## INSTITUTE OF ACTUARIES OF INDIA

## EXAMINATIONS

$27^{\text {th }}$ May 2009
Subject ST6 - Finance and Investment B
Time allowed: Three hours (14.15* pm - 17.30 pm )
Total Marks: 100

## INSTRUCTIONS TO THE CANDIDATE

1. Enter all the candidate and examination details as requested on the front of your answer sheet.
2. $\quad$ You have 15 minutes at the start of the examination in which to read the questions. You are strongly encouraged to use this time for reading only, but notes may be made. You then have three hours to complete the paper.
3. The answers are not expected to be any country or jurisdiction specific However, if Examples/illustrations are required for any answer, the country or jurisdiction from which they are drawn should be mentioned.
4. You must not start writing your answers in the answer sheet until instructed to do so by the supervisor.
5. Mark allocations are shown in brackets.
6. Attempt all questions, beginning your answer to each question on a separate sheet.
7. Candidates should show calculations where this is appropriate.
8. Fasten your answer sheets together in numerical order of questions. This, you may complete immediately after expiry of the examination time.

## AT THE END OF THE EXAMINATION

Please return your answersheets and this question paper to the supervisor seperatly.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.

Q1) CBI and IOB plan to issue Rs. 100 crores in debt and negotiating an interest rate swap that will help them lower their borrowing costs and obtain the preferred type (fixed rate or floating rate) of financing. Both are comparing their balance sheet alternatives with combined balance sheet and swap opportunities. CBI is liability sensitive through three years, and would like to use the debt proceeds to invest in short-term assets to reduce its interest rate risk. IOB is asset sensitive through three years, and would like to use its debt proceeds to invest in fixed-rate-assets to reduce its interest rate risk. CBI can borrow at a $9 \%$ fixed rate for three years or pay the prevailing six-month MIBOR plus $0.8 \%$ on floating-rate debt. IOB can borrow at an $8.45 \%$ fixed rate for three years or the prevailing six-month MIBOR plus $0.55 \%$. A bank is classified as liability (asset) sensitive if its interest rate sensitive or floating rate assets are less (more) than interest rate sensitive or floating rate liabilities
a) Explain whether and why CBI needs fixed-rate or floating-rate funding to meets its objective.
b) Assume that both banks issue three year debt either at fixed-rate or six-month floating rate debt on-balance sheet. They want to combine this with a basic swap to obtain the cheapest form of funding that helps reduce interest rate risk. Using the following swap terms offered by Alpha Limited (a swap dealer); indicate what position each bank should take. Explain how and why it should meet the bank's objectives. Calculate the effective cost of borrowing that each bank ends up with by making suitable assumptions. Assume that the swap terms offered by Alpha Limited are independent of the credit rating of another party. Swap term A (B) implies that Alpha Limited is willing to receive 8.40 \% fixed (six-month MIBOR plus $0.30 \%$ ) in exchange for payment of six-month MIBOR plus $0.30 \%$ ( $8.33 \%$ fixed).

## Basic Swap Terms

| A. | Pay 8.40\% | Receive six-month MIBOR <br> plus $0.30 \%$ |
| :--- | :--- | :--- |
| B. | Pay six-month MIBOR <br> plus $0.30 \%$ | Receive $8.33 \%$ |

c) Do you think such arbitrage situation can exist in the markets as given in the part b? Discuss.

Q2) Following is the balance sheet of Hypothetical Bank Limited.

| Assets | Amount <br> (in Lakhs) | Rate | Macaulay's <br> Duration |
| :--- | ---: | :--- | :--- |
| Cash | Rs. 30,000 | $0 \%$ | 0 |
| Bonds | 120,000 | $7.5 \%$ | 2 Years |
| Commercial <br> Loans | 450,000 | $11.5 \%$ | 1.75 Years |


|  <br> Equity$\quad$Amount <br> (in Lakhs) | Rate | Macaulay's <br> Duration |  |
| :--- | ---: | :--- | :--- |
| Small time <br> deposits | Rs.150,000 | $4 \%$ | 4.2 Years |
| Large CDs | 90,000 | $6.5 \%$ | 1 Year |
| Transaction <br> Account | 300,000 | $3 \%$ | 3.75 Years |
| Equity | 60,000 |  |  |

a) Estimate the change in net worth if all interest rates fall by an average of $2 \%$. The change in net worth is defined as the difference between the market value of all bank assets and the market value of all bank liabilities.
b) How can the firm immunize its interest rate risk such that it has negligible impact on the net worth?

Q3) Given below are the closing prices observed for stock options and futures contracts on XYZ Ltd at NSE as on $13^{\text {th }}$ March 2009.

| Contract | Strike | Close | Contracts | Open_Interest |
| :--- | :--- | :--- | :--- | :--- |
| Call Mar | 140 | 14 | 692 | 428 |
| Call Mar | 120 | 32 | 34 | 25 |
| Call Apr | 140 | 18 | 2 | 2 |
| Put Mar | 140 | 6 | 381 | 183 |
| Fut Mar | - | 148 | 23834 | 14759 |
| Fut Apr | - | 140 | 2194 | 1454 |

The closing share price of XYZ as on $13^{\text {th }}$ March was 153. March settlement date is 26 and April settlement date is 30 . The short term risk free rate is $6 \%$ p.a. compounded annually. Please note that all stock options on NSE are American and settlement mechanism is by cash. Assume that margin money required for Futures is $45 \%$ of the underlying and margin required for selling options is 2 times the premium plus the intrinsic value.
a) Does there exist any arbitrage opportunity between futures and cash?
b) Who can reap the benefit of this arbitrage when short selling is blocked? What can be possible reasons for the market to remain in such a situation for long?
c) Consider this trade - long a March future and short a March deep in the money call option. What is the maximum profit that a person can make? When does he make a loss? Why do you think a buyer exists for such a call option?
d) How the trade is in part c is different from pure arbitrage and what are the extra risks that the person is taking by doing this trade? Does the risk profile differ if he trades using Apr contracts in place of March contracts?
e) Assume that this stock follows GBM with expected return (after allowing for any dividends) of $20 \%$ p.a. and volatility of $50 \%$ p.a. What is the probability that the trade in part c results in a profit?
f) Theoretical Black Scholes price of the April 140 Call option is 19. The theoretical Black Scholes price of the April 140 put option is 5 . What is the market price of the April 140 put option? Assume that they are European to answer this part of the question.

Q4) Consider a European put option on a non-dividend paying stock where the stock price is Rs. 100, the strike price is Rs. 100, the risk-free rate is $6 \%$ per annum (with continuous compounding), the volatility is $32 \%$ per annum with continuous compounding, and the time to maturity is 4 months
a) Calculate $u$ (up movement in the stock price in each step), d (down movement in the stock price in each step), and $p$ (probability of an up movement in the stock price in a risk neutral world in each step) for a two-step tree. Assume that each step is two months long.
b) Value the option using a two-step tree (assuming that each step is two months long). If the actual market put premium is Rs. 1.50, show that in the world of binomial model there exists a replicating portfolio which would provide a certain profit under all states of the world. Ignore transaction costs.
c) If the option is American style, should it be exercised early in the world of binomial models?

Q5) The conversion factor for a bond is set equal to the quoted price that bond would have per rupee of principal on the first day of the delivery month on the assumption that the interest rate for all maturities equals $6 \%$ per annum (with semiannual compounding). The bond maturity and the times to the coupon payment are rounded down to the nearest 3 months for the purposes of calculation. The settlement price of futures contract for the July 2009 bond futures contract is Rs. 120.50.
a) Calculate the conversion factor for a bond maturing on February 1 2030, paying a coupon of $12 \%$ per annum (with semiannual payments).
b) Calculate the conversion factor for a bond maturing on November 1, 2035, paying coupon of $9 \%$ per annum (with semiannual payments).
c) Suppose that the quoted prices of the bonds in (a) and (b) are Rs. 208.00 and Rs. 174.00 respectively on the settlement date of the futures contract (July 28, 2009). Which bond is cheaper to deliver on the settlement date of the futures contract?
d) Assuming that the cheapest-to-deliver bond has been delivered on July 28,2009 , what is the dirty price received for the bond?

Q6) The fund manager of HDFC Asset Management Company Limited has a portfolio worth Rs. 800 million with a beta of 0.80 . The manager is concerned about the performance of the market over the next one month and plans to use 2-month futures contract on NSE Nifty to hedge the risk. The index futures price is currently 2700, and one contract is on 50 times the index.
a) What position should the fund manager take to eliminate all exposure to the market over the next one month?
b) What position should the fund manager take to reduce the beta of the portfolio to 0.5 ?

Q7) The risk-free zero rates (with continuous compounding) and corporate bond zero rates (with continuous compounding) for different maturities are given in Table below:

| Maturity | Risk-Free Zero Rates <br> $(\%)$ | Corporate Bond Zero <br> Rates (\%) |
| :--- | :--- | :--- |
| 1 | 6 | 6.25 |
| 2 | 6 | 6.50 |
| 3 | 6 | 6.70 |
| 4 | 6 | 6.80 |
| 5 | 6 | 7.00 |

a) Assume the recovery rate of $40 \%$, what is the risk-neutral unconditional probability of default between first and second year; between second and third year; between third and fourth year; and between fourth and fifth year in a risk neutral world?
b) Assume the recovery rate of $40 \%$, what is the risk-neutral probability of default per year, conditional on no earlier default?

Q8) A credit default swap requires a quarterly payment at the rate 80 basis points per year. The principal is Rs. 500 crores and the credit default swap is settled in cash. A default occurs after 3 years and 1 month and it is estimated that the price of the cheapest deliverable bond is $30 \%$ of its face value shortly after the default. List the cash flows and their timing for the seller of the credit default swap.

Q9) A fund manager has a well diversified portfolio that has a beta of 1.25 with respect to NSE Nifty index. The portfolio is worth Rs. 600 crores. The value of the NSE Nifty index is 3000 , and the portfolio manager would like to buy insurance against a reduction of more than $4 \%$ in the value of the portfolio over the next three months. One put option contract is on 50 times the index. The risk-free rate of interest is $5 \%$ per annum with quarterly compounding. The dividend yield on the portfolio is $3 \%$ per annum (with quarterly compounding), the dividend yield on NSE Nifty is $2 \%$ per
annum (with quarterly compounding) and the volatility of the index is $25 \%$ per annum (with continuous compounding). Assume that market prices European options using Black-Scholes model.
a) If the fund manager buys traded European put options to provide insurance against a reduction of more than $4 \%$ in the value of the portfolio over the next three months, what strike price of the put option should be used to provide the insurance? If the put option is available for trading with the calculated strike price, how much would the insurance cost?
b) State under what type of market scenarios the fund manager is likely to purchase such an option.
c) Suggest an alternative strategy that the fund manager can follow which is likely to give a similar outcome. Suggest possible reasons for this being not a preferred strategy for the fund manager.

Q10) a) Suppose that the relationship between the implied volatility and the degree of moneyness on BSE Sensex options is expressed by following equation:

$$
I V_{i T, t}=0.50+0.05\left(\frac{S_{t}}{K_{i T, t}}\right)^{2}-0.10\left(\frac{S_{t}}{K_{i T, t}}\right)
$$

Where:
$\mathrm{IV}_{\mathrm{iT}, \mathrm{t}} \quad: \quad$ Implied volatility of an option with an exercise price of $K_{i}$ and time to maturity of $T$ on trading day $t$.
$S_{t} \quad: \quad$ Closing value of BSE Sensex on trading day $t$.
$\mathrm{K}_{\mathrm{iT}, \mathrm{t}} \quad: \quad$ ith exercise price with time to maturity of T available for trading on day $t$.
Describe the nature of above relationship in terms of degree of moneyness of call and put options on BSE Sensex. What this relationship is termed as? In which kind of asset does this kind of relationship between implied volatility and degree of moneyness generally hold?
b) How will you describe the nature of relationship between the implied volatility and the degree of moneyness on BSE Sensex options if it is expressed by following equation:

$$
I V_{i T, t}=0.50+0.01\left(\frac{S_{t}}{K_{i T, t}}\right)
$$

In which kind of asset does this kind of relationship between implied volatility and degree of moneyness generally hold?

Q11) Suppose that $y$ is the yield with continuous compounding on a perpetual bond that pays interest at the rate of Rs. 2 per annum. Assume that interest is paid continuously on the bond, and that y follows the process:

$$
d y=\mu(\alpha-y) d t+\sigma y d Z
$$

Where $\mu, \alpha$ and $\sigma$ are positive constants and dZ is a standard Brownian motion. What is the process followed by the bond price? What is the expected return to the bond holder?
a) Suppose that a stock price S follows geometric Brownian motion with expected return $\mu$ and volatility $\sigma$ : $d S=\mu S d t+\sigma S d Z$
Where dZ is a standard Brownian motion. What is the process followed by $S^{n}$ where n is greater than 1 . Does $S^{n}$ follow geometric Brownian motion?
b) Stock X and stock Y both follow geometric Brownian motion. Changes in value of the two stocks in any short interval of time are uncorrelated with each other. Under what conditions the value of a portfolio consisting of one of stock X and one of stock Y follow geometric Brownian motion? Explain your answer.

