SATHYABAMA INSTITUTE OF SCIENCE AND TECHNOLOGY
DEEMED UNIVERSITY

Course: B.E./B.Tech. Semester: I
Title of the paper: Engineering Mathematics – I Max. Mark: 80
Sub. Code: ET102/ET102A (2002/2003/2004/2005) Time: 3 Hours

PART – A (10 x 2 = 20)
Answer ALL the Questions

1. Show that $\frac{\cos 3\theta}{\cos \theta} = 4\cos^2\theta - 3$.
2. Find the real part of $\sin(x + iy)$.
3. Find the equation of the plane through the point (4, 5, –6) and parallel to the plane $x+3y+5z+6 = 0$.
4. Find the centre and radius of the sphere $x^2+y^2+z^2+12x-2y-6z+30 =0$.
5. Define rank of a matrix.
6. Two eigen values of $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ are equal to 1 each. Find the third eigen value.
7. Evaluate $\int_0^{\pi/2} \sin^3x \cos^6x \, dx$
8. Evaluate $\int_0^a \int_x^a (x^2 + y^2) \, dydx$
9. Prove that $\Gamma(1) = 1$.
10. Evaluate $\beta[5/2 , 7/2]$.

PART – B (5 x 12 = 60)

Answer ALL the Questions

11. (a) Prove that $\cos^6\theta = \frac{1}{32}[\cos6\theta + 6\cos4\theta+15\cos2\theta+10]$.

(b) If $x+iy = \cos(A - iB)$, find the value of $\frac{x^2}{\text{Cosh}^2B} + \frac{y^2}{\sinh^2B}$

(or)

12. (a) Find the real and imaginary parts of $\tan^{-1}(\alpha+i\beta)$.

(b) Prove that $\sin^5\theta \cos^2\theta = \frac{1}{26}[\sin7\theta - 3\sin5\theta + \sin3\theta +5\sin\theta]$

13. (a) Find the image of a point P (-2, 1, 2) with respect to $x - 2y + z = 10$.

(b) Find the shortest distance between the lines.

$$\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1} \quad \text{and}$$

$$\frac{x+1}{7} \qquad \frac{y+1}{-6} \qquad \frac{z+1}{1}$$

(or)

14. (a) Find the equation of the plane through (1, –2, 1) and perpendicular to the planes $3x + y + z - 2=0$ and $x - 2y + z + 4=0$

(b) Find the equation of the sphere that passes through the circle $x^2 + y^2 + z^2 + 3x + y + 2z - 2 = 0$, $x + 3y - 2z + 1 = 0$ and cuts orthogonally the sphere $x^2 + y^2 + z^2 + x - 3z - 2 = 0$.

15. (a) Verify Cayley-Hamilton theorem for the matrix

$$A = \begin{bmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{bmatrix}$$

(b) Find the eigen values and eigen vectors of $\begin{bmatrix} 2 & 2 & -7 \\ 2 & 1 & 2 \\ 0 & 1 & -3 \end{bmatrix}$

(or)

16. Reduce $6x^2 + 3y^2 - 4xy - 2yz + 4xz + 3z^2$ into a canonical form by an orthogonal reduction. Discuss the nature of quadratic form.

17. (a) Prove that $\int_0^{\pi/2} \log \sin x \, dx = \frac{-\pi}{2} \log 2$.

(b) Change the order of integration in $\int_0^a \int_x^a (x^2 + y^2) dy dx$ and hence evaluate.

(or)

18. (a) If $I_n = \int_0^a x^n e^{-x} dx$, prove that $I_n - (n+a) I_{n-1} + a(n-1) I_{n-2} = 0$

(b) Evaluate $\int_0^a \int_0^{\sqrt{a^2 - x^2}} \int_0^{\sqrt{a^2 - x^2 - y^2}} \frac{dz \, dy \, dx}{a^2 - x^2 - y^2 - z^2}$

19. (a) Prove that $\int_0^{\pi/2} \sqrt{\sin \theta} \, d\theta \int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin \theta}} = \pi$

(b) Express $\int_0^1 \frac{dx}{\sqrt{1 - x^4}}$ in terms of gamma function.

20. (a) Prove that $\frac{\beta(m, n+1)}{n} = \frac{\beta(m+1, n)}{m} = \frac{\beta(m, n)}{m+n}$

(b) Prove that $\int_0^\infty \frac{t^2 \, dt}{1+t^4} = \frac{\pi}{2\sqrt{2}}$