SATHYABAMA UNIVERSITY
(Established under section 3 of UGC Act, 1956)

Course & Branch: B.E/ B. Tech – Common to ALL Branches
(Except to Bio Groups)
Title of the paper: Engineering Mathematics - I
Semester: I Max. Marks: 80
Sub.Code: 6C0002 Time: 3 Hours
Date: 12-05-2008 Session: AN

PART – A (10 x 2 = 20)
Answer All the Questions

1. State any two properties of eigen values of a matrix.

2. Use Cayley-Hamilton theorem to find the inverse of the matrix $A = \begin{bmatrix} 7 & 3 \\ 2 & 6 \end{bmatrix}$

3. Find the coefficient of x^n in the expansion of $\frac{1+2x+3x^2}{e^{2x}}$

4. Find the coefficient of x^n in the expansion of $\log(1 + x + x^2 + x^3 + x^4)$

5. Find the radius of curvature for the curve $x = at^2$, $y = 2at$.

6. Write the formula to find the center of the circle of curvature and equation to the circle of curvature.

7. Write the tailor's series expansion of $f(x, y)$ about the point (a, b).

8. State the conditions for $f(x, y)$ to have a maximum or a minimum value.

9. Solve (D^2 + 9) y = sin 3x, where $D \equiv \frac{d}{dx}$.

10. Write the Euler's homogenous linear differential equation of order n.

PART – B (5 x 12 = 60)

Answer All the Questions

11. (a) Find the Eigen values and Eigen vectors of the matrix $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$

(b) Diagonalise the matrix A given above by similarity transformation.

(or)

12. (a) Find the inverse of the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$ by using Cay;ey-Hamilton theorem.

(b) Obtain an orthogonal transformation, which will transform the quadratic form $6x^2 + 3y^2 + 3z^2 - 4xy - 2yz + 4zx$ into a canonical form.

13. (a) If p-q is small compared to p or q, show that $n\sqrt{\frac{p}{q}} = \frac{(n+1)p + (n-1)q}{(n-1)p + (n+1)q}$ $(nearly\ ).Henc\ ,\ find$ $\sqrt[7]{\frac{131}{133}}$.

(b) Show that the coefficient of x^r in the expansion of is $\frac{1}{r!}\left[\frac{1^r}{1!} + \frac{2^2}{2!} + \frac{3^r}{3!} + \ldots\right]$. Hence, show that $\frac{1^3}{1!} + \frac{2^3}{2!} + \frac{3^3}{3!} + \ldots = 5e.$

(or)

14. (a) Find the sum of $\frac{15}{16}+\frac{15}{16}.\frac{21}{24}+\frac{15}{16}.\frac{21}{24}.\frac{27}{32}+...\infty.$

(b) Show that $\frac{3}{10}\left[\log(10)+\frac{1}{2^7}+\frac{1}{2}\frac{3}{2^{14}}+\frac{1}{3}\frac{3^2}{2^{21}}+...\infty\right]=\log 2.$

15. (a) Prove that if the center of curvature of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at one end of the minor axis lies at the other end, then the eccentricity of the ellipse is $\frac{1}{\sqrt{2}}$.

(b) Obtain the equation of the evolute of the curve: x = a(cosθ + θ sinθ); y = a(sinθ - θ cosθ).

(or)

16. (a) Find the envelope of the family of straight lines $y = x \tan \alpha + 2 \sec \alpha$.

(b) Prove that the evolute of the tractrix $x = a(\cos t + \log(\tan\frac{t}{2}))$, $y = a \sin t$ is a catenary.

17. (a) Find and classify the extreme values, if any, of the function $f(x, y) = y^2 + x^2y + x^4$.

(b) A rectangular box open at the top is to have a capacity of 108 cubic meters. Find the dimensions of the box requiring least material for its transaction.

(or)

18. (a) Determine the points on the ellipse, defined by the intersection of the surface $x + y = 1$ and $x^2 + 2y^2 + z^2 = 1$ which are nearest to and farthest from the origin.

(b) Find the maximum and minimum distance from the origin to the curve $5x^2 + 6xy + 5y^2 - 8 = 0$.

19. (a) Solve $x^2 \frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + 5y = x\cos(\log x)$.

(b) Solve the simultaneous equations $\frac{dx}{dt} - y = t, \frac{dy}{dt} = t^2 - x$.

(or)

20. (a) Solve $xy'' - 2(x + 1)y' + (x + 2)y = (x - 2)e^{2x}$, by the method of reduction of order.

(b) Solve by the method of variation of parameters:

$y'' - 2y' + 2y = ex \tan x$.