IES / ISS Exam, 2013

A–HRR–TUBA

STATISTICS

Paper—I

Time Allowed : Three Hours | *Maximum Marks : 200*

INSTRUCTIONS

*The candidates should attempt **FIVE** questions in **ALL** including Question No. 1 and 5 which are compulsory. The remaining **THREE** questions should be answered by choosing at least **ONE** question each from Section A and Section B.*

The number of marks carried by each question is indicated at the end of the question.

*Answers must be written in **ENGLISH**.*

(Symbols and abbreviations are as usual)

Any essential data assumed by candidates for answering questions must be clearly stated.

SECTION—A

1. Answer all of the following : $8 \times 5 = 40$

(a) If the probability for n independent events are $p_1, p_2, \ldots, p_n$, then prove that :

(i) none of the events will occur

(ii) at least one event will occur

(iii) at most one event will occur. 8

(b) Let X have the density function,

$$f(x) = \begin{cases} c(1-x^2), & |x|<1 \\ 0, & \text{otherwise} \end{cases}$$

(i) Find the constant c.

(ii) Find the distribution function.

(iii) Compute $P[X \geq -1/2]$. 8

(c) Compute the factorial moments $\mu_{(r)}$ and the cumulants k_r, r = 1, 2, ..., of Poisson distribution with parameter m. 8

(d) Let X be a random variable with E[X] = 4 and $E[X^2] = 20$. Use Chebyshev's inequality to determine a lower bound for the probability $P[0 < X < 8]$. 8

(e) Show that convergence in probability implies convergence in distribution. 8

2. (a) A fair die is rolled twice. Let A be the event that the first throw shows a number ≤ 2, and B be the event that the second throw shows at least 5. Show that $P(A \cup B) = \frac{5}{9}$. 10

(b) Prove that for among the discrete distributions, the geometric distribution has the lack of memory property. 10

(c) Let X be a continuous random variable and have F(x) as the distribution function. If E[X] exists, then show that :

$$E[X] = \int_0^{\infty} (1 - F(x))\,dx - \int_{-\infty}^{0} F(x)\,dx. \qquad 10$$

(d) For a random vector $(X_1, X_2, \ldots\ldots, X_p)$, let $\text{Cor}(X_i, X_j) = \rho, i \neq j$.

Show that :

$$\rho \geq \frac{1}{(p-1)}. \qquad 10$$

3. (a) Let P(z) be the probability generating function of the random variable X whose probability distribution is $\Pr[X = n] = p_n$, $n = 0, 1, 2, \ldots\ldots$ Find the generating function of (i) $\Pr[X > n]$, (ii) $\Pr[X < n]$ and (iii) $\Pr[X = 2n]$. 10

(b) Find the density function of X whose characteristic function $\phi(t)$ is given by :

$$\phi(t) = \begin{cases} 1 - |t|, & |t| \leq 1 \\ 0, & |t| > 1. \end{cases} \qquad 10$$

 (Contd.)

(c) Let (X, Y) have a joint probability mass function

$$f(x, y) = \frac{xy}{36}, \; x = 1, 2, 3; \;\; y = 1, 2, 3$$

$$= 0 \;\;, \quad \text{elsewhere.}$$

Find the marginal mass functions of X and Y. 10

(d) Find the distribution of the ratio of two iid random variables with density function :

$$f(x) = e^{-x}, \;\; 0 < x < \infty$$

$$= 0 \;\;, \;\; \text{elsewhere.}$$ 10

4. (a) If $X \sim N(0, 1)$, obtain the distribution of X^2. 10

(b) Let $\{X_n\}$ be a sequence of iid random variables with common finite mean $\mu = E(X)$.

Show that :

$$\frac{\rho_n}{n} \xrightarrow{P} \mu \text{ as } n \to \infty.$$ 10

(c) Let $\{X_n\}$ be iid with density function $f(x) = \lambda e^{-\lambda x}$, $x > 0$ and 0, elsewhere. Let $Y_n = \underset{1 \le i \le n}{\text{Min.}} \{X_n\}$ be the first order statistic. Find the limiting distribution of Y_n. 10

(d) State and prove Lindeberg-Levy Central limit theorem. 10

SECTION—B

5. Answer **all** of the following : 8×5=40

(a) For the following frequency distribution on 229 values :

Class	Frequency
0—10	12
10—20	30
20—30	x
30—40	65
40—50	y
50—60	25
60—70	18

the median is found to be 46. Find the values of x and y. 8

(b) Out of 200 persons appeared for an examination, 80% were males and the rest were females. Among 30 married males, 14 were successful; 110 unmarried males were successful. In respect of 10 married females, 4 were successful; 20 unmarried females were successful. Compute Yule's coefficient of association between success in the examination and the marital status for males as well as females. 8

(c) Describe the run test for randomness. For the sequence of outcomes of 14 tosses of a coin,

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test whether the outcomes are in random order. (Given the lower and upper critical values $R_L = 3$, $R_U = 12$ at 0.05 significance level.) 8

(d) Find the missing value in the table by assuming a polynomial form for y :

x	0	1	2	3	4
y	1	2	4	–	16

8

(e) Evaluate the following by taking seven ordinates of the integrand :

$$\int_0^1 \frac{dx}{1+x^2}.$$

8

6. (a) The values of two random samples are arranged below in an increasing order with X denoting the value of sample 1 and Y denoting the value of sample 2 :

X X X Y X Y Y Y Y X X Y X X X Y X X Y Y Y

Use Wald-Wolfowitz run test to test whether the two samples may be regarded as coming from a common population. 10

(b) Given the two regression lines between X and Y, $2Y - X - 50 = 0$, $3Y - 2X - 10 = 0$, compute the means of X and Y and the correlation coefficient between X and Y. 10

(c) From along series of annual river flows, the variance is found out to be 49 units. For a new sample of 25 years, the variance is calculated as 81 units. Can we regard that the sample variance is significant ?

(Given the chi-square value at 5% level of significance as 37.7) 10

(d) Find the weighted arithmetic mean of the first 'n' natural numbers, the weights being the corresponding numbers. 10

7. (a) For a trivariate population, it is given that :

$\sigma_1 = 3, \sigma_2 = 4, \sigma_3 = 5, r_{12} = 0.7, r_{13} = 0.6, r_{23} = 0.4$

Compute all partial and multiple correlation coefficients. 10

(b) Define correlation ratio. Discuss its properties and usefulness. 10

(c) Obtain $100(1 - \alpha)\%$ confidence interval for the ratio of population variances by using two independent random samples from $N(\mu_1, \sigma_1^2)$ and $N(\mu_2, \sigma_2^2)$ under the assumption that the population means are (i) known and (ii) unknown. 10

(d) The memory capacities of nine students were tested before and after some training. The data are given below. Test whether the training was effective :

Student Number	1	2	3	4	5	6	7	8	9
Before Training	10	15	9	3	7	12	16	17	4
After Training	12	17	8	5	6	11	18	20	3

(Given t-value at 8 d.f. = 2.36) 10

8. (a) Prove that independent variables are uncorrelated. Establish that the converse is false. 10

(b) The proportion of defective telephone instruments from the All-India basis was observed to be at 0.01. A new manufacturer of telephone instruments wants to estimate the proportion of defectives in his production lot. How many telephones should be sampled in order to estimate the fraction defective within 0.01 with 90% confidence ? 10

(c) Use Simpson's one-third rule to estimate approximately the area of the cross section of a river 80 feet wide, the depth d (in feet) at a distance x from one bank being given by the following table :

x	:	0	10	20	30	40	50	60	70	80
d	:	0	4	7	9	12	15	14	8	3

10

(d) Solve the equation :

$$\frac{dy}{dx} = 1 - y, \ y = 0 \text{ when } x = 0$$

Use Euler algorithm and tabulate the solution at $x = 0.1, 0.2, 0.3$. 10