# SATHYABAMA UNIVERSITY

(Established under section 3 of UGC Act,1956)

Course & Branch :B.E/B.Tech - AERO/CHEM/CSE/E&C/EIE/
ETCE/IT/M&P/MECH
Title of the Paper :Engineering Mathematics – I    Max. Marks :80
Sub. Code :4ET102A/5ET102A    Time : 3 Hours
Date :11/05/2010    Session :AN

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PART - A    (10 x 2 = 20)

Answer ALL the Questions

1. Expand $\cos^4\theta$ in a series of cosines of multiples of $\theta$.

2. Show that $\tanh^{-1}(x) = \frac{1}{2}\log\left(\frac{1+x}{1-x}\right)$.

3. If a line make angles $\alpha$, $\beta$, $\gamma$ with the coordinate axes, show that $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$.

4. Find the angle between the straight lines $\frac{x}{2} = \frac{y}{-2} = \frac{z}{1}$ *and* $\frac{x-4}{2} = \frac{y-5}{1} = \frac{z+6}{2}$.

5. Find the Eigen values of the matrix $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$.

6. Find the value of $\lambda$, if the equations $x + y - z = 10$, $x - y + 2z = 20$ and $\lambda x - y + 4z = 30$ have a unique solution.

7. Show that $\int_0^\infty \frac{x^2}{\left(a^2 + x^2\right)^4}\,dx = \frac{\pi}{32a^5}$.

8. Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} \frac{dy \quad dx}{\sqrt{a^2 - x^2}}$.

9. Show that $\left[\Gamma\frac{1}{2}\right]^2 = \pi.$

10. Prove that $\dfrac{\beta(m+1,n)}{\beta(m,n+1)} = \dfrac{m}{n}.$

PART – B (5 x 12 = 60)

Answer All the Questions

11. (a) Prove that

$$\frac{Cos7\theta}{Cos\theta} = 64Cos^6\theta - 112Cos^4\theta + 56Cos^2\theta - 7$$

(b) If tan $(\theta + i\phi) = \tan\alpha + i\sec\alpha$,

Prove that $e^{2\varphi} = \pm\cot\left(\dfrac{\alpha}{2}\right) and\, 2\theta = n\pi + \dfrac{\pi}{2} + \alpha.$

(or)

12. (a) Expand $Sin^3\theta . Cos^5\theta$ in a series of sines of multiples of $\theta$.
(b) Separate into real and imaginary parts of $\tanh(x + iy)$.

13. Show that the straight lines whose direction cosines are given by the equations al + bm + cn = 0 and $pl^2 + qm^2 + rn^2 = 0$ are perpendicular if $a^2(q + r) + b^2(r + p) + c^2(p + q) = 0$; are parallel if

$$\frac{a^2}{p} + \frac{b^2}{q} + \frac{c^2}{r} = 0.$$

(or)

14. (a) Find the length of the shortest distance between the pairs of lines $\dfrac{x-10}{1} = \dfrac{y-9}{3} = \dfrac{z+2}{-2}\ and\ \dfrac{x+1}{2} = \dfrac{y-12}{4} = \dfrac{z-5}{1}.$

(b) Show that the plane $2x - 2y + z + 12 = 0$ touches the sphere $x^2 + y^2 + z^2 - 2x - 4y + 2z = 3$ and also find the point of contact

15. (a) Find the value of 'k' such that the following system of equations $kx + y + z = 1;\ x + ky + z = 1;\ x + y + kz = 1$ has
(i) unique solution,
(ii) many solutions and
(iii) no solution

(b) Find the Eigen values and Eigen vectors of the matrix $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$.

(or)

16. (a) Verify Cayley-Hamilton Theorem for the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix}$

(b) Reduce the quadratic form
$q = 3x_1^2 + 2x_2^2 + 3x_3^2 - 2x_1x_2 - 2x_2x_3$ into canonical form by orthogonal reduction.

17. (a) Show that $\int_0^{\infty} \log\left(x + \frac{1}{x}\right) \frac{dx}{\left(1 + x^2\right)} = \pi \log(2).$

(b) Evaluate $\int_0^{\log a} \int_0^{x} \int_0^{x+y} e^{x+y+z}\, dz \quad dy \quad dx.$

(or)

18. (a) If $U_n = \int_0^a x^n e^{-x} dx$,

prove that $U_n - (n + a) U_{n-1} + a(n - 1) U_{n-2} = 0$.

(b) Change the order of integration and hence evaluate $\int_0^a \int_{\frac{x^2}{a}}^{2a-x} xy \quad dy \quad dx.$

19. (a) Evaluate $\int_0^\infty \frac{x^{m-1}}{(1+x^n)^p} dx$ and hence deduce that $\int_0^\infty \frac{x^{m-1}}{(1+x^n)} dx = \frac{\pi}{n} Co\sec\left(\frac{\pi m}{n}\right).$

(b) Show that the volume of the region of space bounded by the co-ordinate planes and the surface $\sqrt{\frac{x}{a}} + \sqrt{\frac{y}{b}} + \sqrt{\frac{z}{c}} = 1 \quad is \frac{abc}{90}.$

(or)

20. (a) Establish the relation between the Beta and Gamma functions. (4)

(b) Find the area of the asteroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$, using Gamma functions. (8)