**TEST CODE: MIII (Objective type) 2006**

## SYLLABUS

**Algebra** — Permutations and combinations. Binomial Theorem. Theory of equations. Inequalities. Complex numbers and De Moivre's theorem. Elementary set theory. Functions and relations. Algebra of Matrices, Determinant, Rank and inverse of a matrix. Solutions of linear equations. Eigenvalues and Eigenvectors of Matrices.

**Coordinate geometry** — Straight lines, Circles, Parabolas, Ellipses and Hyperbolas. Conic sections and their classification. Elements of three dimensional coordinate geometry — Straight lines, planes and spheres.

**Calculus** — Sequences and series. Taylor and Maclaurin series. Power series. Limit and continuity of functions of one or more variables. Differentiation and integration of functions of one variable with applications. Definite integrals. Areas using integrals. Definite integrals as limits of Riemann sums. Maxima and minima. Differentiation of functions of several variables. Double integrals and their applications. Ordinary linear differential equations.

## SAMPLE QUESTIONS

<u>Note:</u> *For each question there are four suggested answers of which only one is correct.*

1. The number of permutations of $\{1, 2, 3, 4, 5\}$ that keep at least one integer fixed is

   (A) 81 (B) 76 (C) 120 (D) 60.

2. A club with $x$ members is organized into four committees such that

   (a) each member is in exactly two committees,
   (b) any two committees have exactly one member in common.

   Then $x$ has

   (A) exactly two values both between 4 and 8
   (B) exactly one value and this lies between 4 and 8
   (C) exactly two values both between 8 and 16
   (D) exactly one value and this lies between 8 and 16.

3. A subset $S$ of the set of numbers $\{2, 3, 4, 5, 6, 7, 8, 9, 10\}$ is said to be *good* if it has exactly 4 elements and their gcd $= 1$. Then the number of good subsets is

(A) 126 (B) 125 (C) 123 (D) 121.

4. In how many ways can three persons, each throwing a single die once, make a score of 11?

(A) 22 (B) 27 (C) 24 (D) 38.

5. Let $(1+x)^n = C_0 + C_1 x + C_2 x^2 + \cdots + C_n x^n$, $n$ being a positive integer. The value of

$$\left(1+\frac{C_0}{C_1}\right)\left(1+\frac{C_1}{C_2}\right)\cdots\left(1+\frac{C_{n-1}}{C_n}\right)$$

is

(A) $(\frac{n+1}{n+2})^n$. (B) $\frac{n^n}{n!}$. (C) $(\frac{n}{n+1})^n$. (D) $\frac{(n+1)^n}{n!}$.

6. $x^2 + x + 1$ is a factor of $(x+1)^n - x^n - 1$, whenever

(A) $n$ is odd

(B) $n$ is odd and a multiple of 3.

(C) $n$ is an even multiple of 3.

(D) $n$ is odd and not a multiple of 3.

7. If the equation $x^4 + ax^3 + bx^2 + cx + 1 = 0$ (where $a, b, c$ are real numbers) has no real roots and if at least one root is of modulus one, then

(A) $b = c$ (B) $a = c$ (C) $a = b$ (D) none of the above holds.

8. The equation $x^6 - 5x^4 + 16x^2 - 72x + 9 = 0$ has

(A) exactly two distinct real roots

(B) exactly three distinct real roots

(C) exactly four distinct real roots

(D) six distinct real roots.

9. The number of real roots of the equation

$$2\cos\left(\frac{x^2+x}{6}\right) = 2^x + 2^{-x}$$

is

(A) 0. (B) 1. (C) 2. (D) infinitely many.

10. Suppose that the roots of the equation

$$x^4 + ax^3 + bx^2 + cx + 1 = 0$$

are all positive real. Consider the two inequalities

$$\text{(I) } a \leq -4, \qquad \text{(II) } b \geq 6.$$

Then
(A) neither (I) nor (II) need be true.
(B) (I) must be true but (II) need not be true.
(C) (II) must be true but (I) need not be true.
(D) both (I) and (II) must be true.

11. If $a_1, a_2, \ldots, a_n$ are positive real numbers, then

$$\frac{a_1}{a_2} + \frac{a_2}{a_3} + \cdots + \frac{a_{n-1}}{a_n} + \frac{a_n}{a_1}$$

is always

(A) $\geq n$ (B) $\leq n$ (C) $\leq n^{1/n}$ (D) none of the above.

12. The maximum possible value of $xy^2z^3$ subject to the conditions $x, y, z \geq 0$ and $x + y + z = 3$, is

(A) 1 (B) $\frac{9}{8}$ (C) $\frac{9}{4}$ (D) $\frac{27}{16}$.

13. Let $X = \dfrac{1}{1001} + \dfrac{1}{1002} + \dfrac{1}{1003} + \cdots + \dfrac{1}{3001}$. Then,

(A) $X < 1$ (B) $X > 3/2$
(C) $1 < X < 3/2$ (D) none of the above holds.

14. The inequality $\dfrac{2 - gx - x^2}{1 - x + x^2} \leq 3$ is true for all values of $x$ if and only if

(A) $1 \leq g \leq 7$ (B) $-1 \leq g \leq 1$
(C) $-6 \leq g \leq 7$ (D) $-1 \leq g \leq 7$

15. The set of complex numbers $z$ satisfying the equation

$$(3 + 7i)z + (10 - 2i)\overline{z} + 100 = 0$$

represents, in the complex plane,
(A) a straight line
(B) a pair of intersecting straight lines
(C) a point
(D) a pair of distinct parallel straight lines.

16. Let $a$ and $c$ be two complex numbers. Then there is at least one complex number $z$ such that

$$|z-a|+|z+a|=2|c|$$

if and only if

(A) $|c|<|a|$ (B) $|c|\leq|a|$ (C) $|a|<|c|$ (D) $|a|\leq|c|$.

17. Let $X$ be a nonempty set and let $\mathcal{P}(X)$ denote the collection of all subsets of $X$. Define $f: X\times\mathcal{P}(X)\to\mathbb{R}$ by

$$f(x,A)=\begin{cases}1 & \text{if } x\in A\\ 0 & \text{if } x\notin A.\end{cases}$$

Then $f(x,A\cup B)$ equals

(A) $f(x,A)+f(x,B)$

(B) $f(x,A)+f(x,B)-1$

(C) $f(x,A)+f(x,B)-f(x,A)\cdot f(x,B)$

(D) $f(x,A)+|f(x,A)-f(x,B)|$

18. The set $\{x:|x+1/x|>6\}$ equals the set

(A) $(0,3-2\sqrt{2})\cup(3+2\sqrt{2},\infty)$

(B) $(-\infty,-3-2\sqrt{2})\cup(-3+2\sqrt{2},\infty)$

(C) $(-\infty,3-2\sqrt{2})\cup(3+2\sqrt{2},\infty)$

(D) $(-\infty,-3-2\sqrt{2})\cup(-3+2\sqrt{2},3-2\sqrt{2})\cup(3+2\sqrt{2},\infty)$

19. For a pair $(A,B)$ of subsets of the set $X=\{1,2,\ldots,100\}$, let $A\triangle B$ denote the set of all elements of $X$ which belong to exactly one of $A$ or $B$. The number of pairs $(A,B)$ of subsets of $X$ such that $A\triangle B=\{2,4,6,\ldots,100\}$ is

(A) $2^{151}$ (B) $2^{102}$ (C) $2^{101}$ (D) $2^{100}$.

20. The set of all real numbers $x$ such that

$$\big|\,|3-x|-|x+2|\,\big|=5$$

is

(A) $[3,\infty)$ (B) $(-\infty,-2]$

(C) $(-\infty,-2]\cup[3,\infty)$ (D) $(-\infty,-3]\cup[2,\infty)$.

21. If $f(x)=\dfrac{\sqrt{3}\sin x}{2+\cos x}$, then the range of $f(x)$ is

(A) the interval $[-1, \sqrt{3}/2]$ (B) the interval $[-\sqrt{3}/2, 1]$

(C) the interval $[-1, 1]$ (D) none of the above.

22. If $f(x) = x^2$ and $g(x) = x \sin x + \cos x$ then
    (A) $f$ and $g$ agree at no points
    (B) $f$ and $g$ agree at exactly one point
    (C) $f$ and $g$ agree at exactly two points
    (D) $f$ and $g$ agree at more than two points.

23. For non-negative integers $m, n$ define a function as follows

$$f(m,n) = \begin{cases} n+1 & \text{if } m = 0 \\ f(m-1, 1) & \text{if } m \neq 0, n = 0 \\ f(m-1, f(m, n-1)) & \text{if } m \neq 0, n \neq 0 \end{cases}$$

Then the value of $f(1,1)$ is

(A) 4 (B) 3 (C) 2 (D) 1.

24. The rank of the matrix $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ a & b & c & d \\ a^2 & b^2 & c^2 & d^2 \\ a^3 & b^3 & c^3 & d^3 \end{bmatrix}$ is less than 4 if and only if
    (A) $a = b = c = d$
    (B) at least two of $a, b, c, d$ are equal
    (C) at least three of $a, b, c, d$, are equal
    (D) $a, b, c, d$ are distinct real numbers.

25. Let $P$ be a square matrix of order greater than 1 and with positive integer entries. Suppose that $P^{-1}$ exists and has integer entries. Then the set of all possible values of the determinant of $P$ is

(A) $\{1\}$. (B) $\{-1,\ 1\}$.

(C) all non-zero integers. (D) all positive integers.

26. The values of $\eta$ for which the following system of equations have a solution

$$\begin{array}{ccccccc} x & + & y & + & z & = & 1 \\ x & + & 2y & + & 4z & = & \eta \\ x & + & 4y & + & 10z & = & \eta^2 \end{array}$$

are

(A) $\eta = 1, -2$ (B) $\eta = -1, -2$ (C) $\eta = 3, -3$ (D) $\eta = 1, 2$.

27. In a rectangle $ABCD$, the co-ordinates of $A$ and $B$ are (1, 2) and (3, 6) respectively and some diameter of the circle circumscribing $ABCD$ has the equation $2x - y + 4 = 0$. Then the area of the rectangle $ABCD$ is

(A) 16 (B) $2\sqrt{10}$ (C) $2\sqrt{5}$ (D) 20.

28. If the tangent at the point $P$ with co-ordinates $(h, k)$ on the curve $y^2 = 2x^3$ is perpendicular to the straight line $4x = 3y$, then
(A) $(h, k) = (0, 0)$
(B) $(h, k) = (1/8, -1/16)$
(C) $(h, k) = (0, 0)$ or $(h, k) = (1/8, -1/16)$
(D) no such point $(h, k)$ exists.

29. If the line $px + qy = r$ is a tangent to the ellipse $a^2x^2 + b^2y^2 = c^2$, then the maximum possible value of $|pq|$ is

(A) $\max\{a, b\} \cdot |r|/c$ (B) $a^2b^2r^2/2c^4$
(C) $abr^2/c^2$ (D) $abr^2/2c^2$.

30. Let $A$ be the matrix $\begin{pmatrix} x & 0 & 3 \\ -3 & y & y \\ 0 & 0 & 1 \end{pmatrix}$. If the determinant of $A^n$ is equal to the determinant of $A$ for all $n \geq 2$, then the locus of the points $(x, y)$ with $xy \neq 0$ is

(A) a parabola (B) an ellipse
(C) a hyperbola (D) none of the above.

31. The circle $C_1 : x^2 + y^2 = 16$ intersects the circle $C_2$ of radius 5 in such a manner that the common chord is a diameter of $C_1$ and has a slope equal to 3/4. Then a possible position of the centre of $C_2$ is

(A) $(-12/5, 9/5)$ (B) $(9/5, -12/5)$
(C) $(12/5, -9/5)$ (D) $(-9/5, 12/5)$.

32. The locus of the foot of the perpendicular from the origin to the planes which contain the point $(1, -1, 1)$ is
(A) $x^2 + y^2 + z^2 + x - y + z = 0$
(B) $x^2 + y^2 + z^2 + x - y + z = 3$

(C) $x^2 + y^2 + z^2 - x + y - z = 0$

(D) $x^2 + y^2 + z^2 - x + y - z = 3$.

33. The equation of the plane that passes through $(1, 4, -3)$ and contains the line of intersection of the planes $3x-2y+4z-7 = 0$ and $x+5y-2z+9 = 0$ is

(A) $11x + 4y + 8z - 3 = 0$

(B) $13x + 4y + 8z - 5 = 0$

(C) $13x + 5y + 9z - 6 = 0$

(D) $11x + 5y + 9z - 4 = 0$.

34. A variable plane passes through a fixed point $(a, b, c)$ and cuts the coordinate axes at $P, Q, R$ (where none of $P, Q, R$ is the origin). The co-ordinates $(x, y, z)$ of the centre of the sphere passing through $P, Q, R$ and the origin satisfy the equation

(A) $\dfrac{a}{x} + \dfrac{b}{y} + \dfrac{c}{z} = 2$

(B) $\dfrac{x}{a} + \dfrac{y}{b} + \dfrac{z}{c} = 3$

(C) $ax + by + cz = 1$

(D) $ax + by + cz = a^2 + b^2 + c^2$

35. The radius of the sphere that passes through the point $(-3, 4, 0)$ and the circle obtained by intersecting the sphere $x^2+y^2+z^2+4x-2y+4z-16 = 0$ with the plane $2x + 2y + 2z + 9 = 0$ is

(A) 5 (B) 3 (C) 2 (D) 1.

36. Let $S_1$ denote a sphere of unit radius and $C_1$ a cube inscribed in $S_1$. Inductively define spheres $S_n$ and cubes $C_n$ such that $S_{n+1}$ is inscribed in $C_n$ and $C_{n+1}$ is inscribed in $S_{n+1}$. Let $v_n$ denote the sum of the volumes of the first $n$ spheres. Then $\lim_{n\to\infty} v_n$ is

(A) $2\pi$. (B) $\frac{8\pi}{3}$. (C) $\frac{2\pi}{13}(9 + \sqrt{3})$. (D) $\frac{6+2\sqrt{3}}{3}\pi$.

37. If $0 < x < 1$, then the sum of the infinite series

$$\frac{1}{2}x^2 + \frac{2}{3}x^3 + \frac{3}{4}x^4 + \cdots$$

is

(A) $\log \dfrac{1+x}{1-x}$

(B) $\dfrac{x}{1-x} + \log(1 + x)$

(C) $\dfrac{1}{1-x} + \log(1-x)$

(D) $\dfrac{x}{1-x} + \log(1-x)$.

38. Let $\{a_n\}$ be a sequence of real numbers. Then $\lim_{n\to\infty} a_n$ exists if and only if

(A) $\lim_{n\to\infty} a_{2n}$ and $\lim_{n\to\infty} a_{2n+2}$ exists

(B) $\lim_{n\to\infty} a_{2n}$ and $\lim_{n\to\infty} a_{2n+1}$ exist

(C) $\lim_{n\to\infty} a_{2n}$, $\lim_{n\to\infty} a_{2n+1}$ and $\lim_{n\to\infty} a_{3n}$ exist

(D) none of the above.

39. Let $\{a_n\}$ be a sequence of non-negative real numbers such that the series $\sum_{n=1}^{\infty} a_n$ is convergent. If $p$ is a real number such that the series $\sum \dfrac{\sqrt{a_n}}{n^p}$ diverges, then

(A) $p$ must be strictly less than $\frac{1}{2}$

(B) $p$ must be strictly less than or equal to $\frac{1}{2}$

(C) $p$ must be strictly less than or equal to 1 but can be greater than $\frac{1}{2}$

(D) $p$ must be strictly less than 1 but can be greater than or equal to $\frac{1}{2}$.

40. In the Taylor expansion of the function $f(x) = e^{x/2}$ about $x = 3$, the coefficient of $(x-3)^5$ is

(A) $e^{3/2}\dfrac{1}{5!}$ (B) $e^{3/2}\dfrac{1}{2^5 5!}$ (C) $e^{-3/2}\dfrac{1}{2^5 5!}$ (D) none of the above.

41. For $x > 0$, let $f(x) = \lim_{n\to\infty} n(x^{\frac{1}{n}} - 1)$. Then

(A) $f(x) + f(1/x) = 1$

(B) $f(xy) = f(x) + f(y)$

(C) $f(xy) = xf(y) + f(x)$

(D) none of the above is true.

42. If $a = \lim_{n\to\infty}(1 + \dfrac{1}{2} + \cdots + \dfrac{1}{n})$ and $b = \lim_{n\to\infty} \dfrac{1}{n}(1 + \dfrac{1}{2} + \cdots + \dfrac{1}{n})$, then

(A) both $a = \infty$ and $b = \infty$

(B) $a = \infty$ and $b = 0$

(C) $a = \infty$ and $b = 1$

(D) none of the above.

43. Consider two series

$$(i) \sum_{n=1}^{\infty} \sin\frac{\pi}{n} \qquad (ii) \sum_{n=1}^{\infty} (-1)^n \cos\frac{\pi}{n}$$

Then

(A) both $(i)$ and $(ii)$ converge.
(B) $(i)$ converges but $(ii)$ diverges.
(C) $(i)$ diverges but $(ii)$ converges.
(D) Both $(i)$ and $(ii)$ diverge.

44. If $0 < c < d$, then the sequence $a_n = (c^n + d^n)^{1/n}$
(A) is bounded and monotone decreasing.
(B) is bounded and monotone increasing.
(C) is monotone increasing, but unbounded for $1 < c < d$.
(D) is monotone decreasing, but unbounded for $1 < c < d$.

45. The limit
$$\lim_{x \to 0+} \log \left( \frac{1+x}{1-x} \right)^{1/x}$$

(A) exists and is equal to 0
(B) exists and is equal to 1
(C) exists and is equal to 2
(D) does not exist.

46. Let
$$f(x,y) = \begin{cases} e^{-1/(x^2+y^2)} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0). \end{cases}$$
Then $f(x,y)$ is
(A) not continuous at $(0,0)$
(B) continuous at $(0,0)$ but does not have first order partial derivatives
(C) continuous at $(0,0)$ and has first order partial derivatives, but not differentiable at $(0,0)$
(D) differentiable at $(0,0)$

47. Let $f(x)$ be the function
$$f(x) = \begin{cases} \dfrac{x^p}{(\sin x)^q} & \text{if } x > 0 \\ 0 & \text{if } x = 0. \end{cases}$$
Then $f(x)$ is continuous at $x = 0$ if

(A) $p > q$
(B) $p > 0$
(C) $q > 0$
(D) $p < q$.

48. $$\lim_{x \to 0} \sin \frac{e^x - x - 1 - x^2/2}{x^2}$$

(A) does not exist
(B) exists and equals 1
(C) exists and equals 0
(D) exists and equals 1/2.

49. If

$$F(x) = \begin{cases} 1/\left[1 + e^{1/(x-2)} + e^{-1/(x-3)^2}\right] & \text{if } x \neq 2 \text{ or if } x \neq 3 \\ 1 & \text{if } x = 2 \\ 1/(1+e) & \text{if } x = 3. \end{cases}$$

Then

(A) $F(x)$ is continuous at $x = 2$, but not at $x = 3$.
(B) $F(x)$ is continuous at $x = 3$, but not at $x = 2$.
(C) $F(x)$ is not continuous at either $x = 2$ or $x = 3$.
(D) $F(x)$ is continuous at both $x = 2$ and $x = 3$.

50. Let $p > 1$ and for $x > 0$, define $f(x) = (x^p - 1) - p(x - 1)$. Then

(A) $f(x)$ is an increasing function of $x$ on $(0, \infty)$
(B) $f(x)$ is a decreasing function of $x$ on $(0, \infty)$
(C) $f(x) \geq 0$ for all $x > 0$
(D) $f(x)$ takes both positive and negative values for $x \in (0, \infty)$.

51. Let $y^2 = F(x)$, where $F(x)$ is a polynomial of degree 3, then

$$2\frac{d}{dx}\left(y^3 \frac{d^2y}{dx^2}\right)$$

equals

| | |
|---|---|
| (A) $F''F'''$ | (B) $3/2FF' + (F'')^2F'''$ |
| (C) $3/2F(F')^2$ | (D) constant times $F$. |

52. The map $f(x) = a_0 \cos|x| + a_1 \sin|x| + a_2|x|^3$ is differentiable at $x = 0$ if and only if

| | |
|---|---|
| (A) $a_1 = 0$ and $a_2 = 0$ | (B) $a_0 = 0$ and $a_1 = 0$ |
| (C) $a_1 = 0$ | (D) $a_0, a_1, a_2$ can take any real value. |

53. $f(x)$ is a differentiable function on the real line such that $\lim_{x\to\infty} f(x) = 1$ and $\lim_{x\to\infty} f'(x) = \alpha$. Then

| | |
|---|---|
| (A) $\alpha$ must be 0 | (B) $\alpha$ need not be 0, but $\lvert\alpha\rvert < 1$ |
| (C) $\alpha > 1$ | (D) $\alpha < -1$. |

54. Let $f$ and $g$ be two differentiable functions such that $f'(x) \leq g'(x)$ for all $x < 1$ and $f'(x) \geq g'(x)$ for all $x > 1$. Then

(A) if $f(1) \geq g(1)$, then $f(x) \geq g(x)$ for all $x$
(B) if $f(1) \leq g(1)$, then $f(x) \leq g(x)$ for all $x$
(C) $f(1) \leq g(1)$
(D) $f(1) \geq g(1)$.

55. $\int_0^{\pi} \min(\sin x, \cos x) dx$ equals

(A) $1 - 2\sqrt{2}$ (B) $1$ (C) $0$ (D) $1 - \sqrt{2}$.

56. The length of the curve $x = t^3$, $y = 3t^2$ from $t = 0$ to $t = 4$ is

(A) $5\sqrt{5} + 1$ (B) $8(5\sqrt{5} + 1)$
(C) $5\sqrt{5} - 1$ (D) $8(5\sqrt{5} - 1)$.

57. Let

$$f(x) = \begin{cases} x & \text{if } x \in [0,2] \\ 0 & \text{if } x \notin [0,2] \end{cases}$$

$$g(y) = \begin{cases} 1 & \text{if } x \in [0,2] \\ 0 & \text{if } x \notin [0,2]. \end{cases}$$

Let $A = \{(x,y) : x + y \leq 3\}$. Then the value of the integral

$$\iint_A f(x)g(y)\ dx\ dy$$

equals

(A) $\frac{9}{2}$ (B) $\frac{7}{2}$ (C) $4$ (D) $\frac{19}{6}$.

58. If $M = \int_0^{\pi/2} \frac{\cos x}{x+2} dx$ and $N = \int_0^4 \frac{\sin x \cos x}{(x+1)^2} dx$,
then the value of $M - N$ is

(A) $\pi$ (B) $\pi/4$ (C) $\frac{2}{\pi - 4}$ (D) $\frac{2}{\pi + 4}$.

59. Let $0 < \alpha < \beta < 1$. Then

$$\sum_{k=1}^{\infty} \int_{\frac{1}{k+\beta}}^{\frac{1}{k+\alpha}} \frac{1}{1+x} dx$$

is equal to

(A) $\log_e \dfrac{\beta}{\alpha}$ (B) $\log_e \dfrac{1+\beta}{1+\alpha}$ (C) $\log_e \dfrac{1+\alpha}{1+\beta}$ (D) $\infty$.

60. The minimum value of the function $f(x,y) = 4x^2 + 9y^2 - 12x - 12y + 14$ is

(A) 1 (B) 3 (C) 14 (D) none of the above.

61. A metal plate of radius 1 is placed on the $XY$ plane with its centre at the origin and its temperature distribution is given by the function

$$T(x,y) = e^x \cos y + e^y \cos x, \quad x^2 + y^2 \leq 1$$

Then the direction in which the temperature increases most rapidly at the centre is towards the point

(A) $(1,0)$ (B) $(0,1)$ (C) $(1/\sqrt{2}, 1/\sqrt{2})$ (D) $(-1/\sqrt{2}, -1/\sqrt{2})$

62. The minimum value of

$$(\sqrt{3}\cos\theta + \sin\theta)(\sin\theta + \cos\theta)$$

in the interval $(0, \pi/2)$ is attained

(A) at exactly one point (B) at exactly two points

(C) at exactly three points (D) nowhere.

63. A man is in a muddy field, 300 feet from the nearest point $A$ of a straight road bordering the field. He wants to walk to a point $B$ on the road 600 feet away from $A$. He can walk 3 feet per second in the muddy field and 5 feet per second on the road. Then the least time in which he can walk to $B$ is

(A) 201.42 sec. (B) 200 sec. (C) 210 sec. (D) 220 sec.

64. A right circular cone is cut from a solid sphere of radius $a$, the vertex and the circumference of the base being on the surface of the sphere. The height of the cone when its volume is maximum is

(A) $4a/3$ (B) $3a/2$ (C) $a$ (D) $6a/5$.

65. The volume of the solid, generated by revolving about the horizontal line $y = 2$ the region bounded by $y^2 \leq 2x$, $x \leq 8$ and $y \geq 2$, is

(A) $2\sqrt{2\pi}$ (B) $28\pi/3$ (C) $84\pi$ (D) none of the above.

66. An inclined plane passing through a diameter of the base of a solid circular cylinder of diameter $D$ cms placed vertically, cuts the curved surface of the cylinder at a maximum height of $H$ cms. Then the volume of the cutout portion in cubic centimeters is

(A) $HD^2/6$ (B) $\pi HD^2/8$ (C) $7\pi HD^2/6$ (D) none of the above.

67. The coordinates of a moving point $P$ satisfy the equations

$$\frac{dx}{dt} = \tan x, \quad \frac{dy}{dt} = -\sin^2 x, \quad t \geq 0.$$

If the curve passes through the point $(\pi/2, 0)$ when $t = 0$, then the equation of the curve in rectangular co-ordinates is

(A) $y = 1/2\cos^2 x$ (B) $y = \sin 2x$

(C) $y = \cos 2x + 1$ (D) $y = \sin^2 x - 1$.

68. Let $y$ be a function of $x$ satisfying

$$\frac{dy}{dx} = 2x^3\sqrt{y} - 4xy$$

If $y(0) = 0$ then $y(1)$ equals

(A) $1/4e^2$ (B) $1/e$ (C) $e^{1/2}$ (D) $e^{3/2}$.

69. Let $f(x)$ be a given differentiable function. Consider the following differential equation in $y$

$$f(x)\frac{dy}{dx} = yf'(x) - y^2. \tag{1}$$

The general solution of this equation is given by

(A) $y = -\dfrac{x+c}{f(x)}$ (B) $y^2 = \dfrac{f(x)}{x+c}$

(C) $y = \dfrac{f(x)}{x+c}$ (D) $y = \dfrac{[f(x)]^2}{x+c}$.

70. Let $y(x)$ be a non-trivial solution of the second order linear differential equation

$$\frac{d^2y}{dx^2} + 2c\frac{dy}{dx} + ky = 0,$$

where $c < 0$, $k > 0$ and $c^2 > k$. Then

(A) $|y(x)| \to \infty$ as $x \to \infty$

(B) $|y(x)| \to 0$ as $x \to \infty$

(C) $\lim_{x \to \pm\infty} |y(x)|$ exists and is finite

(D) none of the above is true.