Signature of Invigilators

1. ....

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(In words)

MATHEMATICAL SCIENCES

Paper II

D/03/1

Name of the Areas/Section (if any) ....

Time Allowed : 75 Minutes] [Maximum Marks : 100

Instructions for the Candidates

1. Write your Roll Number in the space provided on the top of this page.
2. This paper consists of *seventy four* **(74)** multiple choice type questions. Attempt any *fifty* **(50)** questions.
3. Each item has upto four alternative responses marked (A), (B), (C) and (D). The answer should be a capital letter for the selected option. The answer letter should entirely be contained within the corresponding square.

   Correct method $\boxed{A}$ Wrong Method $\boxed{\ }$A or $\boxed{A}$
4. Your responses to the items for this paper are to be indicated on the ICR Answer Sheet under paper II only
5. Read instructions given inside carefully.
6. One sheet is attached at the end of the booklet for rough work.
7. You should return the test booklet to the invigilator at the end of paper and should not carry any paper with you outside the examination hall.

પરીક્ષાર્થીઓ માટેની સૂચનાઓ :

૧. આ પાનાની ટોચમાં દર્શાવેલી જગ્યામાં તમારો રોલ નંબર લખો.

૨. આ પ્રશ્નપત્રમાં **ચવોહતેર (74)** બહુવૈકલ્પિક ઉત્તરોવાળા પ્રશ્નો છે. કોઈપણ **(50)** પ્રશ્નોના જવાબ આપો.

૩. પ્રત્યેક પ્રશ્ન વધુમાં વધુ ચાર બહુવૈકલ્પિક ઉત્તરો ધરાવે છે. જે (A), (B), (C) અને (D) વડે દર્શાવવામાં આવ્યા છે. પ્રશ્નનો ઉત્તર કેપીટલ સંજ્ઞા વડે આપવાનો રહેશે. ઉત્તરની સંજ્ઞા આપેલ ખાનામાં બરાબર સમાઈ જાય તે રીતે લખવાની રહેશે.

   ખરી રીત : $\boxed{A}$ ખોટી રીત : $\boxed{\ }$A , $\boxed{A}$

૪. આ પ્રશ્નપત્રના જવાબ આપેલ ICR Answer Sheetના Paper II વિભાગની નીચે આપેલ ખાનાંઓમાં આપવાના રહેશે.

૫. અંદર આપેલ સૂચનાઓ કાળજીપૂર્વક વાંચો.

૬. આ બુકલેટની પાછળ આપેલું પાનું રફ કામ માટે છે.

૭. પરીક્ષાસમય પૂરો થઈ ગયા પછી આ બુકલેટ જે તે નિરીક્ષકને સોંપી દેવી. કોઈ પણ કાગળ પરીક્ષાખંડની બહાર લઈ જવો નહી.

SEAL

MATHEMATICAL SCIENCES

PAPER-II

Note :—This paper contains **seventy four (74)** multiple choice questions, each carrying **two (2)** marks. Attempt any **fifty (50)** questions.

1. Let

$$a_n = \frac{1}{n+1} + \frac{1}{n+2} + \ldots\ldots + \frac{1}{2n}.$$

(A) $\{a_n\}$ is increasing and unbounded

(B) $\{a_n\}$ is bounded, but not convergent

(C) $\{a_n\}$ is convergent, but not bounded

(D) $\{a_n\}$ is increasing and bounded

2. Let $f : [0, 1] \to [0, 1]$ be continuous.

(A) There exists no $x \in [0, 1]$ such that $f(x) = x$

(B) For some $x \in [0, 1]$, $f(x) = x$

(C) For some $x \in [0, 1]$, $f(x) = 0$

(D) For some $x \in [0, 1]$, $f'(x) = 1$

3. Let

$$\alpha = \int_0^{\pi} \frac{x}{5 + 3\cos x}\, dx \quad \text{and} \quad \beta = \frac{16\alpha}{\pi^2}$$

Which of the following is definitely *true* ?

(A) $1 \le \beta \le 4$

(B) $0 \le \beta \le 2$

(C) $2 \le \beta \le 5$

(D) $1 \le \beta \le 3$

4. Let $f_n(x) = x^n \quad (0 \le x \le 1)$.

(A) $\{f_n\}$ converges uniformly on $[0, 1]$

(B) $\{f_n\}$ converges pointwise on $[0, 1]$ and the limit function is continuous on $[0, 1]$

(C) $\{f_n\}$ converges pointwise on $[0, 1]$, but the limit function is not continuous on $[0, 1]$

(D) $\{f_n\}$ does not converge pointwise on $[0, 1]$

5. If P and Q are points on the unit circle corresponding to complex numbers

$$\frac{1}{2} + i\frac{\sqrt{3}}{2} \text{ and } -\frac{1}{2} + i\frac{\sqrt{3}}{2}$$

respectively, what is the measure of $\angle POQ$?

(A) 30°

(B) 60°

(C) 45°

(D) 90°

6. The constant term in the Laurent series expansion of

$$\frac{1 - \sin z}{z^3}$$

is :

(A) $-\frac{1}{6}$

(B) $\frac{1}{6}$

(C) 1

(D) -1

7. The residue of the function

$$\frac{z^2 - 3z + 4}{(z - 2003)^3}$$

at its pole is :

(A) 2003

(B) 0

(C) -1

(D) 1

8. Let

$$P = \begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix}; \; Q = \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix}; \; R = \begin{bmatrix} 3 & 0 & 2 \\ 4 & 0 & 1 \end{bmatrix} \text{ and } S = \begin{bmatrix} 1 & 2 \\ 6 & 5 \\ -5 & 10 \end{bmatrix}.$$

Then which one of the following expressions is undefined ?

(A) $P + RS + Q^TP$

(B) $(P + Q)^TR + (P - Q)S^T$

(C) $(RS)^{-1} + P^{-1}Q$

(D) $RS^T + PQ^{-1} + P^TQ$

9. Consider a vector space V over a field Q of rational numbers given by

$$V = \{a + b\sqrt{3} \mid a, b \in Q\}.$$

Which of the following is *not* a basis of V ?

(A) $\{1, \sqrt{3}\}$

(B) $\{5, \sqrt{3}\}$

(C) $\{0, 2\sqrt{3}\}$

(D) $\left\{1, \frac{\sqrt{3}}{2}\right\}$

10. Consider the system of equations

$$x + y + 3z = 2$$
$$x + 2y + 4z = 3$$
$$2x + 3y + az = b.$$

Then for what values of a and b the system has infinitely many solutions ?

(A) $a = 7; b = 5$ (B) $a = 5; b = 4$

(C) $a = 5; b = 5$ (D) $a = 7; b = 4$

11. If A and B are two $n \times n$ matrices and A . B = 0, then which one of the following statements is *false* ?

(A) If A is non-singular, then B = 0

(B) If B is non-singular, then A = 0

(C) The given hypothesis implies that either A = 0 or B = 0

(D) If $A \neq 0$ and $B \neq 0$, then Rank of A $< n$ and Rank of B $< n$

12. If A is 3×3 real matrix having eigenvalues 0, $2i$ and $-2i$, then matrix A can be :

(A) symmetric (B) skew-symmetric

(C) orthogonal (D) unitary

13. A batch of 16 microchips contains 4 chips which do not work. A quality control procedure randomly selects 3 distinct chips and rejects the batch if none of the three chips works. The probability that the batch will be accepted by the procedure is :

(A) $\frac{3}{16}$ (B) $\frac{139}{140}$

(C) 0 (D) None of these

14. Suppose E and F are independent events with positive probability. The probability that exactly one of them occurs is given by :

(A) P(E) + P(F) − 2P(E) P(F) (B) P(E) + P(F) − P(E) P(F)

(C) P(E) P(F) (D) min {P(E), P(F)}

15. A discrete random variable X takes values 0, 1 and 2 with probabilities $1 - 3p$, $2p$ and p respectively, where $0 < p < \frac{1}{3}$. If $E(X^2) = \frac{3}{2}$, then the value of E(X) is equal to :

(A) $\frac{1}{2}$

(B) 0

(C) $\frac{1}{3}$

(D) 1

16. A box contains 10 white and 15 black marbles. Ten marbles are selected at random without replacement. Hence the probability that x of the 10 marbles are white for $x = 0, 1, 2, \ldots\ldots, 10$ is given by :

(A) $\dfrac{\binom{10}{x}\binom{15}{10-x}}{\binom{25}{10}}$

(B) $\binom{10}{x}\left(\left(\frac{2}{5}\right)^x\left(\frac{3}{5}\right)^{10-x}\right)$

(C) $\dfrac{\binom{10}{x}}{\binom{25}{10}}$

(D) $\dfrac{\binom{10}{x}}{\binom{25}{x}}$

17. X is a random variable with $E(X) = \mu$ and $V(X) = 4$. Using Chebyshev's inequality upper bound for $P(|X - \mu| \geq 8)$ is equal to :

(A) $\frac{1}{16}$

(B) $\frac{1}{80}$

(C) $\frac{1}{4}$

(D) $\frac{1}{64}$

18. X_1, X_2 and X_3 are independent random variables and probability distribution of X_i for $i = 1, 2, 3$ is :

X_i	1	2
Prob.	$\frac{i}{i+1}$	$\frac{1}{i+1}$

If $Y = \prod_{i=1}^{3} X_i$, then E(Y) is equal to :

(A) 1

(B) 2

(C) $\frac{5}{2}$

(D) 5

19. Consider the L.P.P.

Maximize : $Z = 3x_1 + 9x_2$

Subject to :

$$x_1 + 4x_2 \le 8$$
$$x_1 + 2x_2 \le 4$$
$$x_1, x_2 \ge 0.$$

Then :

(A) the L.P.P. has no feasible solution

(B) the L.P.P. has an unbounded solution

(C) there is redundancy in the problem

(D) None of the above is true

20. Consider the L.P.P.

Maximize : $Z = 2x_1 + x_2$

Subject to :

$$x_1 + 2x_2 \le 5$$
$$x_1 + x_2 \le 4$$
$$x_1, x_2 \ge 0.$$

Then the optimal value is :

(A) 7

(B) 8

(C) 2.5

(D) 10

21. Let **C** be the collection of all bounded subsets of **R** and **D** be the collection of all unbounded subsets of **R**.

(A) Both **C** and **D** are countable

(B) **C** is countable, but **D** is not

(C) If $\mathbf{E} \in \mathbf{C}$, then $\mathbf{E}^{\mathbf{C}} \in \mathbf{D}$

(D) **C** is closed under finite union

22. Let d be the usual metric on **R**.

(A) (**R**, d) is complete, but **R** has no least upper bound property

(B) **R** has least upper bound property, but (**R**, d) is not complete

(C) (**R**, d) is not complete, nor **R** has least upper bound property

(D) (**R**, d) is complete and **R** has the least upper bound property.

23. Let $s_n = -1 + (-1)^2 + (-1)^3 + \ldots\ldots + (-1)^n$ and let

$$\alpha = \underline{\lim}_{n\to\infty} s_n, \ \beta = \overline{\lim}_{n\to\infty} s_n.$$

(A) $\alpha = -1, \beta = 1$

(B) $\alpha = -\infty, \beta = \infty$

(C) $\alpha = 1, \beta = 1$

(D) $\alpha = -1, \beta = 0$

24. Let f be a function from $[0, 1]$ to $\mathbf{R}$. Which of the following is *true* ?

(A) If f has an infinite number of discontinuities in $[0, 1]$, f is not monotonic on $[0, 1]$

(B) If f has a discontinuity of second kind in $[0, 1]$, f is not monotonic on $[0, 1]$

(C) If f has no discontinuity in $[0, 1]$, it is monotonic on $[0, 1]$

(D) If f is monotonic on $[0, 1]$, then f is continuous on $[0, 1]$

25. The horizontal strip in the z-plane bounded by the rays $y = 0, x \leq 0$ and $y = \pi, x \leq 0$ is mapped by $w = e^z$ in the w-plane into :

(A) a vertical strip

(B) a horizontal strip

(C) an oblique strip

(D) a semi-circular disc

26. $f(z) = u + iv$ is an analytic function $u(x, y) = \sin x \cosh y$ and $v(0, 0) = 1$. Then $v(x, y)$ is :

(A) $\cos x \sinh y$

(B) $\cos x \sinh y + 1$

(C) $-\cos x \sinh y$

(D) $-\cos x \sinh y + 1$

27. A Mobius transformation is completely determined by its effect on how many points ?

(A) Two

(B) Three

(C) Four

(D) Five

28. The function e^z fails to be conformal at :

(A) 0

(B) i

(C) $2\pi i$

(D) no point in the z-plane

29. The value of

$$\int_C \frac{z^2 + 3z}{(z - 1)^2}\, dz,$$

where C is $|z| = 2$, is :

(A) $2\pi i$

(B) $10\pi i$

(C) $12\pi i$

(D) 0

30. Consider the following functions defined for all non-zero complex numbers :

$$f_1(z) = z,\ f_2(z) = -z,\ f_3(z) = \frac{1}{z},\ f_4(z) = -\frac{1}{z}.$$

Take the group $G = \{f_1, f_2, f_3, f_4\}$ under the composition of mappings. Then

$$f_2 \circ f_3^{-1} \circ f_4^{-1}$$

is :

(A) f_1

(B) f_2

(C) f_3

(D) f_4

31. G is a finite group. H and K are normal subgroups of G and quotient groups $G|H$ and $G|K$ have orders 6 and 4 respectively. Which of the following cannot be the order of G ?

(A) 12 (B) 18

(C) 24 (D) 36

32. All non-identity elements of a finite cyclic group have the same order. Which of the following cannot be the order of the group ?

(A) 17 (B) 19

(C) 21 (D) 23

33. Let Z $[x]$ be the ring of polynomials with integral coefficients. Let

$$R = \{f(x) \mid f(x) \in Z[x] \text{ and } f(1) = 0 = f(2)\}.$$

Then :

(A) R is not a subring of Z $[x]$

(B) R is a subring but not an ideal of Z $[x]$

(C) R is an ideal but not a maximal ideal of Z $[x]$

(D) R is a maximal ideal of Z $[x]$

34. Which of the following is/are integral domain/s with respect to the usual addition and multiplication of real numbers ?

(1) $R = \left\{a\sqrt{3} \mid a \text{ rational}\right\}$

(2) $S = \left\{a + b\sqrt{7} \mid a, b \text{ rational}\right\}$

(3) $T = \{17a \mid a \text{ integer}\}$

(A) Only R (B) Only S and T

(C) Only R and S (D) Only T and R

35. Let $\mathbf{R}^3$ be a vector space of ordered 3-tuples of real numbers and let S_1 and S_2 be as follows :

$$S_1 = \{(x, y, z) \in \mathbf{R}^3 \mid 2x + 3y + z = 0\} \text{ and}$$
$$S_2 = \{(x, y, z) \in S_1 \mid 3x - y + 2z = 0\}.$$

Then :

(A) Neither S_1 nor S_2 is a subspace of $\mathbf{R}^3$

(B) Both S_1 and S_2 are 2-dimensional subspaces of $\mathbf{R}^3$

(C) S_1 is 2-dimensional subspace of $\mathbf{R}^3$ and S_2 is one-dimensional subspace of $\mathbf{R}^3$

(D) S_1 is a subspace of $\mathbf{R}^3$ but S_2 is not a subspace of $\mathbf{R}^3$.

36. Let V and W be two vector spaces over a field K and $F : V \to W$ be a linear map. Then F is an isomorphism if :

(A) $\dim V < \dim W$ and F is one-one

(B) $\dim V > \dim W$ and F is onto

(C) $\dim V = \dim W$

(D) $\dim V = \dim W$ and F is one-one

37. A linear transformation from $\mathbf{R}^3 \to \mathbf{R}^3$ is represented by a matrix

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}.$$

Then which one of the following equations is valid ?

(A) $2A^{-1} = -A^2 + 6A + 7I$

(B) $A^3 - 6A^2 - 7A + 2I = 0$

(C) $2A^{-1} = -A^2 - 6A - 7I$

(D) $A^4 - 6A^3 + 7A^2 + 2A = 0$

38. Suppose A is an $n \times n$ diagonalizable matrix and S and D are corresponding diagonalizing matrix and diagonal matrix respectively. Then which one of the following statements is *false* ?

(A) Diagonal elements of D are eigenvalues of A and column vectors of S are corresponding eigenvectors

(B) All eigenvalues of A must be distinct

(C) A has n linearly independent eigenvectors

(D) S also diagonalizes A^k for any +ve integer k

39. Which of the following differential equations is linear ?

(A) $xy'' + yy''' + \cos x = 0$

(B) $e^{xy}y' + 2y = \sin x$

(C) $x^{10}y'' + \dfrac{(\ln x + \tan x)^2}{x^3 + 10}y' + y = f(x)$

(D) $(y')^2 + 2y = x$

40. Which of the following differential equations is *not* exact ?

(A) $2xy\,dx + (x^2 + 3y^2)\,dy = 0$

(B) $3x(xy - 2) + (x^3 + 2y)\,dy = 0$

(C) $\{xy \sin(xy) + \cos(xy)\}\,y\,dx + \{xy \sin(xy) - \cos(xy)\}\,x\,dy = 0$

(D) $x^2y^3dx + x^3y^2dy = 0$

41. The singular solution of the differential equation

$$y = x\frac{dy}{dx} + a\left(\frac{dx}{dy}\right)$$

is :

(A) $x^2 = 2ay$

(B) $x^2 = 4ay$

(C) $y^2 = 4ax$

(D) $y^2 = 2ax$

42. If $D \equiv \dfrac{d}{dx}$, then

$$\frac{1}{(1 + D^2)}(x^2 - x + 2)$$

is :

(A) $x^2 + 2x$

(B) $x^2 - x$

(C) $x^2 + x$

(D) $x^2 - 2x$

43. Which of the following is a solution of the partial differential equation

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = u\ ?$$

(A) $u = y^2f(x/y)$

(B) $u = x^2f(y/x)$

(C) $u = yf(y/x)$

(D) $u = xf(y^2/x)$

44. The differential equation

$$x\frac{\partial^2 z}{\partial x^2} + 3\frac{\partial^2 z}{\partial x \partial y} + y\frac{\partial^2 z}{\partial y^2} + \frac{\partial z}{\partial y} = 0$$

is :

(A) elliptic for $xy < \frac{9}{4}$ and hyperbolic for $xy > \frac{9}{4}$

(B) elliptic for $xy \leq \frac{9}{4}$ and hyperbolic for $xy > \frac{9}{4}$

(C) elliptic for $xy > \frac{9}{4}$ and hyperbolic for $xy < \frac{9}{4}$

(D) elliptic for $xy > \frac{9}{4}$ and hyperbolic for $xy \leq \frac{9}{4}$

45. If in the bivariate normal distribution with parameters

$$(\mu_x, \mu_y, \sigma_x^2, \sigma_y^2, \rho), \sigma_x \neq \sigma_y \text{ and } \rho = 0$$

the distribution is :

(A) symmetrical normal

(B) degenerate normal on a line

(C) degenerate normal at a point

(D) not symmetric normal

46. If $\rho_{XY} = 1$, then :

(A) Y is proportional to X

(B) Y is inversely proportional to X

(C) Y is same as X

(D) X, Y are perfectly linearly related

47. Let A and B be events in a sample space. Then which of the following statements is *false* ?

(A) A and B are independent if and only if A and B′ are independent

(B) A and B′ are independent if and only if A′ and B are independent

(C) A and A′ are independent if A is a null set

(D) A and A′ are independent only if A is a null set.

48. Which of the following *cannot* be obtained as a consequence of central limit theorem ?

(A) Binomial distribution tends to normal

(B) Student's t distribution tends to normal

(C) Chi-square distribution tends to normal

(D) Gamma distribution tends to normal

49. Let X be a r.v. with $E(X) = \mu$ and $V(X) = 1$. Then which of the following is *true* ?

(A) $P[|X - \mu| \leq 2] \leq \frac{1}{4}$ (B) $P[|X - \mu| \leq 2] \geq \frac{3}{4}$

(C) $P[|X - \mu| \geq 2] \geq \frac{1}{4}$ (D) $P[X - \mu \geq 2] \leq \frac{1}{8}$

50. The joint probability distribution of X and Y is given below :

X \ Y	−1	0	1
−2	1/8	1/8	1/8
0	1/8	0	1/8
2	1/8	1/8	1/8

Which of the following is *true* ?

(A) X and Y are identically distributed

(B) X and Y are not independent

(C) X is a symmetric r.v. but Y is not

(D) $E(XY) = 0$

51. Which of the following is called Markov inequality ?

(A) $(E(X))^2 > E(X^2)$

(B) $(E(XY))^2 > E(X^2)\, E(Y^2)$

(C) $P(|X| > C) \leq E(|X|)/C,\ C > 0$

(D) $E(|X + Y|) \leq E(|X|) + E(|Y|)$

52. Let X_1, X_2, ..., X_n be i.i.d. exponential random variable. Then $\min\limits_{1 \le i \le n} X_i$ follows :

(A) uniform distribution

(B) exponential distribution

(C) normal distribution

(D) Chi-square distribution

53. If X has a uniform distribution over (0, 1), then the joint distribution of X and Y = 1 − X is :

(A) degenerate at (1/2, 1/2)

(B) concentrated on a line

(C) uniform over the square $(0, 1) \times (0, 1)$

(D) degenerate at (1, 1)

54. If X_1 and X_2 are two independent Poisson variates with parameters λ_1 and λ_2 respectively, then consider the two statements :

S_1 : $\dfrac{X_1 + X_2}{2}$ is distributed as Poisson with $\dfrac{\lambda_1 + \lambda_2}{2}$

S_2 : $X_1 + X_2$ is distributed as Poisson with $(\lambda_1 + \lambda_2)$

Choose your answer from the following :

(A) Both S_1 and S_2 are correct

(B) Only S_1 is correct

(C) Only S_2 is correct

(D) Neither S_1 nor S_2 is correct

55. The Chi-square distribution is a particular case of :

(A) Gamma distribution

(B) Beta I distribution

(C) Exponential distribution

(D) Beta II distribution

56. If $X_{(1)}$, $X_{(2)}$, ...$X_{(n)}$ is an ordered sample from $U(\theta, \theta + 1)$, then which of the following statements is *not* true ?

(A) $(X_{(1)}, X_{(n)})$ is sufficient statistic for θ

(B) $(X_{(1)}, X_{(n)})$ is complete sufficient statistic for θ

(C) $(X_{(1)}, X_{(n)})$ is minimal sufficient statistic for θ

(D) $(X_{(1)}, X_{(n)})$ is minimal sufficient but not complete for θ

57. While testing the hypothesis $H_0 : \mu = \mu_0$ against $H_1 : \mu \neq \mu_0$ on the basis of a random sample from normal distribution $N(\mu, \sigma^2)$, the H_0 was rejected at 5% level of significance. The corresponding P value is :

(A) $> .05$

(B) $< .05$

(C) $> .025$

(D) $.025 < P < .05$

58. $X_1, X_2, \ldots\ldots X_n$ be i.i.d. with p.d.f.

$$f(x) = \frac{\theta}{x^{\theta+1}} \qquad x > 1, \theta > 1$$

$$= 0 \qquad \text{otherwise}$$

The moment estimator of θ is :

(A) $\bar{x}$

(B) $\frac{1}{\bar{x}}$

(C) $\frac{\bar{x}}{\bar{x} - 1}$

(D) $\frac{\bar{x} - 1}{\bar{x}}$

59. $X_1, X_2, \ldots\ldots X_n$ be i.i.d. $N(\mu, \sigma^2)$. The most powerful test for testing $H_0 : \sigma^2 = \sigma_0^2$ *Vs.* $H_1 : \sigma^2 = \sigma_1^2$ is based on the statistic :

(A) $\Sigma(X_i - \bar{X})^2$

(B) ΣX_i^2

(C) $\Sigma(X_i - \mu)^2$

(D) $\sqrt{\Sigma X_i^2}$

60. (X_i, Y_i) $i = 1\ldots\ldots\ldots n$ is a random sample of n pairs from the p.d.f.

$$f(x, y) = e^{-\left(\theta x + \frac{y}{\theta}\right)} \qquad x > 0, y > 0, \theta > 0$$

$$= 0 \qquad \text{otherwise}$$

Which of the following estimators is sufficient for θ ?

(A) $\left(\Sigma\frac{1}{X_i}, \Sigma Y_i\right)$

(B) $(\Sigma X_i, \Sigma Y_i)$

(C) $\left(\Sigma X_i, \Sigma\frac{1}{Y_i}\right)$

(D) $\left(\Sigma\frac{1}{X_i}, \Sigma\frac{1}{Y_i}\right)$

61. Let r be the correlation coefficient between X and Y. Then the correlation coefficient between (1 + X) and (1 − 2Y) is :

(A) $1 - 2r$ (B) $-r$

(C) r (D) $2r$

62. Let (x_i, y_i) $i = 1......n$ be a random sample from a bivariate normal distribution with parameters 0, 0, σ_1, σ_2 and ρ. The hypothesis $H_0 : \sigma_1^2 = \sigma_2^2$ can be tested using :

(A) Chi-square test (B) Student's t-test

(C) z-test (D) None of these

63. Chi-square test of goodness of fit was first proposed by :

(A) R.A. Fisher (B) A. Wald

(C) K. Pearson (D) C.R. Rao

64. In the two-way classification model

$$y_{ij} = \mu + \alpha_i + \beta_j + \gamma_{ij} + e_{ij}; \qquad i = 1......r$$
$$j = 1......c$$

the degrees of freedom for the error are :

(A) $(r - 1)(c - 1)$ (B) $r(c - 1)$

(C) $c(r - 1)$ (D) None of these

65. Consider the elementary inventory model with deterministic demand, cost of carrying inventory C_I, cost of ordering C_0 and cost of a unit item C. If supply is instantaneous, the optimal order quantity depends on :

(A) all the 3 costs (B) only C_I and C_0

(C) only C_I and C (D) only C_0 and C

66. Customers arrive at a service station in a Poisson fashion with 4 customers per hour. Then the probability of no arrivals between noon and 12.30 P.M. is :

(A) $1 - e^{-2}$ (B) e^{-2}

(C) e^{-4} (D) $1 - e^{-4}$

67. In the standard M|M|1 model the waiting time of a customer who waits is :

(A) an exponential r.v.

(B) a gamma r.v.

(C) a mixture of an exponential and a degenerate r.v.

(D) a Poisson r.v.

68. Consider the problem of minimising $-w_1 + w_2$ subject to the constraints

$$w_1 - w_2 \geq 1, \; w_1 \geq 0, \; w_2 \geq 0.$$

Which of the following statements is *false* ?

(A) The minimising problem has an unbounded solution

(B) The problem has a unique solution

(C) (1, 0) is a feasible solution of the given problem

(D) The convex set of feasible solutions of the problem is unbounded

69. Consider a transportation problem with 4 sources of supply of an item and 4 places at which demand for the item exists :

Supply : $a_1 = 10, a_2 = 4, a_3 = 5, a_4 = 2$

Demand : $D_1 = 10, D_2 = 7, D_3 = 5, D_4 = 7.$

Then to balance the model we have to introduce :

(A) a dummy demand location with demand 8

(B) a dummy demand location with demand -8

(C) a dummy source with availability 8

(D) a dummy source with unlimited availability

70. Consider the game defined by

2	0
0	2

The value of the game is :

(A) 0 (B) 2

(C) 3 (D) 1

71. Consider the following two statements :

A : The stratified sampling is always more efficient than SRSWOR.

R : By stratifying the population we get groups of utmost homogeneity and therefore, within strata variance is minimised.

Choose your answer from the following :

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is not the correct explanation of A

(C) A is true but R is false

(D) A is false but R is true

72. The two-stage sampling design is used in large scale surveys because :

(i) it provides more efficient estimator for population mean per element than one based on unistage design.

(ii) it is easier to collect information on second stage units.

(iii) the sampling frame of ultimate stage units is not readily available.

(iv) the cost of collecting information in two-stage is less as compared to that of unistage design.

Choose your answer from the following code :

(A) (i) and (ii) only
(B) (i) and (iii) only
(C) (i) and (iv) only
(D) (ii), (iii) and (iv) only

73. The number of replications in an experiment is based on :

(i) precision required

(ii) availability of experimental material.

(iii) heterogeneity of experimental material.

Choose your answer from the following code :

(A) (i) and (ii) only
(B) (i) and (iii) only
(C) (ii) and (iii) only
(D) (i), (ii) and (iii)

74. According to Prof. R.A. Fisher, if s^2 is the estimate of error based on $n > 3$ degrees of freedom, then the amount of information available from the experimental data is :

(A) $\frac{n+1}{n-1}\frac{1}{s^2}$
(B) $\frac{n+1}{n-3}\frac{1}{s^2}$
(C) $\frac{n-1}{n-3}\frac{1}{s^2}$
(D) $\frac{n-1}{n+3}\frac{1}{s^2}$

ROUGH WORK

ROUGH WORK

SEAL