AMIETE – ET/CS/IT (OLD SCHEME)

Code: AE01/AC01/AT01
Time: 3 Hours

DECEMBER 2009

Subject: MATHEMATICS-I
Max. Marks: 100

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: (2 x 10)

a. The value of limit $\lim_{(x,y)\to(1,0)} \frac{(x-1)\sin y}{y \ln x}$ is

(A) 0 **(B)** 1
(C) -1 **(D)** limit does not exist

b. If $u(x,y) = \cos^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right), 0 < x, y < 1$ then

(A) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = -\frac{1}{2}\cot u$ **(B)** $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\cot u$
(C) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = -\frac{1}{2}\tan u$ **(D)** $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u$

c. The value of the integral $\int_0^{\frac{\pi}{2}} \int_0^2 r\, dr\, d\theta$ is

(A) π **(B)** $\frac{\pi}{2}$
(C) $\frac{\pi}{4}$ **(D)** 0

d. The value of integral $\iint_R e^{x^2} dxdy$ where **R** is the region given by R: $2y \le x \le 2,\ 0 \le y \le 1$ equal to

(A) $\frac{1}{2}\left(e^2 - 1\right)$ **(B)** $-\frac{1}{2}\left(e^2 - 1\right)$

(C) $\frac{1}{4}\left(e^4 - 1\right)$ **(D)** $\frac{1}{2}\left(e^4 - 1\right)$

The solution of the differential equation y dx – x dy +$e^{1/x}$ dx = 0, is given by

(A) y + x $e^{1/x}$ = cy **(B)** y + x $e^{2/x}$ = cy
(C) y + x $e^{1/x}$ = cx **(D)** x + x $e^{1/x}$ = cy

e. The particular integral of the differential equation $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 4y = 65\sin 2x$ is

(A) $\frac{13}{2}\cos 2x$ **(B)** $\frac{13}{2}\sin 2x$
(C) $-\frac{13}{2}\cos 2x$ **(D)** $-\frac{13}{2}\sin 2x$

f. The product of the eigen values of $\begin{pmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{pmatrix}$ is equal to

(A) 6 **(B)** -8
(C) 4 **(D)** -6

g. Let T be a linear transformation from R^3 into R^2 defined by the relation Tx=Ax, A= $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$. The value of Tx when x is given by $[3\ 4\ 5]^T$

(A) $\begin{bmatrix} 62 \\ 26 \end{bmatrix}$ **(B)** $\begin{bmatrix} 26 \\ 62 \end{bmatrix}$
(C) $\begin{bmatrix} 65 \\ 25 \end{bmatrix}$ **(D)** $\begin{bmatrix} 25 \\ 65 \end{bmatrix}$

h. The value of $P(x) = 2P_2(x) + 4P_1(x) + 5P_0(x)$ as a polynomial in x is equal to

(A) $3x^2$-4x-4 **(B)** $3x^2$+4x-4
(C) $3x^2$-4x+4 **(D)** $3x^2$+4x+4

i. The value of the $J_3(x)$ is

(A) $\left(\frac{8}{x^2}-1\right)J_1(x)-\frac{4}{x}J_0(x)$ **(B)** $\left(\frac{8}{x^2}-1\right)J_1(x)+\frac{4}{x}J_0(x)$

(C) $\left(\frac{8}{x^2}+1\right)J_1(x)-\frac{4}{x}J_0(x)$ **(D)** $\left(\frac{8}{x^2}+1\right)J_1(x)+\frac{4}{x}J_0(x)$

Answer any FIVE Questions out of EIGHT Questions.
Each Question carries 16 marks.

Q.2 a. Show that the function $f(x,y)=\begin{cases}\frac{x^2+y^2}{|x|+|y|}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$ is continuous at (0, 0) but its partial derivatives f_x and f_y does not exist at (0, 0). **(8)**

b. If $u=a^3x^2+b^3y^2+c^3z^2$ where $x^{-1}+y^{-1}+z^{-1}=1$, show that the stationary value of u is given by $x=\left(\sum a\right)/a,\ y=\left(\sum b\right)/b,\ z=\left(\sum c\right)/c$. **(8)**

Q.3 a. Expand f(x, y) = $\tan^{-1}(y/x)$, in powers of (x-1) and (y-1) upto third degree terms. Hence compute f (1.1, 0.9) approximately. **(8)**

b. Evaluate $\iint_D xy\sqrt{1-x-y}\,dxdy$ where D is the region bounded by x = 0, y = 0, x + y =1, using the transformation x + y = u, y = uv. **(8)**

Q.4 a. Solve the differential equation $3x(1-x^2)y^2\frac{dy}{dx}+(2x^2-1)y^3=ax^3$. **(8)**

b. Solve by the method of undetermined coefficients, $y''-y=e^{3x}\cos 2x-e^{2x}\sin 3x$. **(8)**

Q.5 a. Find the general solution of the equation $y''-3y'+2y=xe^{3x}+\sin 2x$. **(8)**

b. Reduce the matrix $A=\begin{bmatrix}-1 & 2 & -2\\ 1 & 2 & 1\\ -1 & -1 & 0\end{bmatrix}$ to the diagonal form. **(8)**

Q.6 a. If the following system has non-trivial solution, prove that a + b + c = 0 or a = b = c; ax + by + cz = 0, bx + cy + az = 0, cx + ay + bz = 0. **(8)**

b. Prove that the matrix $A = \begin{bmatrix} (1+i)/2 & (-1+i)/2 \\ (1-i)/2 & (1+i)/2 \end{bmatrix}$ is unitary and find A^{-1} **(8)**

Q.7 a. Test for consistency the following system of equations, and if consistent, solve them: $x_1 + 2x_2 - x_3 = 3;\ 3x_1 - x_2 + 2x_3 = 1;\ 2x_1 - 2x_2 + 3x_3 = 2;\ x_1 - x_2 + x_3 = -1$ **(8)**

b. If u = log ($x^3 + y^3 + z^3$ - 3xyz) show that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x+y+z)^2}$. **(8)**

Q.8 a. Find the power series solution about the origin of the equation $xy'' + y' + xy = 0$. **(11)**

b. Prove that $J_n''(x) = \frac{1}{4}\left[J_{n-2}(x) - 2J_n(x) + J_{n+2}(x)\right]$ **(5)**

Q.9 a. Show that $\int_{-1}^{1} (1-x^2) P_m'(x) P_n'(x) dx = 0$. **(8)**

b. Solve $\left\{ y\left(1 + \frac{1}{x}\right) + \cos y \right\} dx + (x + \log x - x \sin y) dy = 0$. **(8)**