BE4-R3: PRINCIPLES OF MODELING AND SIMULATION

NOTE:

> 1. **Answer question 1 and any FOUR questions from 2 to 7.**
> 2. **Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) The cdf of a discrete random variables is given by $F(X) = \frac{x(x+1)}{k(k+1)}, \quad x = 1,2,\ldots,k$

Using inverse transform technique, develop a procedure for generating random variable X.

b) Name entities, attributes, activities, events and state variables for
  - i) Communication system
  - ii) Banking system

c) Every simulation must have a stopping time, which defines how long the simulation will run. Discuss ways to stop a simulation.

d) One of the important steps in the development of a useful model of input data is to identify a probability distribution to represent the input process. Discuss methods for selecting input distributions when data are available.

e) Let X and Y be independent exponential random variables with mean 1. Explain how can you use Monte Carlo technique to estimate $E[e^{XY}]$.

f) Consider the M/M/∞ queuing system with infinite number of servers. In this queuing system every arriving customer is assigned to its own server of rate μ. Therefore if there are n customers, the aggregate output rate is nμ. Obtain equilibrium state probability and the mean queue size.

g) As an aid in the validation process, a three-step approach is widely followed
  - i) Build a model that has high face validity.
  - ii) Validate model assumptions.
  - iii) Compare the model input-output transformations to that for real systems.

Explain the above-mentioned steps.

(7x4)

2.

a) It is often necessary in practice to develop a simulation model (perhaps for demonstration purposes or a preliminary study) before any process data are available.
  - i) Discuss ways to obtain information about a process even if data are not available.
  - ii) Which statistical distributions are generally used as input models when data are not available?

b) Verification and validation of simulation models is of great importance. What is the purpose of model verification? Compare and contrast the issues involved in validation and verification.

(9+9)

3. Tankers carrying crude oil arrive at a singe unloading dock, supplying a storage tank that in turn feeds a refinery through a pipeline. An unloading tanker delivers oil to the storage tank at a specified constant rate. The storage tank supplies oil to the refinery at a specified rate. Unloading the tanker is considered complete when the level of oil in the tanker is less than 5% of its capacity, but unloading must be temporarily stopped if level of the oil in the storage tank reaches its capacity.

i) Justify that the description of the above model would require combined discrete-continuous simulation.

ii) Identify the discrete events for this model.

iii) State the continuous state variables here. How will you model their dynamics?

iv) Develop a model that describes the situation of arriving tankers forming a queue when the dock is busy.

v) Describe the fundamental interaction that can occur between discretely changing and continuously changing state variables.

(3+3+3+5+4)

4.

a) You are given the multiplicative congruential generator x_0=1 and x_{n+1}= 7 x_n (modulo 13) for n=0,1,2,…

i) Calculate x_n for n=1,2,…,12

ii) How often does each integer between 1 and 12 appear in the sequence in part i) above?

iii) Without performing additional calculations, indicate how x_{13}, x_{14},… will compare with x_1, x_2,…, x_{12}

b) Apply the inverse transform method to generate random observations from the uniform distribution between –10 and 40 by using the random numbers 0.0965, 0.5692, 0.6658.

i) Apply this method graphically.

ii) Apply the method algebraically.

c) What are pseudo random numbers? Briefly discuss the frequency test for testing whether they form random sequences.

(7+6+5)

5. Consider a single run of a simulation model whose purpose is to estimate a steady state or long-run characteristic of the system. Suppose that the single run produces observations Y_1,Y_2,… Here Y_i denotes the total time job i takes in a job shop.

i) How do you obtain the steady state average time a job takes in the shop?

ii) How does the simulation analyst decide when to stop the simulation? Discuss the considerations in this regard.

iii) What steps are needed for reducing initialization bias in steady-state simulation?

iv) For making initialization bias negligible, how does one decide the extent of data to be deleted?

(3+5+5+5)

6.

a) Though in terms of specifics, the simulation packages differ, generally they have many common characteristics. Describe some of them.

b) What is embedded simulation?

c) When do you think simulation is inappropriate?

(8+4+6)

7.

a) Consider a single finite buffer single server system with arrival and service rates such that equilibrium probabilities of state are

n	0	1	2	3	≥ 4
p(n)	0.415	0.277	0.185	0.123	0.0

Take μ=10. Calculate mean throughput and mean number of customers in the queuing system and mean delay.

b) Is the following table realizable for a finite buffer state-independent M/M/1 system? Why or why not?

n	0	1	2	3	≥ 4
p(n)	0.4	0.3	0.2	0.1	0.0

c) The demand rate of a particular item is 12000 units per year. The set up cost per run is Rs 350 and the holding cost is Re 0.20 per unit per month. If no shortages are allowed and the replacement is instantaneous, determine (i) the optimum lot size and (ii) the optimum scheduling period.

(6+6+6)