

**INDIAN STATISTICAL INSTITUTE**

**STUDENTS' BROCHURE**

**B. MATH.(HONS.) PROGRAMME**

2008-09

8th MILE MYSORE ROAD

BANGALORE 560 059



INDIAN STATISTICAL INSTITUTE  
B. MATH.(HONS.) PROGRAMME

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# 1 GENERAL INFORMATION

## 1.1 Scope

The B. Math.(Hons.) degree programme offers comprehensive instruction in basic mathematics along with rudimentary courses in Probability, Statistics, Computing and Physics. It is so designed that on successful completion, the students would be able to pursue higher studies in the areas of Mathematics, Statistics, Computer Science, Mathematical Physics etc. or take up a career in applications of Mathematics. The students successfully completing the requirements for the B. Math. (Hons.) degree will automatically be admitted to the M. Math. programme.

## 1.2 Duration

The total duration of the B. Math.(Hons.) programme is three years (six semesters). An academic year, consisting of two semesters with a recess in-between, usually starts in July and continues till May. The classes are generally held only on the weekdays from 9.20/10.15 a.m. to 5.30 p.m. The time-table preferably will not have an off day in the beginning or the end of the week. There is a study-break of one week before the semestral examinations in each semester.

## 1.3 Centre

The B. Math. (Hons.) programme is offered at the Bangalore centre only.

## 1.4 Course Structure

The B. Math.(Hons.) programme has twenty eight one-semester credit courses, as given in the curriculum below in Section 2. Besides the above courses, a non-credit course on *Writing of Mathematics* is offered in the first semester of the first year.

## 1.5 Examinations

There are two examinations in each course: mid-semestral and semestral (final). The composite score in a course is a weighted average of the scores in the mid-semestral and semestral examinations, home-assignments, practical record-book, project work, etc. (announced at the beginning of the semester) For courses other than project / dissertation, the minimum weight given to the semestral examination is 50%.

The minimum composite score required for passing a course is 35%. This also applies to the (non-credit) *Writing of Mathematics* course offered in the first semester of the first year of B. Math.

There is a provision of backpaper examinations in all the courses. If the composite score of a student falls short of 45% in a credit course, or 35% in a non-credit course the student may take a back-paper examination to improve the score. A student is REQUIRED to take a backpaper examination if the composite score is less than 35%. At most one back-paper examination is allowed in a particular course. The maximum a student can score in a back-paper examination is 45%.

The ceiling on the total number of backpaper examinations is as follows: **4 in the first year, 4 in the second year, 2 in the final year.** A student may take more than the allotted quota of backpaper examinations in a given academic year, and decide at the end of that academic year which of the backpaper examination scores should be disregarded.

If a student misses the mid-semester or the semester examination due to medical or family emergency, the Teachers' Committee may, on an adequately documented representation from the student, allow him/her to take a supplementary examination in the course. The supplementary semester examination is held at the same time as the back-paper examinations for that semester and a student taking supplementary semester examination is not allowed to take a backpaper examination in the course. The maximum a student can score in a supplementary examination is 60%.

## 1.6 Scores

The composite score in a course is a weighted average of the scores in the mid-semester and semester examinations, home-assignments, and the practical record book (and/or project work) in that course. In the case of all courses which involve field work, some weight is given to the field reports also. The semester examination normally has a weight of at least 60%. The weights are announced beforehand by the Dean of Studies, or the Class Teacher, in consultation with the teacher concerned.

The minimum composite score to pass a credit or non-credit course is 35%.

When a student takes back-paper examination in a credit course, his/her final score in that course is the higher of the back-paper score and the earlier composite score, subject to a maximum of 45%.

When a student takes supplementary semester examination in a course, the maximum he/she can score in that examination is 60%. The score in the supplementary examination is used along with other scores to arrive at the composite score.

## 1.7 Satisfactory Conduct

Here and in what follows, copying in the examination, rowdyism or some other breach of discipline or unlawful/unethical behaviour etc. are regarded as unsatisfactory conduct.

## Examination Guidelines for Students

1. Students are required to take their seats according to the seating arrangement displayed. If any student takes a seat not allotted to him/her, he/she may be asked by the invigilator to hand over the answer script (i.e., discontinue the examination) and leave the examination hall.
2. Students are not allowed to carry inside the examination hall any mobile phone with them, even in switched-off mode. Calculators, books and notes will be allowed inside the examination hall only if these are so allowed by the teacher(s) concerned i.e., the teacher(s) of the course, or if the question paper is an open-note/ book one. Even in such cases, these articles cannot be shared.
3. No student is allowed to leave the examination hall without permission from the invigilator(s). Further, students cannot leave the examination hall during the first 30 minutes of any examination. Under no circumstances, two or more students writing the same paper can go outside together.
4. Students should ensure that the main answer booklet and any extra loose sheet bear the signature of the invigilator with date. Any discrepancy should be brought to the notice of the invigilator immediately. Presence of any unsigned or undated sheet in the answer script will render it (i.e., the unsigned or undated sheet) to be cancelled, and this may lead to charges of violation of the examination rules.
5. Any student caught cheating or violating examination rules for the first time will get Zero in that paper. If the first offence is in a backpaper examination the student will get Zero in the backpaper. (The other conditions for promotion, as mentioned in Section 1.8 in Students brochure, will continue to hold.) Students will not receive direct admission to the M. Math programme.
6. Any student caught cheating or violating examination rules for the second/third time will be denied promotion in that year. This means that
  - (i) a student in the final year of any programme and not already repeating, will have to repeat the final year without stipend;
  - (ii) a student in the final year of any programme and already repeating, will have to discontinue the programme;
  - (iii) a student not in the final year of any programme will have to discontinue the programme even if he/she was not repeating that year.

Any student caught cheating or violating examination rules twice or more will be denied further admission to any programme of the Institute.

The students shall observe the normal discipline of the Institute and shall not indulge in cheating in the examinations, rowdyism or any other act of indiscipline or unlawful/unethical/ indecent behaviour. There are also specific attendance requirements that the students are

expected to meet, details of which are mentioned in Section 1.11. Violations of these are likely to attract punishments such as withholding promotion/award of degree, withdrawing stipend and/or expulsion from the hostel/Institute.

Ragging is banned in the Institute and anyone found indulging in ragging will be given punishment such as expulsion from the Institute, or, suspension from the Institute/ classes for a limited period and fine. The punishment may also take the shape of (i) withholding Stipend/Fellowship or other benefits, (ii) withholding results, (iii) suspension or expulsion from hostel and the likes. Local laws governing ragging are also applicable to the students of the Institute. Incidents of ragging may be reported to the police.

## 1.8 Promotion

A student passes a semester of the programme only when he/she secures composite score of 35% or above in every course and his/her conduct has been satisfactory. If a student passes both the semesters in a given year, the specific requirements for promotion to the following year are as follows:

*First Year to Second Year:* Average composite score in all the credit courses taken in the first year is not less than 45%.

*Second Year to Third Year:* Average composite score in all the credit courses taken in the second year is not less than 40%.

No student is allowed to repeat B. Math.(Hons.) First Year or Second Year. Repetition of a year is allowed only in the final year of the programmes. The scores obtained during the repetition of the final year are taken as the final scores in the final year. A student is given only one chance to repeat the final year of the programme.

## 1.9 Final Result

At the end of the third academic year the overall average of the percentage composite scores in all the credit courses taken in the three-year programme is computed for each student. Each of the credit courses carries a total of 100 marks, while Statistics Comprehensive carries 200 marks. The student is awarded the B. Math.(Hons.) degree in one of the following categories according to the criteria he/she satisfies, provided his/her conduct is satisfactory, and he/she passes all the semesters.

### **B. Math.(Hons.) - First Division with distinction –**

- (i) The overall average score is at least 75%,
- (ii) average score in the sixteen core courses is at least 60%, and
- (iii) the number of composite scores less than 45% is at most four.

### **B. Math.(Hons.) - First Division**

- (i) Not in the First Division with distinction
- (ii) The overall average score is at least 60% but less than 75%,
- (iii) average score in the sixteen core courses is at least 60%, and
- (iv) the number of composite scores less than 45% is at most six

### **B. Math.(Hons.) - Second Division**

- (i) Not in the First Division with distinction or First Division,
- (ii) the overall average score is at least 45%,
- (iii) average score in the sixteen core courses is at least 45%, and
- (iv) the number of composite scores less than 45% is at most eight.

If a student has satisfactory conduct, passes all the courses but does not fulfill the requirements for the award of the degree with Honours, then he/she is awarded the B. Math. degree without Honours. A student fails if his/her composite score in any credit or non-credit course is less than 35%.

\* The sixteen core courses are Algebra I, II, III, IV; Analysis I, II, III, IV; Probability I, II; Optimization, Complex Analysis, Differential Equations, Introduction to Differential Geometry, Introduction to Representation Theory and Combinatorics & Graph Theory.

The students who fail, along with the students who secure B. Math. degree without Honours and have at most eight composite scores (in credit courses) less than 45% in the first two years, are allowed to repeat the final year of the B. Math.(Hons.) programme without stipend and contingency grant. The scores obtained during the repetition of the third year are taken as the final scores in the third year.

### **1.10 Award of Certificates**

A student passing the B. Math. degree examination is given a certificate which includes (i) the list of all the credit courses taken in the three-year programme along with their respective composite scores and (ii) the category (Hons. First Division with Distinction or Hons. First Division or Hons. Second Division or Pass) of his/her final result.

The Certificate is awarded in the Annual Convocation of the Institute following the last semestral examinations.

### **1.11 Class-Teacher**

One of the instructors is designated as the Class Teacher. Students are required to meet their respective Class Teachers periodically to get their academic performance reviewed, and to discuss their problems regarding courses.

## 1.12 Attendance

Every student is expected to attend all the classes. If a student is absent, he/she must apply for leave to the Dean of Studies or Academic Coordinator. Failing to do so may result in disciplinary action. Inadequate attendance record in any semester would lead to reduction of stipend in the following semester; see Section 1.13.

A student is also required to furnish proper notice in time and provide satisfactory explanation if he/she fails to take any mid-semester or semester examination.

## 1.13 Stipend

Stipend, if awarded at the time of admission, is valid initially for the first semester only. The amount of stipend to be awarded in each subsequent semester depends on academic performance, conduct, and attendance, as specified below, provided the requirements for continuation in the academic programme (excluding repetition) are satisfied; see Sections 1.6 and 1.7.

### 1. *Performance in course work*

If, in any particular semester, (i) the composite score in any course is less than 35%, or (ii) the composite score in more than one course (two courses in the case of the first semester of the first year) is less than 45%, or (iii) the average composite score in all credit courses is less than 45%, no stipend is awarded in the following semester.

If all the requirements for continuation of the programme are satisfied, the average composite score is at least 60% and the number of credit course scores less than 45% is at most one in any particular semester (at most two in the first semester of the first year), the full value of the stipend is awarded in the following semester.

If all the requirements for continuation of the programme are satisfied, the average composite score is at least 45% but less than 60%, and the number of credit course scores less than 45% is at most one in any particular semester (at most two in the first semester of the first year), the stipend is halved in the following semester.

All composite scores are considered after the respective back-paper examinations. Stipend is fully withdrawn as soon as the requirements for continuation in the academic programme are not met.

### 2. *Attendance*

If the overall attendance in all courses in any semester is less than 75%, no stipend is awarded in the following semester.

### 3. *Conduct*

The Dean of Studies or the Class Teacher, at any time, in consultation with the respective Teachers' Committee, may withdraw the stipend of a student fully for a specific period if his/her conduct in the campus is found to be unsatisfactory.

Note: Once withdrawn, stipends may be restored in a subsequent semester based on improved performance and/or attendance, but no stipend is restored with retrospective effect.

Stipends are given after the end of each month for eleven months in each academic year. The first stipend is given two months after admission with retrospective effect provided the student continues in the B. Math.(Hons.) programme for at least two months.

Contingency grants can be used for purchasing a scientific calculator and other required accessories for the practical class, text books and supplementary text books and for getting photocopies of required academic material. All such expenditure should be approved by the respective Class Teacher. No contingency grants are given in the first two months after admission. Every student is required to bring a scientific calculator for use in the practical classes. Calculators can be purchased with contingency grants.

#### **1.14 ISI Library Rules**

Any student is allowed to use the reading room facilities in the library and allowed access to the stacks. B. Math.(Hons.) students have to pay a security deposit of Rs.200/- in order to avail the borrowing facility. A student can borrow at most three books at a time.

Any book from the Text Book Library (TBL) collection may be issued out to a student only for overnight or week-end provided at least one copy of that book is left in the TBL. Only one book is issued at a time to a student. *Fine is charged if any book is not returned by the due date stamped on the issue-slip.* The library rules, and other details are posted in the library.

#### **1.15 Hostel Facility**

The Institute has hostel for students in the Bangalore campus. However, it may not be possible to accommodate all degree/diploma students in the hostels. Limited medical facilities are available free of cost at Bangalore campus.

#### **1.16 Expenses for the Field Training Programmes**

All expenses for the necessary field training programmes are borne by the Institute, as per the Institute rules.

### **1.17 Change of Rules**

The Institute reserves the right to make changes in the above rules, course structure and the syllabi as and when needed.

## 2 B. MATH.(HONS.) CURRICULUM

All the courses listed below except the course on Writing of Mathematics are allocated four *lecture* sessions per week.

### First Year

#### Semester I

1. Analysis I (Calculus of one variable)
2. Probability Theory I
3. Algebra I (Groups and Rings)
4. Physics I (Mechanics of particles and fluids)
5. Writing of Mathematics (non-credit course)

#### Semester II

1. Analysis II (Metric spaces and Multivariate Calculus)
2. Probability Theory II
3. Algebra II (Linear Algebra)
4. Physics II (Thermodynamics and Optics)

### Second Year

#### Semester I

1. Analysis III (Vector Calculus)
2. Algebra III (Rings and Modules)
3. Statistics I
4. Physics III  
(Electromagnetism and Electrodynamics)
5. Computer Science I (Programming)

#### Semester II

1. Analysis IV  
(Introduction to Function Spaces)
2. Algebra IV (Field Theory)
3. Statistics II
4. Optimization
5. Computer Science II (Numerical Methods)

### Third Year

#### Semester I

1. Complex Analysis
2. Introduction to Differential Geometry
3. Introduction to Differential Equations
4. Statistics III
5. Elective subject I

#### Semester II

1. Combinatorics and Graph theory
2. Introduction to Representation Theory
3. Physics IV  
(Modern Physics and Quantum Mechanics)
4. Elective subject II
5. Elective subject III

### 3 ELECTIVE COURSES

Elective subjects can be chosen from the following list:

1. Topology
2. Introduction to Algebraic Geometry
3. Introduction to Algebraic Number Theory
4. Differential Geometry II
5. Introduction to Differential Topology
6. Introduction to Dynamical systems
7. Probability III (Introduction to Stochastic Processes)
8. Statistics IV
9. Statistics V
10. Mathematics of Computation
11. Computer Science III (Data Structures)
12. Computer Science IV (Design and Analysis of Algorithms)

### 4 BRIEF SYLLABI OF THE B. MATH.(HONS.) COURSES

#### 4.1 Mathematics courses

**Algebra I (Groups and Rings)** : Basic set theory: Equivalence relations and partitions. Zorn's lemma. Axiom of choice. Principle of induction. Groups, subgroups, homomorphisms. Modular arithmetic, quotient groups, isomorphism theorems. Groups acting on sets. Sylow's theorems. Permutation groups. Rings, ideals. Fields. Ring homomorphisms, quotient rings, adjunction of elements. Polynomial rings.

**Algebra II (Linear Algebra)** : Matrices and determinants. Linear equations. Vector spaces over fields. Bases and dimensions. Direct sums and quotients of vector spaces. Linear transformations and their matrices. Eigen values and eigen vectors. Characteristic polynomial and minimal polynomial. Bilinear forms, inner products, symmetric, hermitian forms. Unitary and normal operators. Spectral theorems.

**Algebra III (Rings and Modules)** : Review of quotient rings, adjunction of elements. Chinese remainder theorem and applications. Factorisation in a ring. Irreducible and prime elements, Euclidean domains, Principal Ideal Domains, Unique Factorisation Domains. Field of fractions, Gauss's lemma. Noetherian rings, Hilbert basis theorem. Finitely generated modules over a PID and their representation matrices. Structure theorem for finitely generated abelian groups. Rational form and Jordan form of a matrix.

**Algebra IV (Field Theory)** : Finite Fields. Field extensions, degree of a field extension. Ruler and compass constructions. Algebraic closure of a field. Transcendental bases. Galois theory in characteristic zero, Kummer extensions, cyclotomic extensions, impossibility of solving quintic equations. Galois theory in positive characteristic (separability, normality). Separable degree of an extension.

**Introduction to Representation Theory:**

Introduction to multilinear algebra: Review of linear algebra, multilinear forms, tensor products, wedge product, Grassmann ring, symmetric product. Representation of finite groups: Complete reducibility, Schur's lemma, characters, projection formulae. Induced representation, Frobenius reciprocity. Representations of permutation groups.

Reference Texts:

1. M. Artin: *Algebra*.
2. S. D. Dummit and M. R. Foote: *Abstract Algebra*.
3. I. N. Herstein: *Topics in Algebra*.
4. K. Hoffman and R. Kunze: *Linear Algebra*.
5. W. Fulton and J. Harris: *Representation Theory*, Part I.

**Analysis I (Calculus of one variable)** : The language of sets and functions - countable and uncountable sets (see also Algebra 1). Real numbers - least upper bounds and greatest lower bounds. Sequences - limit points of a sequence, convergent sequences; bounded and monotone sequences, the limit superior and limit inferior of a sequence. Cauchy sequences and the completeness of  $\mathbf{R}$ . Series - convergence and divergence of series, absolute and conditional convergence. Various tests for convergence of series. (Integral test to be postponed till after Riemann integration is introduced in Analysis II.) Connection between infinite series and decimal expansions, ternary, binary expansions of real numbers, calculus of a single variable - continuity; attainment of supremum and infimum of a continuous function on a closed bounded interval, uniform continuity. Differentiability of functions. Rolle's theorem and mean value theorem. Higher derivatives, maxima and minima. Taylor's theorem - various forms of remainder, infinite Taylor expansions.

**Analysis II (Metric spaces and Multivariate Calculus)** : The existence of Riemann integral for sufficiently well behaved functions. Fundamental theorem of Calculus. Calculus of several variables: Differentiability of maps from  $\mathbf{R}^m$  to  $\mathbf{R}^n$  and the derivative as a linear map. Higher derivatives, Chain Rule, Taylor expansions in several variables, Local maxima and minima, Lagrange multiplier.

Elements of metric space theory - sequences and Cauchy sequences and the notion of completeness, elementary topological notions for metric spaces i.e. open sets, closed sets, com-

pact sets, connectedness, continuous and uniformly continuous functions on a metric space. The Bolzano - Weierstrass theorem, Supremum and infimum on compact sets,  $\mathbf{R}^n$  as a metric space.

**Analysis III (Vector Calculus)** : Multiple integrals, Existence of the Riemann integral for sufficiently well-behaved functions on rectangles, i.e. product of intervals. Multiple integrals expressed as iterated simple integrals. Brief treatment of multiple integrals on more general domains. Change of variables and the Jacobian formula, illustrated with plenty of examples. Inverse and implicit functions theorems (without proofs). More advanced topics in the calculus of one and several variables - curves in  $\mathbf{R}^2$  and  $\mathbf{R}^3$ . Line integrals, Surfaces in  $\mathbf{R}^3$ , Surface integrals, Divergence, Gradient and Curl operations, Green's, Stokes' and Gauss' (Divergence) theorems. Sequence of functions - pointwise versus uniform convergence for a function defined on an interval of  $\mathbf{R}$ , integration of a limit of a sequence of functions. The Weierstrass's theorem about uniform approximation of a continuous function by a sequence of polynomials on a closed bounded interval.

Reference Texts:

1. T.M. Apostol: *Mathematical Analysis*.
2. S. Dineen: *Multivariate Calculus and Geometry*.

**Analysis IV (Introduction to Function Spaces)** : Review of compact metric spaces.  $C([a, b])$  as a complete metric space, the contraction mapping principle. Banach's contraction principle and its use in the proofs of Picard's theorem, inverse & implicit function theorems. The Stone-Weierstrass theorem and Arzela-Ascoli theorem for  $C(X)$ . Periodic functions, Elements of Fourier series - uniform convergence of Fourier series for well behaved functions and mean square convergence for square integrable functions.

Reference Texts:

1. T.M. Apostol: *Mathematical Analysis*.
2. R. R. Goldberg: *Methods of Real Analysis*.

### **Differential Equations:**

Ordinary differential equations - first order equations, Picard's theorem (existence and uniqueness of solution to first order ordinary differential equation), Second order linear equations - second order linear differential equations with constant co-efficients, Systems of first order differential equations, Equations with regular singular points, Introduction to power series and power series solutions, Special ordinary differential equations arising in physics and some special functions (eg. Bessel's functions, Legendre polynomials, Gamma functions). Partial differential equations - elements of partial differential equations and the three equations of physics i.e. Laplace, Wave and the Heat equations, at least in 2 - dimen-

sions. Lagrange's method of solving first order quasi linear equations.

Reference Texts:

1. G.F. Simmons: *Differential Equations*.
2. R. Haberman: *Elementary applied partial differential equations*.
3. R. Dennemeyer: *Introduction to partial differential equations and boundary value problems*.

### **Complex Analysis:**

Holomorphic functions and the Cauchy-Riemann equations, Power series, Functions defined by power series as holomorphic functions, Complex line integrals and Cauchy's theorem, Cauchy's integral formula. Representations of holomorphic functions in terms of power series. Zeroes Liouville's theorem, The fundamental theorem of algebra, The maximum modulus principle, Schwarz's lemma, The argument principle, The open mapping property of holomorphic functions. The calculus of residues and evaluation of integrals using contour integration.

Reference Texts:

1. D. Sarason: *Notes on Complex Function Theory*.
2. T. W. Gamelin: *Complex Analysis*.

### **Introduction to Differential Geometry:**

Curves in two and three dimensions, Curvature and torsion for space curves, Existence theorem for space curves, Serret-Frenet formula for space curves, Inverse and implicit function theorems, Jacobian theorem, Surfaces in  $R^3$  as two dimensional manifolds, Tangent space and derivative of maps between manifolds, First fundamental form, Orientation of a surface, Second fundamental form and the Gauss map, Mean curvature and scalar curvature, Integration on surfaces, Stokes formula, Gauss-Bonnet theorem.

Reference Texts:

1. M.P. do Carmo: *Differential Geometry of Curves and Surfaces*.
2. A. Pressley: *Elementary Differential Geometry*.

**Combinatorics and Graph Theory:** Review of finite fields, mutually orthogonal Latin squares and finite projective planes, Desargue's theorem, t-designs and their one point extensions. Review of group actions, transitive and multiply transitive actions, Mathieu groups and Witt designs, Fisher's inequality, symmetric designs. Graphs, Hamilton Cycles and Euler Cycles, Planar Graphs, vector spaces and matrices associated with Graphs, Flows in Directed Graphs, Connectivity and Menger's Theorem, Matching, Tutte's 1-Factor Theorem.

Reference Texts:

1. P. J. Cameron and J.H. Van Lint: *Graphs, codes and designs*.
2. D. R. Hughes and F. Piper: *Projective planes*, Graduate texts in Mathematics 6.
3. B. Bollobas: *Graph Theory* (Chapters I - III).

### Optimization

Linear Programming: Basic notions; fundamental theorem of LP; the simplex algorithm; duality and applications; LP and game theory, Karmarkar's algorithm. Constrained Optimization Problems: Equality constraints, Lagrange multipliers; Inequality constraints, Kuhn-Tucker theorem; Illustrations (including situations where the above can fail), convexity.

Reference Texts:

1. H. Karloff: *Linear Programming*.
2. S-C. Fang and S. Puthenpura: *Linear optimization and extensions Theory and algorithms*.
3. R. K. Sundaram: *A first course in optimization Theory*.
4. S. Tijs: *Introduction to game theory*.

## 4.2 Probability courses

**Probability I** : Orientation, Combinatorial probability and urn models, Independence of events, Conditional probabilities, Random variables, Distributions, Expectation, Variance and moments, probability generating functions and moment generating functions, Standard discrete distributions (uniform, binomial, Poisson, geometric, hypergeometric), Independence of random variables, Joint and conditional discrete distributions. Univariate densities and distributions, standard univariate densities (normal, exponential, gamma, beta, chi-square, cauchy). Expectation and moments of continuous random variables. Transformations of univariate random variables. Tchebychev's inequality and weak law of large numbers.

**Probability II** : Joint densities and distributions. Transformation of variables (assuming Jacobian formula). Distributions of sum, maxima, minima, order statistics, range etc. Multivariate normal (properties, linear combinations) and other standard multivariate distributions (discrete and continuous) as examples. Standard sampling distributions like  $t$ ,  $\chi^2$  and  $F$ . Conditional distributions, Conditional Expectation. Characteristic functions: properties, illustrations, inversion formula, continuity theorem (without proof). Central Limit Theorem for i.i.d. case with finite variance. Elements of modes of convergence of random variables and the statement of the strong law of large numbers.

Reference Texts:

1. K. L. Chung: *Elementary Probability Theory*.
2. P. G. Hoel, S.C. Port and C.J. Stone : *Introduction to Probability Theory*.
3. R. Ash : *Basic Probability Theory*.
4. W. Feller : *Introduction to Probability Theory and its Applications*, Volume 1.
5. W. Feller : *Introduction to Probability Theory and its Applications*, Volume 2.
6. P. Billingsley : *Probability and Measure*.

### 4.3 Statistics courses

#### Statistics I

Introduction; Descriptive Statistics; Sampling Distributions, Introduction to Statistics with examples of its use, Descriptive statistics, Graphical representation of data, Basic distributions, properties, fitting, and their uses, Distribution theory for transformations of random vectors, Sampling distributions based on normal populations  $-t$ ,  $\chi^2$  and  $F$  distributions.

#### Statistics II

Theory and Methods of Estimation and Hypothesis testing, Point and interval estimation, Sufficiency, Exponential family, Bayesian methods, Moment methods, Least squares, Maximum likelihood estimation, Criteria for estimators, UMVUE, Large sample theory: Consistency; asymptotic normality, Confidence intervals, Elements of hypothesis testing; Neyman-Pearson Theory, UMP tests, Likelihood ratio and related tests, Large sample tests.

#### Statistics III

Multivariate normal distribution. Linear models, Regression and Analysis of variance, General linear model, Matrix formulation, Estimation in linear model, Gauss-Markov theorem, Estimation of error variance, Testing in the linear model, Regression, Partial and multiple correlations, Analysis of variance, Multiple comparisons. Stepwise regression, Regression diagnostics.

### 4.4 Computer Science Courses

#### Computer Science I (Programming) Recommended Language: C

Basic abilities of writing, executing, and debugging programs. Basics: Conditional statements, loops, block structure, functions and parameter passing, single and multi-dimensional arrays, structures, pointers. Data Structures: stacks, queues, linked lists, binary trees. Simple algorithmic problems: Some simple illustrative examples, parsing of arithmetic expressions, matrix operations, searching and sorting algorithms.

## Computer Science II (Numerical Methods)

Introduction to Matlab (or appropriate package) and Numerical Computing: Number representations, finite precision arithmetic, errors in computing. Convergence, iteration, Taylor series. Solution of a Single Non-linear Equation: Bisection method. Fixed point methods. Newton's method. Convergence to a root, rates of convergence. Review of Applied Linear algebra: Vectors and matrices. Basic operations, linear combinations, basis, range, rank, vector norms, matrix norms. Special matrices. Solving Systems of equations (Direct Methods): Linear systems. Solution of triangular systems. Gaussian elimination with pivoting. LU decomposition, multiple right-hand sides. Nonlinear systems. Newton's method. Least Squares Fitting of Data: Fitting a line to data. Generalized least squares. *QR* decomposition. Interpolation: Polynomial interpolation by Lagrange polynomials. Alternate bases: Monomials, Newton, divided differences. Piecewise polynomial interpolation. Cubic Hermite polynomials and splines. Numerical Quadrature: Newton - Cotes Methods: Trapezoid and Simpson quadrature. Gaussian quadrature. Adaptive quadrature. Ordinary Differential Equations: Euler's Method. Accuracy and Stability. Trapezoid method. Runge - Kutta method. Boundary value problems and finite differences.

Reference Texts:

1. B. Kernighan and D. Ritchie: *The C Programming Language*.
2. J. Nino and F. A. Hosch: *An Introduction to Programming and Object Oriented Design using JAVA*.
3. G. Recketenwald: *Numerical Methods with Matlab*.
4. Shilling and Harries: *Applied Numerical methods for engineers using Matlab and C*.
5. S. D. Conte and C. De Boor: *Elementary Numerical Analysis: An Algorithmic Approach*.
6. S. K. Bandopadhyay and K. N. Dey: *Data Structures using C*.
7. J. Ullman and W. Jennifer: *A first course in database systems*.

## 4.5 Physics Courses

### Physics I (Mechanics of Particles and Fluids)

Newton's laws of motion: Concept of inertial frame of reference: Conservation laws (energy, linear momentum and angular momentum) in mechanics for a single particle as well as for a system of particles; Motion of a system with variable mass; Frictional forces; Centre of mass and its motion: Simple collision problems; Torque; Moment of Inertia (parallel and perpendicular axes theorems) and Kinetic energy of a rotating rigid body. Newton's laws of gravitation; Kepler's laws of planetary motion; elements of variational calculus. Viscosity of liquid, streamline and turbulent flows (simple examples); critical velocity and Reynold's number; Euler Equation for incompressible fluids; Equation of law; surface tension and surface energy; Angle of contact; Gravity waves and ripples in ideal fluids; Navier Stoke's

equation, concept of homogeneous deformation (strain) and stress. Hooke's law; Interrelations of elastic constants for an isotropic solid, elastic waves normal modes of vibration group & phase velocities. + 5 experiments (10 hours)

Reference Texts:

1. R. Resnick and D. Halliday: *Physics*.
2. L. D. Landau and E. M. Lifshitz: *Short course on Mechanics*.
3. H. Goldstein: *Classical Mechanics*.

### **Physics II (Thermodynamics and Optics)**

Kinetic theory of gases : Perfect gas equation; Maxwell's law of distribution of molecular speeds; average root mean square and most probable speed; Principle of equipartition of energy; Van der Waal's equation and deduction of critical constants, phase transitions (first & second order phase changes), equilibrium of phases (Gibb's phase rule). Thermal conductivity; thermal diffusivity; Fourier equation of heat conduction and its solution for rectilinear flow of heat. State function, exact and inexact differentials; First law of thermodynamics and its applications; Isothermal and adiabatic changes; reversible, irreversible and cyclic processes; Second law of thermodynamics; Carnot's cycle and its efficiency; Absolute scale of temp.; Entropy (its physical interpretation); Joule Thompson effect, third law of thermodynamics, Maxwell's relations. Optics: Light as a ray, laws of reflection, refraction, prisms and lenses. Light as a scalar wave, superposition of waves and interference, Young's double slit experiment, Newton's rings. Diffraction: Fraunhofer and Fresnel. Polarisation of light - transverse nature of light wave.

Reference Texts:

1. F. Reif: *Statistics and Thermal Physics*.
2. C. Kittel: *Thermal Physics*.
3. M. W. Zemansky: *Thermodynamics*.
4. A. Jenkins and H. E. White: *Optics*.

### **Physics III (Electromagnetism and Electrodynamics)**

Vectors, Vector algebra, Vector Calculus (Physical meaning of gradient, divergence & curl); Gauss's divergence theorem; Theorems of gradients and curls. Electrostatics, Coulomb's law for discrete and continuous charge distribution; Gauss's theorem and its applications; Potential and field due to simple arrangements of electric charges; work and energy in electrostatics; Dielectrics, Polarization; Electric displacement; Capacitors (Paralleleplates); Electrical images. Magnetostatics: Magnetic field intensity (H), Magnetic induction (B), Biot-Savart's law; Ampere's law; comparison of electrostatics & magnetostatics. Electrodynamics: Ohm's law, Electromotive force, Faraday's law of electromagnetic induction; Lorentz force, Maxwell's equations. Electromagnetic theory of light and wave optics. Elec-

tronics: Semiconductors; pn junctions; transistors; zenor diode, IV characteristics. +5 experiments (10 hours)

Reference Texts:

1. D. J. Giffths: *Introduction to Electrodynamics*.
2. J. R. Reitz, F. J. Milford and W. Charisty: *Foundations of Electromagnetic theory*.
3. D. Halliday and R. Resnick: *Physics II*.

**Physics IV (Modern Physics and Quantum Mechanics):** Special theory of Relativity: Michelson-Morley Experiment, Einstein's Postulates, Lorentz Transformations, length contraction, time dilation, velocity transformations, equivalence of mass and energy. Black body Radiation, Planck's Law, Dual nature of Electromagnetic Radiation, Photoelectric Effect, Compton effect, Matter waves, Wave-particle duality, Davisson-Germer experiment, Bohr's theory of hydrogen spectra, concept of quantum numbers, Frank-Hertz experiment, Radioactivity, X-ray spectra (Mosley's law), Basic assumptions of Quantum Mechanics, Wave packets, Uncertainty principle, Schrodinger's equation and its solution for harmonic oscillator, spread of Gaussian wave packets with time.

**A list of possible physics experiments:**

1. Determination of the coefficient of viscosity of water by Poiseuille's method (The diameter of the capillary tube to be measured by a travelling vernier microscope).
2. Determination of the surface tension of water by capillary rise method.
3. Determination of the temp. coefficient of the material of a coil using a metric bridge.
4. To draw the frequency versus resonant length curve using a sonometer and hence to find out the frequency of the given tuning fork.
5. Study of waves generated in a vibrating string and vibrating membrane.
6. Determination of wave length by Interference & Diffraction.
7. One experiment on polarized light.
8. Experiments on rotation of place of polarization - chirality of media.
9. Elasticity: Study of stress-strain relation and verification of Hooke's law of elasticity, Measurement of Young's modulus.
10. Faraday Experiment: Pattern in fluid and granular materials under parametric oscillation.
11. Determination of dispersion rotation of Faraday waves in liquid (water/glycerol) and to compute the surface tension of the liquid.
12. Determination of the moment of a magnet and the horizontal components of Earth's magnetic field using a deflection and an oscillation magnetometer.
13. Familiarization with components, devices and laboratory instruments used in electronic systems.
14. To study the characteristics of a simple resistor-capacitor circuit.

15. Transistor Amplifier: To study a common emitter bipolar junction transistor amplifier.
16. Diodes and Silicon controlled rectifiers: To study the operational characteristics of diodes and silicon controlled rectifier.
17. Logic circuits: Combinational logic and binary addition.

## 4.6 Writing of Mathematics Course

### Writing of Mathematics (non-credit course)

The aim of this (non-credit) course is to improve the writing skills of students while inculcating an awareness of mathematical history and culture. The instructor may choose a book, like the ones listed below, and organize class discussions. Students will then be assigned five formal writing assignments (of 8 to 10 pages each) related to these discussions. These will be corrected, graded and returned.

Reference Texts:

1. J. Stillwell: Mathematics and Its History, Springer UTM.
2. W. Dunham, Euler: The Master of Us All, Mathematical Association of America.
3. W. Dunham: Journey Through Genius, Penguin Books.
4. M. Aigner and M. Ziegler: Proofs From The Book, Springer.
5. A. Weil: Number Theory, An Approach Through History from Hammurapi to Legendre.

## 4.7 Elective Courses

### 1. Topology:

Topological spaces, quotient topology. Separation axioms, Urysohn lemma. Connectedness and compactness. Tychonoff's theorem, one point compactification, Arzela-Ascoli Theorem. Fundamental group, Covering spaces.

Reference Texts:

1. J. Munkres: *Topology a first course*.
2. M. A. Armstrong: *Basic Topology*.
3. G. G. Simmons: *Introduction to Topology and Modern Analysis*.
4. K. Janich: *Topology*.

### 2. Introduction to algebraic geometry

Prime ideals and primary decompositions, Ideals in polynomial rings, Hilbert basis theorem, Noether normalisation theorem, Hilbert's Nullstellensatz, Projective varieties, Algebraic curves, Bezout's theorem, Elementary dimension theory.

Reference Texts:

1. M. Atiyah and I.G. MacDonald: *Commutative Algebra*.
2. J. Harris: *Algebraic Geometry*.
3. I. Shafarevich: *Basic Algebraic Geometry*.
4. W. Fulton: *Algebraic curves*.
5. M. Ried: *Undergraduate Commutative Algebra*.

### **3. Introduction to Algebraic Number Theory:**

Number fields and number rings, prime decomposition in number rings, Dedekind domains, definition of the ideal class group, Galois theory applied to prime decomposition and Hilbert's ramification theory, Gauss's reciprocity law, Cyclotomic fields and their ring of integers as an example, the finiteness of the ideal class group, Dirichlet's Unit theorem.

Reference Texts:

1. D. Marcus: *Number fields*.
2. G. J. Janusz: *Algebraic Number Theory*.

### **4. Differential Geometry II:**

Manifolds and Lie groups, Frobenius theorem, Tensors and Differential forms, Stokes theorem, Riemannian metrics, Levi-Civita connection, Curvature tensor and fundamental forms.

Reference Texts:

1. S. Kumaresan: *A course in Differential Geometry and Lie Groups*.
2. T. Aubin: *A course in Differential Geometry*.

### **5. Introduction to Differential Topology:**

Manifolds. Inverse function theorem and immersions, submersions, transversality, homotopy and stability, Sard's theorem and Morse functions, Embedding manifolds in Euclidean space, manifolds with boundary, intersection theory mod 2, winding numbers and Jordan-Brouwer separation theorem, Borsuk-Ulam fixed point theorem.

Reference Texts:

1. V. Guillemin and Pollack: *Differential Topology* (Chapters I, II and Appendix 1, 2).
2. J. Milnor: *Topology from a differential viewpoint*.

### **6. Introduction to Dynamical systems**

Linear maps and linear differential equation: attractors, foci, hyperbolic points; Lyapunov stability criterion, Smooth dynamics on the plane: Critical points, Poincare index, Poincare-Bendixon theorem, Dynamics on the circle: Rotations: recurrence, equidistribution, Invertible transformations: rotation number, Denjoy construction, Conservative systems:

Poincare recurrence. Newtonian mechanics.

Reference Texts:

1. B. Hasselblatt and A. Katok: *A first course in dynamics*.
2. M. Brin, G. Stuck: *Introduction to dynamical systems*.
3. V. I. Arnold: *Geometrical methods in the theory of Ordinary Differential Equations*.

### **7. Probability III (Introduction to Stochastic Processes)**

Discrete Markov chains with countable state space. Classification of states-recurrences, transience, periodicity. Stationary distributions, reversible chains. Several illustrations including the Gambler's Ruin problem, queuing chains, birth and death chains etc. Poisson process, continuous time markov chain with countable state space, continuous time birth and death chains.

Reference Texts:

1. P. G. Hoel, S. C. Port and C. J. Stone: *Introduction to Stochastic Processes*.
2. S. M. Ross: *Stochastic Processes*.
3. J. G. Kemeny, J. L. Snell and A. W. Knapp: *Finite Markov Chains*.
4. D. L. Isaacsen and R. W. Madsen: *Markov Chains, Theory and Applications*.

### **8. Statistics IV :**

Analysis of Discrete data: Nonparametric methods: Decision theory, Goodness of fit tests, Multiway contingency tables, Odds ratios, Logit model, Wilcoxon test, Wilcoxon signed rank test, Kolmogorov test. Elements of decision theory : Bayes and minimax procedures.

Reference Texts:

1. G. K. Bhattacharya and R. A. Johnson: *Statistics : Principles and Methods*.
2. P. J. Bickel and K. A. Doksum: *Mathematical Statistics*.
3. E. J. Dudewicz and S. N. Mishra: *Modern Mathematical Statistics*.
4. V. K. Rohatgi: *Introduction to Probability Theory and Mathematical Statistics*.

### **9. Statistics V:**

**Sample Surveys (1/2 Semester):** Scientific basis of sample surveys. Complete enumeration vs. sample surveys. Principal steps of a sample survey; illustrations, N.SS., Methods of drawing a random sample. SRSWR and SRSWOR: Estimation, sample size determination. Stratified sampling; estimation, allocation, illustrations. Systematic sampling, linear and circular, variance estimation. Some basics of PPS sampling, Two-stage sampling and Cluster sampling. Nonsampling errors. Ration and Regression methods.

Reference Texts:

1. W. G. Cochran: *Sampling Techniques*.
2. M. N. Murthy: *Sampling Theory and Methods*.
3. P. Mukhopadhyay: *Theory and Methods of Survey Sampling*.

**Design of Experiments (1/2 semester):** The need for experimental designs and examples, basic principles, blocks and plots, uniformity trials, use of completely randomized designs. Designs eliminating heterogeneity in one direction: General block designs and their analysis under fixed effects model, tests for treatment contrasts, pairwise comparison tests; concepts of connectedness and orthogonality of classifications with examples; randomized block designs and their use. Some basics of full factorial designs. Practicals using statistical packages.

Reference Texts:

1. A. Dean and D. Voss: *Design and Analysis of Experiments*.
2. D. C. Montgomery: *Design and Analysis of Experiments*.
3. W. G. Cochran and G. M. Cox: *Experimental Designs*.
4. O. Kempthorne: *The Design and Analysis of Experiments*.
5. A. Dey: *Theory of Block Designs*.

## 10. Mathematics of Computation

Models of computation (including automata, PDA). Computable and non-computable functions, space and time complexity, tractable and intractable functions. Reducibility, Cook's Theorem, Some standard NP complete Problems: Undecidability.

## 11. Computer Science III (Data Structures)

Fundamental algorithms and data structures for implementation. Techniques for solving problems by programming. Linked lists, stacks, queues, directed graphs. Trees: representations, traversals. Searching (hashing, binary search trees, multiway trees). Garbage collection, memory management. Internal and external sorting.

## 12. Computer Science IV (Design and Analysis of Algorithms) :

Efficient algorithms for manipulating graphs and strings. Fast Fourier Transform. Models of computation, including Turing machines. Time and Space complexity. NP-complete problems and undecidable problems.

Reference Texts:

1. A. Aho, J. Hopcroft and J. Ullmann: *Introduction to Algorithms and Data Structures*.
2. T. A. Standish: *Data Structure Techniques*.
3. S. S. Skiena: *The algorithm Design Manual*.

4. M. Sipser: *Introduction to the Theory of Computation*.
5. J.E. Hopcroft and J. D. Ullmann: *Introduction to Automata Theory, Languages and Computation*.
6. Y. I. Manin : *A Course in Mathematical Logic*.