions ['Some students are Indians' and 'Some students are not Indians'] made complementary pairs.
Please note that sometimes a pair of statements may be complementary although it may not appear so. Consider the given example,
Ex. 4: Some books are hooks.
Some hooks are not books.
Explanation: Here the two sentences do not have the same subject and predicate. 'Books' is the subject of the first sentence and the predicate of the second while 'hooks' is the predicate of the
first sentence and the subject of the second. Hence the sentences do not appear to be complementary. But if we convert the first sentence from 'Some books are hooks' to 'Some hooks are books', the two sentences have the same subject and predicate now, and being an 'I and O' pair they are complementary.
Note: This step asks you to check for a complementary pair. If you do find a complementary pair you should choose "either of them follows". This step is applicable to only those conclusions which do not follow from step II or III. Thus, the choice "Either of them follows" should be chosen if
(i) None of the given two conclusions are found to be correct, and
(ii) the two conclusions form a complementary pair.
The rule is explained by way of the following examples:
Ex. 5: Statements: Some stones are radios.
Some radios are chairs.
Conclusions: No stones are chairs.
Some chairs are stones.
Explanation: You may check that none of the given conclusions is correct. But the two conclusions form a complementary pair because they are 'E and I' type. Hence the choice 'either of them follows' is correct.
Ex. 6: Statements: Some stones are radios.
Some radios are chairs.
Conclusions: No stones are chairs.
Some chairs are not stones.
Explanation: You may check that none of the given conclusions is correct. The answer conclusions do not form a complementary pair [E-O pair is not complementary]. Hence the choice 'None of them follows' is correct.
Ex. 7: Statements: Some stones are radios.
All radios are chairs.
Conclusions: Some stones are chairs. Conclusions: Some stones are chairs.
Some stones are not chairs.
Explanation: We see that the answer conclusions form a complementary pair. But this does not mean that the choice "either of the two follows" is correct. This is because the conclusion 'Some stones are chairs' is correct.
I hope that this thorough and exhaustive analysis will be of immense help to the readers and they will never be confused for the choice "either of them follows".

## Summary of the Analytical Method

Now, the discussion of the analytical method is complete. For the readers' benefit, I once again summarise the steps to do a syllogism problem.
Step I: Properly align the sentences.
Step II: Use the table to draw conclusion.
Step III: Check for immediate inferences.
Step IV: Check for complementary pairs if steps II and III fail.
Each of the steps given above has been elaborately and clearly explained before. The reader is advised to go through those details once again.

## Three-Statement Syllogism

Three-statement syllogism problems are not more difficult than the usual two-statement syllogism. It may,
of course, be a little more time-taking but it is not more difficult.

## Step I:

The first step in solving a three-statement syllogism problem involves "carefully choosing the two relevant statements out of the three given statements". You should perform the first step in the following manner:
(i) Take a given conclusion.
(ii) See the subject and the predicate of this given conclusion.
(iii) Now see which of the two given statements has this subject and predicate.
(iv) A. If there is one term common between the two statements chosen in the previous part [(iii)], these two statements are your relevant statements.
B. If there is no term common between the two statements chosen in the previous part [(iii)], then all the three statements are your relevant statements. In this case you will have to apply a chain-like formula.

## Step II:

In syllogism, there are four types of statements, viz A,
$E, I$ and $O$. When two statements are given and they
are arranged in such a way that the predicate of the
first sentence and the subject of the second sentence
is the same, then the following six formulae are
applicable.
$A+A=A \quad I+A=I \quad I+E=O$
$A+E=E \quad E+A=O^{*} \quad E+I=O^{*}$
Note: O* means "O reversed." In this case the predicate and the subject of the conclusion appear in reverse order. For example, "No bomb is comb" + (Some combs are bullets" is of the form $\mathrm{E}+1$ and it will give a conclusion "O reversed", ie "Some bullets are not bombs". It will not be "Some bombs are not bullets". Thus, in the second step you should apply the formulae to get conclusions. The second step should be performed in the following order:
(i) Take a given conclusion. See its subject and predicate. Now, by using the first step, find out how many and which statements are relevant for this conclusion.
(ii) I. If two statements are relevant for a given conclusion, write them in such an order that the predicate of the first sentence and the subject of the second sentence are the same.
II. If three statements are relevant for a given conclusion then write them as a chain. Arrange them in such a way that the predicate of the first sentence and the subject of the second sentence are the same, and the predicate of the second sentence and the subject of the third sentence are the same. For example,
A. No hook is a cook.
B. All books are docks.
C. Some docks are hooks.

These should be rearranged as a chain below:

(iii) Now apply the formulae. Note that there are six formulae only. This means that there are only six types of cases in which a conclusion is possible. In any other type of cases you may write "no conclusion". For example, A $+\mathrm{I}=$ no conclusion. Also, note that when three statements are simultaneously relevant, you have to write them in a chain and use the formula repeatedly. For example, if you get $I+A+E$, you should write it as $(\mathrm{I}+\mathrm{A})+\mathrm{E} \Rightarrow \mathrm{I}+\mathrm{E}(\because \mathrm{I}+\mathrm{A}=\mathrm{I}) \Rightarrow \mathrm{O}(\because \mathrm{I}+\mathrm{E}=\mathrm{O})$. Thus you get, $I+A+E=O$. Similarly,you may see that $A+E+I=(A+E)+I=$ $(\mathrm{E})+\mathrm{I}=\mathrm{O}^{*}$ or O reversed.
(iv) Now compare the given conclusion with the result of the formula that you have applied. If they match, the given conclusion is true. If they do not match, it is false.

## Step III:

In certain cases, a conclusion follows directly from one given statement only. This is called immediate inference. Also, in some cases, two given answer choices make a complementary pair, and in such cases "either of them follows" should be chosen. Therefore, in the third step you should do the following:
(i) Check for immediate inference: Take a given conclusion. If it has already been marked as a valid conclusion after step II then leave it. Otherwise,
check if it is an immediate inference of any of the three given statements.

## (ii) Check for complementary pair:

(a) Check if any two given conclusions have the same subject and the same predicate.
(b) If yes, then check that none of them has been marked as a valid conclusion after step II or as a case of immediate inference.
(d) If none of them has been marked as a valid conclusion then they will form a complementary pair if they are an A-O or an IO or an I-E pair.
(d) If they do make a complementary pair then mark the choice "either of the two follows".

## A Summary of the Method

The entire process of solving three-statement syllogism is performed in three simple steps. These three steps have been already described above. To make it clearer, we are giving below the gist of these three steps:

Take a given conclusion.
Perform step I. In other words, find out the relevant statements to test this conclusion.
Perform step II. In other words, use the formulae to test a given conclusion. [If more than two statements are relevant for a given conclusion, use the chainlike formulae]. If the given conclusion has been rejected in step II.
Perform step III: (i). In other words check for immediate inference. If the given conclusion has still not been accepted.
Perform step III: (ii). In other words check for complementary pair. [Note: You do not need to perform Step III (i), if a conclusion has already been accepted in step II. Again, you do not need to perform step III (ii), if a given conclusion has already been accepted in step III (i)].


Ex. 1: Statements: (A) All docks are boxes.
(B) Some cars are docks.
(C) No boxes are chocolates.

Conclusions:
I. Some cars are boxes.
II. No chocolates are docks.
III. Some cars are chocolates.
IV. Some cars are not chocolates.

1) Only I and IV follow
2) Only I and II follow
3) Either III or IV, and I follow
4) I, IV and II follow
5) Either III or IV, and I and II follow

## Explanations:

Conclusion I. We first take conclusion I. Here, the subject is 'cars' and the predicate is 'boxes'. Now, we see that 'cars' is in statement $B$ and 'boxes' is in statement A. Now look at statements A and B. Do they have a common term? Yes, the common term is 'docks'. Therefore, A and B are our two relevant statements.

Now we perform step II. For this, we first arrange A and B in such a way that the predicate of the first sentence is the subject of the second. This is done as:


Now we have, $I+A=I$. So the conclusion will be of type I; it will be "Some cars are boxes." Hence, conclusion I is valid. (Now, we do not need to perform step III because the conclusion has been accepted in step II itself.)
Conclusion II. Now we take conclusion II. Here, the subject is 'chocolates' and the predicate 'docks'. Now we see that 'chocolates' appears in statement 'C'. But 'docks' appears in both ' A ' and ' $B$ '. So, which of the two should we take? We should take ' $A$ ' because (Note this) there is a term common between A and C . We would not take $B$ because there is no term common between $B$ and $C$.

Now we perform step II. For this we write the sentences in such a way that the predicate of the first sentence is the subject of the second. This is done as:

All docks are boxes. (A)


No boxes are chocolates. (E)

Now, we have: $\mathrm{A}+\mathrm{E}=\mathrm{E}$. Hence, the conclusion will be "No docks are chocolates". Now the given conclusion is: "No chocolates are docks". But "No docks are chocolates" automatically implies "No chocolates are docks". Hence this conclusion is also valid. [Again we do not need to perform step III because the conclusion has been accepted in step II itself.]
Conclusion III. Now we take conclusion III. Here, the subject is 'cars' and the predicate 'chocolates'. We see that 'cars' appears in 'B' while 'chocolates' appears in ' $C$ '. Now, is there any common term between ' $B$ ' and ' $C$ '? We see that there is no term common between ' $B$ ' and ' $C$ '. So we will have to use a chainlike formula in this case because all the three statements are relevant here.

So we should perform step II. For this we should write the three statements in such a way that the predicate of the first sentence is the subject of the second and the predicate of the second sentence is the subject of the third. This is done as:

Some cars are docks. (I)


Now, we have the chainlike formula:
$\mathrm{I}+\mathrm{A}+\mathrm{E}=(\mathrm{I}+\mathrm{A})+\mathrm{E}=(\mathrm{I})+\mathrm{E}=\mathrm{O}$. Thus a valid conclusion would be of type $O$; that is "Some cars are not chocolates". But this is conclusion IV. Hence, conclusion IV is valid. But conclusion III is not accepted in step II. So we perform step III (i). We see that "Some cars are not chocolates" does not follow from any of the three given statements alone. So this test fails. Now, we perform step III (ii). For this, we will have to first search any other conclusion that has the same subject; and predicate. We see that conclusion IV fulfils this condition because the subject and the predicate of conclusions III and IV are the same: 'cars' and 'chocolates' respectively. Again, conclusion III is of type I and conclusion IV is of type O and we know that an I-O pair makes a complementary pair.

Still, we would not choose the choice "either of them follows" unless it is made sure that step II and step III (i) fail on conclusion IV too. But we see that conclusion IV is accepted in step II. So, despite the fact that conclusion III and IV make a complementary pair, we do not accept the choice "either of them follows" because, conclusion IV is accepted in step II.

## Conclusion IV.

As already explained, it is a valid conclusion.
Answer: 4 (I, II and IV follow).

## Guicker Approach

Till now, we have presented step-by-step, exhaustive solutions to the questions. This may give an indication that this method is very lengthy. But, actually it is not so. In fact, once you have solved ten such questions, the method will become very easy for you. First of all, you should do step I mentally. After a certain time you won't take more than a couple of seconds for this step. Similarly, after some practice, you will develop the ability of doing
step II and step III mentally, too. Once you attain the skill you will be able to solve 5 questions of three-statement syllogism in less than 4 minutes. To convince you that this method is really fast, I am presenting another example. In this example, I am only writing the relevant steps and omitting unnecessary details.
Ex. 2:Statements: A. All birds are swans.
B. Some skirts are birds.
C. All swans are ducks.

Conclusions: I. Some ducks are birds.
II. Some skirts are ducks.
III. All birds are ducks.
IV. Some birds are not ducks.

1) Only I and II follow
2) Only II and III follow
3) Either III or IV follows
4) I, II and either III or IV follow 5) I, II and III follow

## Solution:

Conclusions I, III, IV: Subject and predicate are "ducks" and "birds". Hence, the relevant statements are A and C. Now, All birds are swans + All swans are ducks $=A+A=A=$ All birds are ducks. Hence, III is valid. Again, "All birds are ducks" gives "Some ducks are birds". Hence I is valid. Ill and IV make complementary pair but we do not choose "either III or IV follows" because III has already been accepted.
Conclusion II: Subject $=$ 'skirts' and predicate $=$ 'ducks'. 'Skirts' appears in B while 'ducks' appears in C. But there is no term common between B and C . Hence, all three statements are relevant. Now, rearrange the three statements. Some skirts are birds + All birds are swans + All swans are ducks $=1+\mathrm{A}+\mathrm{A}$ $=(I+A)+A=(I)+A=I=$ Some skirts are ducks. Hence, conclusion II is valid.
Answer: 5 (I, II, III follow).

## The Euler's Circles or Venn-Diagrams

There is a pictorial way of representing the propositions, formulated by Euler, an ancient mathematician. Suppose that the proposition is trying to relate the subject (S) with the predicate (P). Then there are four ways in which the relation could be made according to the four propositions:


Fig. I All S are P .

Type-I


Fig. III
Some S are P.

Type-E


Fig. II
No S are P .
Type-0


Fig. IV
Some $S$ are not $P$.

Fig. I clearly represents 'All S are P'. This is denoted by the fact that the whole circle denoting $S$ (denoting 'all S') lies within the circle denoting P. Similarly, Fig. II represents 'No $S$ are $P$ ' because the circles denoting $S$ and $P$ do not intersect at all. Fig. III, similarly, represents the proposition "Some S are P" because some part of the circle
denoting S as indicated by the shaded area of S (representing 'some S') lies within the circle denoting P. Slightly more attention-seeking is the representation for the $\mathbf{O}$ proposition "Some S are not P". For this, take a closer look at Fig. IV. The figure shows that some portion of the circle S has no intersection with the circle P , while the remaining portion of the circle S is left incomplete and it is uncertain whether this portion touches P or not. The verbal interpretation of this figure would be: "there are some S that are definitely not P while there may be some S that might be P or might not be P ".

The case may be better understood by taking the following two sets.

Let $\mathrm{S}=\{$ Green, Red, Blue, Madan, Mohan\} and let $\mathrm{P}=$ \{names\}. Now, in this case, there are some S (Green, Red, Blue) that are not names and therefore not P. While there are some S (Madan, Mohan) that are names and are, therefore, P. Therefore, here, we have "Some S are not P" while "Some S are P". The correct pictorial representation for such a case would be like Fig. Ill, that is:

"Some S are not P". ["Some S are P".]
Fig. V
Now, consider another case.
Let, $S=\{$ even numbers\}, that is, $S-\{2,4,6,8, \ldots$.$\} , and$ let $P=\{$ odd numbers $\}$, that is, $P=\{1,3,5,7, \ldots$.$\} . Now, here$ again, we may find "Some $S$ ", say $\{2,4,8\}$ that are not odd numbers and, therefore, not P. Therefore, the proposition "Some S are not P" is true in this case as well. But on a closer scrutiny we find that there is no element in S which is an odd number or which is a P. In fact, here, we have "No S are P ". The correct pictorial representation for such a case would be like figure II, that is,


Fig. VI
Fig. IV is a general representation for the statement "Some S are not P". This proposition gives no clue whether the remaining S are there in P or not. The dotted portion of Fig. IV represents this lack of information only. If it is further known that the remaining $S$ are in $P$ then the dotted portion should be drawn to intersect with the circle P and it would take the shape of Fig. V, while if it is known, on the other hand, that the remaining $S$ are not in P either, then the dotted portion too, should be outside the circle $P$ and hence the representation would take the form of Fig. VI.

Quite similar to this, there could be the case of the statement "Some S are P". This statement is as ambiguous as the statement "Some S are not P ". The reader may analyse this statement on the same lines as discussed above. He should be able to appreciate the fact that "Some S are P " may have two aspects. One, where the remaining S are not P , and two, where the remaining S are also P . Correspondingly, there could be two representations, viz:

```
"Some S are P".
["Some S are not P".]
and
"Some S are P"
["All S are P".]
["All S are P".]
```



Fig. VII
Based on the discussions made so far, we tabulate the concepts developed in the following table.

Table 2: Euler's Circles and representation of the four propositions

| If the Type of the given proposition is | Then its pictorial representation is |  |  |
| :---: | :---: | :---: | :---: |
| A <br> All $S$ are $P$ | Always | $\mathrm{P}$ |  |
| $\begin{gathered} \mathrm{E} \\ \text { No } \mathbf{S} \text { are } \mathbf{P} \end{gathered}$ | Always <br> S |  |  |
| Some $\mathbf{S}$ are $P$ | Either S $\left.\begin{array}{l}\text { Some S are P. } \\ \text { [Some } \mathrm{S} \text { are not P] }\end{array}\right]$ |  |  |
|  | Or, | Some S are P [All S are P ] |  |
|  | Or, | Some S are P [All P are S ] |  |
| OSome $S$ are not $P$ | Either |  |  |
|  | Or, | Some S are not P [All P are S ] |  |
|  | Or, <br> S | Some S are not P <br> [No S are P ] |  |

## Determining the Hidden Proposition

The reader must have noticed that there are some sentences in the previous example that are on standard patterns as given in the first table. But some other sentences are not on the standard patterns. We should know, therefore, how to find out the hidden propositions in such sentences.

## Some A-type propositions not beginning with 'All'

(i) All positive propositions beginning with 'every', 'each', 'any', are $\mathbf{A}$-type propositions.

## Examples:

(a) Every man makes sandwiches.
(All men make sandwiches.)
(b) Each of them has a share of profit.
(All (of them) have a share of profit.)
(c) Any one could kill a mosquito.
(All (men) can kill mosquitoes.)
(ii) A positive sentence with a particular person as its subject is always an $\mathbf{A}$-type proposition.

## Examples:

(a) He should be awarded a gold medal.

He (is a man) who should be awarded a (gold medal). Subject Predicate.
(b) Baba Ramdev is a controversial personality.
(iii) A positive sentence with a very definite exception is also of A-type.
Example:
All students except Ram have failed.
(All except Ram (are the students) who have failed.) Subject

Predicate

## Some E-type propositions not beginning with 'No'

(i) All negative sentences beginning with 'no one', 'none', 'not a single', etc are E-type propositions.
Examples:
(a) None can escape from Tihar.
(No man is one who/can escape from Tihar). Subject Predicate
(b) Not a single player is present. (No player is present.)
(ii) A sentence with a particular person as its subject but a negative sense is an E-type proposition.

## Examples:

(a) He does not deserve a gold medal.
(He (is not a man) who deserves a gold medal.) Subject Predicate
(b) Baba Ramdev is not a controversial personality.
(iii) A negative sentence with a very definite exception is also of $\mathbf{E}$-type.

## Example:

No student except Ram has failed.
(iv) When an interrogative sentence is used to make an assertion, this could be reduced to an E-type proposition.

## Examples:

(a) Is there any sanity left in the world? (No sanity is left in the world.)
(b) Is there any person who can cheat himself? (None can cheat himself.)

## Some I-type propositions not beginning with 'Some'

(i) Positive propositions beginning with words such as 'most',
'a few', 'mostly', 'generally', 'almost', 'frequently', 'often' are to be reduced to the I-type.

## Examples:

(a) Girls are usually feminine.
(Some girls are feminine.)
(b) Students are frequently short-tempered.
(Some students are short-tempered.)
(c) Almost all the books have been sold. (Some books have been sold.)
(d) A few dollars are left in my pocket. (Some dollars are left in my pocket.)
(e) Most of the paper is handmade. (Some (of the) paper is handmade.)
(ii) Negative propositions beginning with words such as 'few', 'seldom', 'hardly', 'scarcely', 'rarely', 'little' etc are to be reduced to the I-type.

## Examples:

(a) Few men are not corruptible. (Some men are corruptible.)
(b) Seldom are people not jealous. (Some people are jealous.)
(c) Rarely is a rich man not worried. (Some rich (men) are worried.)
(iii) A positive sentence with an exception which is not definite, is reduced to I-type proposition.

## Examples:

(a) All students except three have passed. (Some students have passed.)
(b) All students except a few are present. (Some students are present.)

## Some O-type propositions not beginning with "Some...not"

(i) All negative propositions beginning with words such as 'all', 'every', 'any', 'each' etc are to be reduced to O-type propositions.
Examples:
(a) All men are not rich.
(Some men are not rich.)
(b) Every one is not present. (Some are not present.)
(c) All that glitters is not gold. (Some glittering objects are not gold.)
(ii) Negative propositions with words as 'most', 'a few', 'mostly', 'generally', 'almost', frequently' are to be reduced to the O-type.

## Examples:

(a) Girls are usually not feminine.
(Some girls are not feminine.)
(b) Students are not frequently short-tempered. (Some students are not short-tempered.)
(c) Almost all the books have not been sold. (Some books have not been sold.)
(d) Most of the paper is not handmade. (Some (of the) paper is not handmade.)
(iii) Positive propositions with beginning words such as few, 'seldom','hardly"scarcely','rarely', 'little'etc are to be reduced to the $\mathbf{O}$-type.
Examples:
(a) Few men are corruptible. (Some men are not corruptible.)
(b) Seldom are people jealous. (Some people are not jealous.)
(c) Rarely is a rich man worried. (Some rich men are not worried.)
(iv) A negative sentence with an exception, which is not definite, is to be reduced to the O-type.

## Examples:

(a) No students except two have passed.
(Some students have not passed.)
(b) No students except a few are absent.
(Some students are not absent.)


