Signature of Invigilators

1. ....

2. ....

Roll No. [ ][ ][ ][ ][ ]

(In figures as in Admit Card)

MATHEMATICAL SCIENCES

Paper II

Roll No. ....

....

(In words)

D—0102

Name of the Areas/Section (if any) ....

Time Allowed : 75 Minutes]

[Maximum Marks : 100

SEAL

Instructions for the Candidates

1. Write your Roll Number in the space provided on the top of this page.
2. Each item has upto four alternative responses marked (A), (B), (C) and (D). The answer should be a capital letter for the selected option. The answer letter 'A' should entirely be contained within the corresponding square.

   Correct method $\boxed{A}$ Wrong Method $\boxed{}$A or $\boxed{A}$
3. Your responses to the items for this paper are to be indicated on the ICR Answer Sheet under paper II only
4. Read instructions given inside carefully.
5. One sheet is attached at the end of the booklet for rough work.
6. You should return the test booklet to the invigilator at the end of paper and should not carry any paper with you outside the examination hall.

પરીક્ષાર્થીઓ માટે સૂચનાઓ :

૧. આ પૃષ્ઠના ઉપલા ભાગે આપેલી જગ્યામાં તમારી ક્રમાંક સંખ્યા (રોલ નંબર) લખો.

૨. પ્રત્યેક વિગતના (A), (B), (C) અને (D) એવા ચાર સંભવિત ઉત્તરો આપવામાં આવ્યા છે. તમે સ્વીકારેલા વિકલ્પનો ઉત્તર કેપિટલ (પહેલી એબીસીડી) અક્ષરમાં આપવાનો રહેશે.તમારા ઉત્તર આપેલા ચોરસમાં સરખી રીતે લખવા.

   સાચી પદ્ધતિ : $\boxed{A}$ ખોટી પદ્ધતિ : $\boxed{}$A , $\boxed{A}$

૩. આ પ્રશ્નપત્રના ઉત્તરો આઈસીઆરના ઉત્તરપત્રકમાં Paper II ની નીચે લખવાના રહેશે.

૪. અંદર આપેલી સૂચનાઓ ધ્યાનથી વાંચો.

૫. આ ઉત્તરપોથીને અંતે આપેલું પૃષ્ઠ કાચા કામ માટે છે.

૬. પ્રશ્નપત્ર લખાઈ રહે એટલે આ ઉત્તરપોથી તમારા નિરીક્ષકને આપી દેવી. પરીક્ષાખંડની બહાર કોઈપણ પ્રશ્નપત્ર લઈ જવું નહીં.

MATHEMATICAL SCIENCES

Paper II

Note :—This paper contains *seventy* (**70**) multiple-choice questions, each question carrying **two** (**2**) marks. Attempt any **50** questions.

1. Let $a_n = n \sin\left(\frac{\pi}{n}\right)$, $n = 1, 2, \ldots\ldots$ Then $\lim_{n\to\infty} a_n = \ldots\ldots\ldots$

   (A) 0 (B) ∞

   (C) π (D) 1.

2. $\int_0^3 (x - [x])\,dx = \ldots\ldots\ldots$

   (A) 0 (B) $\frac{1}{2}$

   (C) 1 (D) $\frac{3}{2}$.

3. The series $\sum_{n=1}^{\infty} \frac{nx^3}{n^3 + x^3}$ is :

   (A) not convergent in [0, 2]

   (B) convergent but not uniformly convergent in [0, 2]

   (C) uniformly convergent in [0, 2], but the sum function is not continuous in [0, 2]

   (D) uniformly convergent in [0, 2] and the sum function is continuous in [0, 2].

4. Let $f(z) = \frac{z^3}{(z-\pi)^3\,(z+5)^2}$ and let C be $|z| = 3$. Then $\int_C f(z)\,dz = 0$,

   because :

   (A) the residue is 0 at its only pole within C

   (B) the sum of the residues at its two poles within C is 0

   (C) $f(z)$ is analytic on and within C

   (D) $|f(z)| < \frac{M}{|z^2|}$ for some constant M.

5. Let $f(z) = \dfrac{z^7 + z^6 + 1}{z}$. Let $I_j = \int_{C_j} f(z)dz$. Here C_1 is the circle $|z| = 1$ and C_2 is the ellipse whose equation in R^2 is $\dfrac{x^2}{4} + \dfrac{y^2}{9} = 1$, both oriented counterclockwise. Then :

(A) $I_1 = 0 = I_2$
(B) $I_1 = 2\pi i = I_2$
(C) $I_1 = 2\pi i$, $I_2 = 0$
(D) $I_1 = 2\pi i$, I_2 does not exist.

6. Let $A = \begin{bmatrix} 1 & 4 \\ 3 & 12 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}$. Then the inverse of BA :

(A) is the matrix $\begin{bmatrix} 8/7 & -4 \\ -2/7 & 1 \end{bmatrix}$
(B) is $\begin{bmatrix} -1/7 & 1/2 \\ -1/28 & -1/8 \end{bmatrix}$
(C) does not exist
(D) is $\begin{bmatrix} -1/2 & 1/7 \\ 2 & -4/7 \end{bmatrix}$.

7. If (1, 1) is an eigen vector of $A = \begin{bmatrix} 2 & 5 \\ 3 & k \end{bmatrix}$, then one of the eigenvalues of A is :

(A) 0
(B) -1
(C) 1
(D) 2.

8. Let Δ be the determinant $\begin{vmatrix} x+a & b & c & d \\ a & x+b & c & d \\ a & b & x+c & d \\ a & b & c & x+d \end{vmatrix}$.

Which of the following is a pair of factors of Δ ?

(A) $x+a+b+c+d$ and $x-a+b-c+d$
(B) $x+a+b+c+d$ and x^3
(C) $x-a+b-c+d$ and x^3
(D) $x+a+b+c+d$ and $x+a-b+c-d$.

9. The linear equations :

$$2x + 3y + 4z = 0$$
$$5x - y + 2z = 0$$
$$9x + 5y + kz = 0$$

have :

(A) no solution for any value of k

(B) a unique solution for $k = 10$

(C) at least one solution for every value of k

(D) infinite number of solutions if $k \neq 10$.

10. Suppose X_1 and X_2 are independent Bernoulli r.v.s with $EX_1 = p_1$ and $EX_2 = p_2$. Then X_1X_2 follows :

(A) Bernoulli distribution with mean p_1p_2

(B) Bernoulli distribution with mean $p_1 + p_2$

(C) Binomial distribution with $n = 2$ and $p = p_1p_2$

(D) Bernoulli distribution with mean $(1 - p_1)(1 - p_2)$.

11. If X is a binomial (n, p) r.v. and Y is a Poisson r.v. with EX = EY, then :

(A) $P(X = 0) = P(Y = 0)$

(B) $P(X = 0) < P(Y = 0)$

(C) $P(X = 0) > P(Y = 0)$

(D) nothing can be said about $P(X = 0)$ and $P(Y = 0)$ because n is not known.

12. A family has 4 children. The probability of a child being a boy is 1/2. The sex of each child is determined independently of the previous child/children. Let :

α = P(no two consecutive children are of the same sex)

β = P(the first 3 children are girls).

Then :

(A) $\alpha > \beta$

(B) $\alpha < \beta$

(C) $\alpha = \beta$

(D) nothing can be said about α and β.

13. If X is a standard normal r.v., then $P(X \leq x) \cdot P(X \geq x)$ is maximum at the point θ, where :

(A) $\theta = 1$ (B) $\theta < 0$

(C) $\theta > 1$ (D) $\theta = 0$.

14. If X has the standard normal distribution and Y is a discrete r.v. independent of X, then the distribution of $X^2 + Y^2$ is :

(A) discrete

(B) absolutely continuous

(C) chi-square

(D) neither continuous nor discrete.

15. Suppose $E(X^2) = 0.5$. Then $P(-1 \leq X \leq 1)$ must be :

(A) less than or equal to 0.5

(B) greater than or equal to 0.5

(C) exactly equal to 0.5

(D) none of the above.

16. Suppose X and Y are independent Poisson r.v.s with $EX = 12$ and $EY = 6$. Then :

(A) $\frac{X + Y}{2}$ follows Poisson distribution with mean 9

(B) $X - Y$ follows Poisson distribution with mean 6

(C) $\frac{X}{2} + Y$ follows Poisson distribution with mean 12

(D) none of the above holds.

17. A, B and C are independent events with $P(A) = P(B) = P(C) = \frac{1}{3}$. Then $P(A \cup B \cup C)$ equals :

(A) $\frac{20}{27}$ (B) $\frac{19}{27}$

(C) $\frac{18}{27}$ (D) 1.

18. Consider the L.P.P. :

Maximize : $Z = 2x_1 + x_2$

Subject to the constraints :

$$5x_1 + 10x_2 \leq 10$$
$$x_1 + x_2 \geq 1$$
$$x_1, x_2 \geq 0.$$

Then :

(A) the problem has no bounded solution

(B) the problem has more than one optimal solution

(C) the problem has a unique optimal solution

(D) the constraints are inconsistent.

19. Consider the L.P.P. :

Maximize : $x + 2y + 2z$

Subject to :

$$x + y + z \leq 7$$
$$2x + y + z \leq 10$$
$$x \geq 0, y \geq 0, z \geq 0.$$

Then :

(A) $x = 3, y = 2, z = 2$ is an optimal solution

(B) $x = 0, y = 7/2 = z$ is an optimal solution

(C) optimal solution for this problem is unique

(D) none of the above is true.

20. Consider a L.P.P. with the constraints :

$$2x_1 + 3x_3 \leq 15$$
$$x_1 + x_2 \geq 6.$$

We rewrite the above constraints as :

$$2x_1 + 3x_3 + y = 15$$
$$x_1 + x_2 - z = 6.$$

Then which of the following is *true* ?

(A) y is called a slack variable and z is called a surplus variable

(B) y is called a surplus variable and z is called a slack variable

(C) Both y and z are called slack variables

(D) Both y and z are called surplus variables.

21. Let E be an uncountable subset of **R** and let

$$E = \bigcup_{n=1}^{\infty} A_n,$$

where each A_n is a subset of **R**. Then :

(A) at least one A_n contains a rational number

(B) at least one A_n contains an irrational number

(C) at least one A_n contains an open interval

(D) no A_n is countable.

22. Let

$$f(x) = \begin{cases} x, & x \in [0, 1] \text{ and } x \text{ is rational,} \\ -x, & x \in [0, 1] \text{ and } x \text{ is irrational.} \end{cases}$$

Then :

(A) all discontinuities of f in $[0, 1]$ are of the first kind

(B) all discontinuities of f in $[0, 1]$ are of the second kind

(C) f is continuous in $[0, 1]$

(D) f is discontinuous at all points in $[0, 1]$.

23. Let

$$f(x) = \begin{cases} x + 1 & \text{if} \quad -1 \le x \le 0, \\ -x + 1 & \text{if} \quad 0 < x \le 1. \end{cases}$$

Then :

(A) f is continuous on $[-1, 1]$, is of bounded variation on $[-1, 1]$ and its total variation on $[-1, 1]$ is 2

(B) f is of bounded variation on $[-1, 1]$ and its total variation on $[-1, 1]$ is 1

(C) f is continuous on $[-1, 1]$, but is not of bounded variation

(D) f is of bounded variation on $[-1, 1]$ but is not continuous.

24. Let d be the discrete metric on **R**, i.e. $d(x, y) = 0$, if $x = y$ and $d(x, y) = 1$ if $x \neq y$. Let

$$a_n = \frac{1}{n}, \; n = 1, 2, \ldots$$

Then :

(A) $\{a_n\}$ is neither bounded, nor convergent in $(\mathbf{R}, d)$

(B) $\{a_n\}$ is both bounded and convergent in $(\mathbf{R}, d)$

(C) $\{a_n\}$ is convergent but not bounded in $(\mathbf{R}, d)$

(D) $\{a_n\}$ is bounded, but not convergent in $(\mathbf{R}, d)$.

25. Which of the following functions has a zero at $z = \dfrac{29\pi}{4}$?

(A) $\sin z + \cos z$ (B) $\sin z - \cos z$

(C) $\sin 2z$ (D) $\sin \dfrac{z}{2}$.

26. A Mobius transformation is completely determined by its effect on how many points ?

(A) Two (B) Three

(C) Four (D) Five.

27. Which of the following is the Taylor series of $\sin 2z$ in powers of $(z - \pi)$?

(A) $1 - \dfrac{(z-\pi)^2}{2!} + \dfrac{(z-\pi)^3}{3!} - \dots$ (B) $2(z-\pi) + \dfrac{4}{3}(z-\pi)^3 + \dots$

(C) $(z-\pi) + \dfrac{4}{3!}(z-\pi)^3 + \dots$ (D) $\dfrac{(z-\pi)^2}{2!} + \dfrac{4(z-\pi)^3}{3!} + \dots$

28. If

$$x + \frac{1}{x} = 2\cos\theta \text{ and } x - \frac{1}{x} = 2i\sin\theta,$$

then $x^n + \dfrac{1}{x^n}$ is equal to :

(A) $2^n \cos\theta$ (B) $2(\cos\theta)^n$

(C) $2\cos n\theta$ (D) $(2\cos\theta)^n$.

29. $\sin \dfrac{1}{z}$ has :

(A) no singularities in the plane

(B) has only a pole at 0

(C) has an essential singularity only at 0

(D) has essential singularities at $z = \pm n\pi$ for all integers n.

30. Let R be the ring of all 2×2 matrices with integer entries. Let

$$S = \left\{ \begin{bmatrix} 0 & a \\ 0 & b \end{bmatrix} \middle| \; a, b \text{ integers} \right\}$$

Then :

(A) S is not an ideal of R

(B) S is only a left ideal of R

(C) S is only a right ideal of R

(D) S is both a left and a right ideal of R.

31. Which of the following are correct ?

(1) Every ideal of the ring Z of all integers is principal

(2) Every principal ideal of Z is a prime ideal

(3) Every prime ideal of Z is a maximal ideal

(A) Only (1) and (2)

(B) Only (2) and (3)

(C) Only (3) and (1)

(D) All of these.

32. H is a subgroup of G and every coset of H in G is a subgroup of G. Then :

(A) H = {e}

(B) H = G

(C) G must have prime order

(D) H must have prime order.

33. G is a group and H_1 and H_2 are subgroups of orders 24 and 60. What is the maximum possible order of the subgroup $H_1 \cap H_2$?

(A) 1

(B) 3

(C) 12

(D) 24.

34. Let $X_1, X_2, \ldots\ldots, X_p$ be p vectors with n components each, and let M be the matrix with these p vectors as row vectors. Then the p vectors $X_1, X_2, \ldots\ldots, X_p$ are :

(A) linearly independent if M has rank less than p

(B) linearly independent if M has rank less than p, and they are linearly dependent if M has rank p

(C) linearly dependent if M has rank p

(D) linearly independent if M has rank p, and they are linearly dependent if M has rank less than p.

35. Let $F : R^3 \to R$ be a map defined by $F(x, y, z) = 3x - 2y + z$. Then :

(A) F is not a linear map

(B) F is linear and the kernel of F has dimension 1

(C) the kernel of F is a vector space of dimension greater than 2

(D) the kernel of F is the set of all vectors in R^3 which are orthogonal to the vector $3\hat{i} - 2\hat{j} + \hat{k}$.

36. Let $F : R^2 \to R^2$ be the linear map such that $F(x, y) = (3x - y, 4x + 2y)$. Then :

(A) F is neither injective nor surjective

(B) F is injective but not surjective

(C) F is not injective but it is surjective

(D) F is an isomorphism.

37. The matrix associated with the linear transformation $F : R^2 \to R^2$ given by $F(x, y) = (4x + y, -x + 2y)$:

(A) is skew-symmetric

(B) has an eigenvalue whose algebraic multiplicity is equal to its geometric multiplicity

(C) has an eigenvalue whose algebraic multiplicity is not equal to its geometric multiplicity

(D) is diagonalizable.

38. The singular solution of the differential equation

$$x^6\left(\frac{dy}{dx}\right)^2 - 2x\frac{dy}{dx} - 4y = 0,$$

is given by :

(A) $2x^2y^2 = -1$

(B) $4x^4y = -1$

(C) $2xy^4 = -1$

(D) $4x^4y^4 = -1.$

39. Using the method of variation of parameters the particular integral y_p of the differential equation

$$\frac{d^2y}{dx^2} + 9y = \operatorname{cosec} 3x,$$

can be expressed as $y_p = A(x)\cos 3x + B(x)\sin 3x$, where $A(x)$ and $B(x)$ are given by :

(A) $A(x) = \int \sin 3x \operatorname{cosec} 3x\, dx,\ B(x) = -\int \cos 3x \operatorname{cosec} 3x\, dx$

(B) $A(x) = -\frac{1}{3}\int \sin 3x \cos 3x\, dx,\ B(x) = \int \frac{\cos 3x \operatorname{cosec} 3x}{3} dx$

(C) $A(x) = -\frac{1}{3}\int \sin 3x \operatorname{cosec} 3x\, dx,\ B(x) = \frac{1}{3}\int \cos 3x \operatorname{cosec} 3x\, dx$

(D) $A(x) = \int \sin 3x \cos 3x\, dx,\ B(x) = \int \cos 3x \operatorname{cosec} 3x\, dx.$

40. The roots of the auxiliary equation for a homogeneous linear differential equation with real, constant coefficients that has $y = 4 + 2x^2 - e^{-3x}$ as a particular solution, are :

(A) $m = 0, 0, 0, -3$

(B) $m = 0, 0, 2, -3$

(C) $m = 4, 0, 0, 3$

(D) $m = 0, 0, 0, 3.$

41. The form of the particular solution y_p that can be examined for the solution of the differential equation

$$(D^2 + 3D)\, y = -18x$$

is :

(A) $y_p = C_1 + C_2x + C_3e^{-3x}$

(B) $y_p = C_1 + C_2x + C_3e^{3x}$

(C) $y_p = C_1 + C_2x + C_3x^2 + C_4e^{3x}$

(D) $y_p = C_1 + C_2x + C_3x^2 + C_4e^{-3x}$.

42. The differential equation

$$(x^3 + y^3)\, dx + 3xy^2 dy = 0$$

is :

(A) exact, homogeneous and linear

(B) exact, non-homogeneous and linear

(C) inexact, homogeneous and non-linear

(D) exact, homogeneous and non-linear.

43. The general solution of the differential equation

$$\frac{\partial z}{\partial x} + 2xy^3 \frac{\partial z}{\partial y} = z^3$$

is of the form $F(u, v) = 0$, where $u(x, y, z) = C_1$ and $v(x, y, z) = C_2$, are given by :

(A) $u(x, y, z) = \frac{x^2}{2} + \frac{1}{y^2}$, $v(x, y, z) = x + \frac{1}{z^3}$

(B) $u(x, y, z) = x^2 + \frac{1}{2y^2}$, $v(x, y, z) = x + \frac{1}{2z^2}$

(C) $u(x, y, z) = \frac{x^2}{2} + \frac{1}{y^2}$, $v(x, y, z) = x - \frac{1}{2z^2}$

(D) $u(x, y, z) = x^2 - \frac{1}{2y^2}$, $v(x, y, z) = x - \frac{1}{2z^2}$.

44. The complete integral of

$$z\left(\frac{\partial z}{\partial x}\right)\left(\frac{\partial z}{\partial y}\right) = \frac{\partial z}{\partial x} + \frac{\partial z}{\partial y}$$

is given by :

(A) $\dfrac{z^2}{2(1+a)} = x + \dfrac{1}{a}y + b$ (B) $z^2(1+a) = 2x + \dfrac{1}{a}y + 2b$

(C) $z^2(1+a) = ax + 2y + 2b$ (D) $\dfrac{z^2}{(1+a)} = 2x + \dfrac{1}{a}y + b$.

45. Consider the following data set :

(x_i, y_i), $i = 1, 2, 3, 4, 5.$

i	1	2	3	4	5
x	10	11	12	13	14
y	31	29	27	25	23

Then :

(A) the correlation coefficient r_{xy} between X and Y is negative but not -1

(B) $r_{xy} = -1$

(C) $r_{xy} = 0$

(D) none of the above is true.

46. For a bivariate data set $\{(x_i, y_i), i = 1, 2, ..., n\}$ the correlation coefficient r was found to be -0.8. Then :

(A) both the regression coefficients are negative

(B) one regression coefficient is positive while the other is negative

(C) the product of the regression coefficients is -0.64

(D) both the regression coefficients are positive.

47. Let P (E) = 0.25 and P (F) = 0.80. Then which of the following statements must be true ?

I : $P(E \cap F) \leq 0.25$

II : $P(E \cap F) \geq 0.20$

III : $P(E \cap F) \geq 0.05$

(A) I only
(B) I and II only
(C) I and III only
(D) II and III only.

48. X is a random variable with E (X) = 3 and E (X^2) = 13. Then the lower bound for $P(-2 < X < 8)$ using Chebyshev's inequality is :

(A) 0.05
(B) 0.40
(C) 0.95
(D) 0.84.

49. The moment generating function of a degenerate random variable is $m(t) = 2^t$ for real t. Then the rth moment of X about the origin is equal to :

(A) ln 2
(B) $(\ln 2)^r$
(C) $\frac{(\ln 2)^r}{r!}$
(D) 1.

50. Which of the following statements is correct ?

I : The standard deviation of a random variable can take any value in the set R of real numbers.

II : The partial correlation coefficient $r_{12.3}$ is defined as

$$\frac{r_{12} - r_{13}r_{23}}{(1-r_{13}^2)(1-r_{23}^2)}.$$

III : For a random variable X with mean $\mu \neq 0$, the value of real constant b for which $E(X-b)^2$ is minimum is 0.

IV : For a random variable X, the value of real constant a for which $E|X-a|$ is minimum is equal to the median of the distribution of X.

(A) IV
(B) I
(C) II
(D) III.

51. In answering a question on a multiple choice test, an examinee either knows the answer with probability p $(0 < p < 1)$ or he guesses with probability $1 - p$. Suppose that the probability of answering the question correctly is 1 for an examinee who knows the answer and $1/m$ for one who guesses, m (> 1) being the number of multiple choice answers.

Assuming that an examinee answers a question correctly, the probability that he knows the answer is :

(A) $\dfrac{p}{1 + (m - 1)p}$ (B) $\dfrac{m - 1}{m}$

(C) $\dfrac{mp}{1 + (m - 1)p}$ (D) $\dfrac{mp}{1 + (m + 1)p}$.

52. Let the p.d.f. of random variable Y be $f(y)$. Then the p.d.f. of the random variable $X = |Y|$ in the region $[0, \infty)$ is :

(A) $g(y) = f(y) + f(-y)$ (B) $g(y) = f(y) - f(-y)$

(C) $g(y) = \dfrac{f(y) + f(-y)}{2}$ (D) $g(y) = \dfrac{f(y) - f(-y)}{2}$.

53. Let X follow $N_p(0, \Sigma)$ distribution. Consider the following two statements :

S_1 : Every linear combination $\underset{\sim}{l}'\underset{\sim}{x}$ follows univariate normal distribution,

S_2 : Every quadratic form $X'AX$ follows chi-square distribution.

Then :

(A) only S_2 is true (B) only S_1 is true

(C) both are true (D) both are false.

54. Let $X_1, X_2, \ldots\ldots, X_n$ be a random sample from a p.d.f. $f(\cdot)$ with c.d.f. $F(\cdot)$. Let $Y_1 \le Y_2 \le \ldots\ldots\ldots \le Y_n$ denote corresponding ordered statistics. Then

$$f_{Y_\alpha}(y) \text{ for } \alpha = 1, 2, \ldots\ldots, n$$

is :

(A) $\binom{n}{\alpha} (F(y))^{\alpha} (1 - F(y))^{n-\alpha} \cdot f(y)$ (B) $\dfrac{n!}{(\alpha - 1)!(n - \alpha)!} F(y)^{\alpha - 1} (1 - F(y))^{n-\alpha} f(y)$

(C) $\binom{n}{\alpha} (F(y))^{\alpha} (1 - F(y))^{n-\alpha}$ (D) $\dfrac{n!}{(\alpha - 1)!(n - \alpha)!} (F(y))^{\alpha - 1} \cdot (1 - F(y))^{n-\alpha}$.

55. Let random variable X follow exponential with mean $1/\theta$ ($\theta > 0$). Then the median of the distribution is :

(A) $\theta \ln 2$

(B) $\frac{1}{\theta} \ln 2$

(C) $\frac{\theta}{2} \ln 2$

(D) $e^{-\frac{1}{2}\theta}$.

56. Let X be distributed as beta variate with

$$P_{a,b}(x) = \frac{1}{\beta(a, b)} x^{a-1}(1 - x)^{b-1} \qquad \begin{array}{c} 0 < x < 1 \\ a, b > 0 \end{array}$$

Let $Y = -\log_e X$. Then the distribution of Y, if $b = 1$, is :

(A) exponential distribution with mean $\frac{1}{a}$

(B) exponential distribution with mean a

(C) exponential distribution with mean $\frac{1}{a + 1}$

(D) exponential distribution with mean $a + 1$.

57. Consider the two statements :

(a_1) Locally most powerful test is to be used when the UMP test for the problem does not exist.

(a_2) For those distributions which do not possess monotone likelihood ratio, the UMP does not exist. We, therefore, search for the test which has larger power than any other test, of same size, in the localised region.

Choose your answer from the following :

(A) Both (a_1) and (a_2) are correct and (a_2) is the correct explanation of (a_1)

(B) Both (a_1) and (a_2) are correct but (a_2) is not the correct explanation of (a_1)

(C) (a_1) is true but (a_2) is false

(D) (a_1) is false but (a_2) is true.

58. In the sequential testing, the Wald's fundamental identity provides the expression for :

(A) OC function only when $E(z) \neq 0$

(B) ASN function only when $E(z) \neq 0$

(C) both OC and ASN functions only when $E(z) \neq 0$

(D) both OC and ASN functions when $E(z) = 0$ as well as $E(z) \neq 0$.

59. The multiple correlation coefficient measures the strength of relationship between :

(A) two sets of random variables

(B) two random variables

(C) two random variables after eliminating the effect of other variables

(D) a random variable and a set of other random variables.

60. Let X be an observation from a normal distribution with mean zero and variance σ^2.

Which of the following statements is *false* ?

(A) X is sufficient for σ

(B) $\left(\sqrt{\frac{\pi}{2}}\right) |X|$ is unbiased for σ

(C) $\left(\sqrt{\frac{\pi}{2}}\right) |X| + aX$ is biased for σ for any real $a \neq 0$

(D) Unbiased estimator for σ^2 is X^2.

61. Let X be distributed as $N(\theta, 1)$ and let for $\theta = 0$, $P(|X| > 1.96) = 0.05$. The best critical region based on X for testing the null hypothesis $H_0 : \theta = 0$ versus the alternative hypothesis $H_1 : \theta > 0$ is given by $X > 1.96$. If $P(X > 1.96 | H_0) = \alpha$, then the value of α is :

(A) 0.01

(B) 0.025

(C) 0.05

(D) 0.10.

62. In order to test randomness of a series of observations the appropriate non-parametric test is :

(A) Sign test
(B) Mann-Whitney test
(C) Run test
(D) Wilcoxon rank test.

63. In M/M/1 standard queueing model, which of the following r.v.s has a distribution which is a mixture of a degenerate and gamma distributions ?

(A) The number of customers in the queue
(B) The number of customers in the system
(C) The time spent by a customer outside the service counter
(D) The time spent by a customer in the system.

64. In the deterministic elementary inventory model with fixed demand and instantaneous supply, the EOQ formula is independent of :

(A) holding cost
(B) ordering cost
(C) cost of the item
(D) none of these.

65. Consider the two person zero sum game with pay-off a_{ij} to player 1 if player 1 chooses strategy i and the player 2 chooses strategy j, $i = 1, 2, \ldots\ldots, m$; $j = 1, 2, \ldots\ldots, n$. Suppose :

$$a_{rk} = \max_i \min_j a_{ij} = \min_j \max_i a_{ij}.$$

Consider the two statements :

S1 : If player 2 chooses k, then player 1 cannot get more than a_{rk},

S2 : If player 1 chooses r, then he is sure of getting at least a_{rk}.

Then :

(A) S1 is true but S2 is false
(B) S2 is true but S1 is false
(C) both S1 and S2 are true
(D) both S1 and S2 are false.

66. Suppose that a variable x_j in a L.P.P. is unrestricted in sign. Then the jth constraint of the dual L.P.P. is :

(A) unrestricted in sign (B) a strict inequality

(C) a strict equality (D) is unbounded.

67. For estimating proportion of population units possessing a particular attribute based on a SRSWR sample, the estimator for the $V(\hat{p})$ is given by :

(A) $\hat{V}(\hat{p}) = \frac{\hat{p}(1-\hat{p})}{n}$ (B) $\hat{V}(\hat{p}) = \frac{\hat{p}(1-\hat{p})}{n-1}$

(C) $\hat{V}(\hat{p}) = \frac{\hat{p}(1-\hat{p})}{n-3}$ (D) $\hat{V}(\hat{p}) = \frac{\hat{p}(1-\hat{p})}{n(n-1)}$.

68. The two-stage sampling designs are preferred to corresponding unistage sampling designs because :

(A) these provide more efficient estimators than that of unistage designs

(B) unistage designs are not suitable for large scale surveys

(C) the sampling frame of ultimate units is not readily available

(D) it requires less number of trained persons than unistage designs.

69. The value of error S.S. in the following A.V. table for CRD is :

Source	d.f.	S.S.	M.S.S.
Treatments	4		$3p$
Error			p
Total	9	425	

(A) 300 (B) 200

(C) 125 (D) 150.

70. Which of the following is *not* a basic principle to be adopted in designs of experiments ?

(A) Fractional replication (B) Randomization

(C) Local control (D) Maximum number of replications.

ROUGH WORK

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