IES/ISS Exam, 2013

A–HRR–TUBD

STATISTICS—IV

Time Allowed : Three Hours	Maximum Marks : 200

INSTRUCTIONS

Candidates should attempt FIVE questions in ALL including Question Nos. 1 and 5, which are compulsory and attempt remaining THREE questions by choosing at least ONE question each from Sections A and B.

The number of marks carried by each question is indicated at the end of the question.

All parts and sub-parts of a question are to be attempted together in the answer-book.

Attempt of a part/question shall be counted in chronological order. Unless struck off, attempt of a part/question shall be counted even if attempted partly. Any page or portion of the page left blank in the answer-book must be clearly struck off.

Answers must be written only in ENGLISH.

Symbols and abbreviations are as usual.

If any data is required to be assumed for answering a question, it may be suitably assumed, indicating this clearly.

Any graphs/sketches are to be drawn on the answer-book itself, wherever required.

Section—A

1. Attempt *all* of the following : 8×5=40

(a) Show that a Markov chain is irreducible if and only if each state can be reached from every other state. 8

(b) If $\{X_t, t \geq 0\}$ is a Furry-Yule process with parameter λ and initial condition $P(X_0 = r) = 1$, where r is some natural number, then for $s < t$, obtain the joint distribution of the random variables X_s and X_t. 8

(c) Define—

(i) basic feasible solution;

(ii) non-degenerate basic feasible solution.

Establish that the set of all feasible solutions to the LPP is a convex set. 8

(d) Illustrate the following methods of generating random numbers from a given distribution—*(i)* inverse transformation, *(ii)* composition, *(iii)* convolution and *(iv)* acceptance and rejection methods, indicating clearly where a particular method is more suitable. 8

(e) An airline company has drawn up a new flight schedule involving five flights. To assist in allocating five pilots to the flights, it has asked them to state their preference scores by giving each flight a number out of 10. The higher the number, the greater is the preference. Certain of these flights are unsuitable to

some pilots owing to domestic reasons which are marked with 'x' :

↓ *Pilot*	*Flight No.*				
	1	*2*	*3*	*4*	*5*
A	8	2	×	5	4
B	10	9	2	8	4
C	5	4	9	6	×
D	3	6	2	8	7
E	5	6	10	4	3

What should be the allocation of pilots to flights in order to meet as many preferences as possible? 8

2. *(a)* State and prove Chapman-Kolmogorov equation. 10

(b) Define simple random walk. Obtain the first passage times in such a random walk. 10

(c) In an M|M|1 queueing system in a steady state, show that the inter-departure times are independently distributed exponentially random variables with mean $1/\lambda$, where λ is the parameter of the input (Poisson) process. 10

(d) Prove that if the sum of two independent renewal processes is a Poisson process, then both renewal processes must be Poisson processes. 10

3. *(a)* Sunshine Electronics Ltd. assembles 3 types of Printed Circuit Boards (PCBs) used in personal computers such as motherboards, graphic cards and memory cards. The company wants to determine how many each of these boards should be produced next month. The component parts are placed on the board by a computer-controlled insertion machine called Dyna Assembler. The assembled boards are connected to Test Rig to verify all functioning of the boards. All the three types of boards must be processed and tested. The time required for these two processes for each of the products is given below :

Production Processing Time (in minutes)			
	Mother-board	*Graphic Card*	*Memory Card*
Dyna Assembler	5	3	4
Test Rig	600	720	560

To assemble these 3 products next month, 65 hours of Dyna Assembler time and 4800 hours of Test Rig time are available. It is not necessary to use all the available time but it should not exceed the available hours.

As per accounting department, the direct production cost of using Dyna Assembler is ₹ 360 per hour and for Test Rig is ₹ 12 per hour (no matter what product is being produced in these machines). The material cost and

revenue of each of these products are given below :

Revenue and Material Cost (in ₹/unit)			
	Mother-board	*Graphic Card*	*Memory Card*
Revenue	750	650	600
Material Cost	250	210	200

The company faces tremendous demand for its products and is confident that it can sell everything it produces at the prices that are given. However, at least 20 graphic cards be produced in the next planning period (next month). The marketing department wants to ensure that the company should produce at least one memory card for every 20 motherboards that are produced next month.

Determine production plan to decide how many motherboards, graphic cards and memory cards are to be produced optimally during next month. 10

(b) Examine whether TPP is a particular case of LPP. If so—

(i) formulate TPP as an LPP;

(ii) mention the reason(s) why simplex procedure is not used to find optimal solution of TPP.

Further write down the algorithm of (1) VAM and (2) MODI methods in solving TPP. 10

(c) Generate 10 random numbers from the following density function, assuming ρ and μ values : 10

$$f(\theta) = \frac{1}{2\pi}[1 + 2\rho\cos(\theta - \mu)]$$

$$|\rho| < \frac{1}{2};\ \theta \in [0,\ 2\pi[$$

$$\mu \in [0,\ 2\pi[$$

[*Note* : Random sampling numbers table is given in Page No. 11.]

(d) Write down the following algorithms : 10

(i) Two-phase simplex method

(ii) Charnes' perturbation method

4. (a) At time '0', a blood culture starts with one red cell. At the end of one minute, the red cell dies and is replaced by one of the combinations with probabilities as indicated—2 red cells with probability 1/4, 1 red cell and 1 white cell with probability 2/3, 2 white cells with probability 1/12. Each red cell lives for one minute and gives birth to offsprings in the same way as the parent cell. Each white cell lives for one minute and dies without reproducing. Assuming that individual cell behaves independently, what is the probability that the entire culture dies out eventually? 10

(b) Write notes on the following : 10

(i) Divergent birth process

(ii) General birth process

(c) Generate ten pseudorandom numbers between 0 and 1, using—

(i) mixed congruential generator;

(ii) multiplicative congruential generator;

(iii) composite generator;

(iv) combined linear congruential generator. 10

(d) A company is considering a selective inventory control using the following data :

Item No.	:	1	2	3	4	5	6
Requirement (in no. of units)	:	6000	61200	16800	3000	55800	22680
Unit Cost (in ₹)	:	4	0·05	2·10	6	0·20	0·50
Item No.	:	7	8	9	10	11	12
Requirement (in no. of units)	:	26640	14760	20520	90000	29940	24660
Unit Cost (in ₹)	:	0·65	0·40	0·40	0·10	0·30	0·50

Carry out A, B, C analysis. 10

Section—B

5. Attempt *all* of the following : 8×5=40

(a) Discuss the sources, uses and limitations of demographic data in India. 8

(b) Define CBR, GFR and ASFR, and indicate why each is considered an improvement on the preceding measure of fertility. 8

(c) Describe a method of fitting Makeham's graduation formula using most of the available data. 8

(d) Write an algorithm to implement binary multiplication on a digital system. 8

(e) Explain *(i)* multiplication and *(ii)* division of normalized floating-point numbers. 8

6. (a) Distinguish between projection and forecast. Discuss the component method of population projection. 10

(b) Describe King's method for constructing an abridged life table. 10

(c) Show that the age distribution of two stable populations would be identical if they are such that their mortalities at all ages differ by a constant at all ages. 10

(d) Give a short account of the main findings of the Population Census of India, 2011. 10

7. (a) Draw a flow chart to find correlation coefficient and hence draw flow chart of the main program to find partial and multiple correlation coefficients of 3 variables. 10

(b) Illustrate the following : 10

(i) Batch processing

(ii) Real-time processing

(iii) Sequential processing

(iv) Indexed sequential processing

(v) Random files processing

(c) Explain the following functions of any operating system : 10

(i) Processor

(ii) Memory

(iii) Device

(iv) File managements

(d) Illustrate the following conversion procedures : 10

(i) Decimal to binary numbers

(ii) Binary to octal numbers

(iii) Octal to hexadecimal numbers

(iv) Hexadecimal to binary numbers

(v) Binary to decimal numbers

8. (a) For the following fertility data of a country, calculate GRR and NRR : 10

Age (in years)	*Age-specific Fertility Rate*	*Female Life Table Stationary Population*
15–19	0·0696	4180
20–24	0·2346	4123
25–29	0·1897	4063
30–34	0·1143	4001
35–39	0·0611	3934
40–44	0·0285	3860
45–49	0·0101	3763

The sex ratio at birth for the country may be assumed to be 105 males to 100 females.

(b) What do you understand by estimation of demographic measures from incomplete data? Explain how stable population concept is used in estimating birth and death rates of a population. 10

(c) Explain the steps to solve an LPP problem using solver of MS-Excel and illustrate with suitable example. 10

(d) Explain the steps to solve ANOVA one-way classification using MS-Excel. 10

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TABLE [for Question No. 3 *(c)*]

RANDOM SAMPLING NUMBERS							
4652	3819	8431	2150	2352	2472	0043	3488
9031	7617	1220	4129	7148	1943	4890	1749
2030	2327	7353	6007	9410	9179	2722	8445
0641	1489	0828	0385	8488	0422	7209	4950
8479	6062	5593	6322	9439	4996	1322	4918
9917	3490	5533	2577	4348	0971	2580	1943
6376	9899	9259	5117	1336	0146	0680	4052
7287	0983	3236	3252	0277	8001	6058	4501
0592	4912	3457	8773	5146	2519	3931	6794
6499	9118	3711	8838	0691	1425	7768	9544
0769	1109	7909	4528	8772	1876	2113	4781
8678	4873	2061	1835	0954	5026	2967	6560
0178	7794	6488	7364	4094	1649	2284	7753
3392	0963	6364	5762	0322	2592	3452	9002
0264	6009	1311	5873	5926	8597	9051	8995
4089	7732	8163	2798	1984	1292	0041	2500
9376	7365	7987	1937	2251	3411	6737	0367
3039	3780	2137	7641	4030	1604	2517	9211
8971	8653	1855	5285	5631	2649	6696	5475
0373	4153	5199	5765	2067	6627	3100	5716
9092	4773	0002	7000	7800	2292	2933	6125
2464	1038	3163	3569	7155	2029	2538	7080
3027	6215	3125	5856	9543	3660	0255	5544
5754	9247	1164	3283	1865	5274	5471	1346
4358	3716	6949	8502	1573	5763	5046	7135
7178	8324	8379	7365	4577	4864	0629	5100
5035	5939	3665	2160	6700	7249	1738	2721
3318	0220	3611	9887	4608	8664	2185	7290
9058	1735	7435	6822	6622	8286	8901	5534
7886	5182	7595	0305	4903	3306	8088	3899
3354	8454	7386	1333	5345	6565	3159	3991
3415	7671	0846	7100	1790	9449	6285	2525
3918	5872	7898	6125	2268	1898	0755	6034
6138	9045	6950	8843	6533	0917	6673	5721
3825	1704	2835	4677	4637	7329	3156	3291
1349	0417	9311	9787	1284	0769	8422	1077
4234	0248	7760	6504	2754	4044	0842	9080
6880	3201	7044	3657	5263	0374	7563	6599
0714	5008	5076	1134	5342	1608	5179	0967
3448	6421	3304	0583	1260	0662	7257	0766
5711	7343	7539	3684	9397	5335	4031	1486
2588	3301	0553	2427	3598	2580	7017	9176
8581	4253	7404	5264	5411	3431	3092	8573
8475	6322	3949	9675	6533	1133	8776	2216
0272	5624	8549	5552	7469	2799	2822	9620
7383	7795	7939	2652	4456	6993	2950	8573
5126	2089	7729	0945	3901	4445	7117	8186
2064	3760	0939	7319	5939	3432	2030	4752
9315	8185	7805	6294	7072	6491	4012	1016
6814	8752	3462	6001	3302	3895	7371	3432