Code: DE-01 / DC-01 **Subject: MATHEMATICS - I**

JUNE 2007

Time: 3 Hours **Max. Marks: 100**

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q. 1. must be written in the space provided for it in the answer book supplied and nowhere else.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or best alternative in the following: (2x10)

a. If ${}^{n}c_{12} = {}^{n}c_{8}$, then n is equal to

(A) 8 **(B)** 12
(C) 16 **(D)** 20

b. $\lim_{x \to 0} \frac{2\sin x - \sin 2x}{x^3}$ is equal to

(A) 0 **(B)** 1
(C) 2 **(D)**
3

c. If the point P(x, y) is equidistant from the points $A(a+b, b-a)$ and $B(a-b, a+b)$, then

(A) bx = ay **(B)** ax = by
(C) x = y **(D)** x + y = 0

d. The area of the triangle formed by the lines y = a + x, y = a – x, y = 0, where a > 0, is

(A) 1 **(B)** a
(C) a^2 **(D)** zero

e. If $\frac{x}{y} + \frac{y}{x} = 2$ and $Y \neq x$, then $\frac{dy}{dx}$ is equal to

(A) 1 **(B)** 2
(C) -1 **(D)** -2

f. $\int \frac{\text{Cos}2x}{\text{Cos}^2 x \, \text{Sin}^2 x} dx$ is equal to

(A) sec x + cosec x **(B)** $\sec x - \operatorname{cosec} x$
(C) $-\sec x \ \operatorname{cosec} x$ **(D)** $\sec x \ \operatorname{cosec} x$

g. The area bounded by the parabola $y^2 = 4ax$ and its latus rectum is

(A) a^2 **(B)** $\frac{2}{3}a^2$
(C) $\frac{4}{3}a^2$ **(D)** $\frac{8}{3}a^2$

h. The solution of differential equation $\frac{dy}{dx} = e^{x-y} + 2xe^{-y}$ is

(A) $y = xe^{x-y} + x^{z-y}e + c$ **(B)** $e^y = e^x + x^2 + c$
(C) $y = e^{x-y} + x^{z-y}e + c$ **(D)** $e^{-y} = e^x + 2x + c$

i. Value of $\sin^{-1} x + \cos^{-1} x$ is

(A) 2π **(B)** π
(C) $\frac{\pi}{2}$ **(D)** $\frac{\pi}{4}$

j. Value of $(\sin 3A - \sin A)\cos A - (\cos 3A + \cos A)\sin A$ is

(A) 0 **(B)** 1
(C) $\frac{1}{2}$ **(D)** $\frac{1}{\sqrt{2}}$

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. The sum of first p terms of an A.P. is the same as the sum of its first q terms. Find the sum of its first (p + q) terms. **(8)**

b. For what value of n are the coefficients of second, third and fourth terms in the expansion of $(1+x)^n$ in A.P.? **(8)**

Q.3 a. Solve for θ the equation $\text{Sin } m\theta + \text{Sin } n\theta = 0$, where $m \neq n$. **(8)**

b. If a, b, c be the sides opposite to the angles A, B, C for a triangle ABC, show that

$$\frac{a+b}{c} = \frac{\text{Cos}\,\frac{A-B}{2}}{\text{Sin}\,\frac{C}{2}}.$$

(8)

Q.4 a. Derive the formula for the angle between the straight lines $y = m_1 x + c_1$ and $y = m_2 x + c_2$. **(8)**

b. Find the equation of a straight line which is perpendicular to 2x – 5y = 30 and the sum of its intercepts on the coordinate axes is 7. **(8)**

Q.5 a. Find the equation of the circle concentric with the circle $2x^2 + 2y^2 + 8x + 10y - 39 = 0$ and having its area equal to 16π. **(8)**

b. Find the centre, eccentricity, foci and length of the latus rectum of the ellipse $4x^2 + 9y^2 - 8x + 36y + 4 = 0$. **(8)**

Q.6 a. Differentiate from the first principle the function y = tan x. **(8)**

b. Evaluate $\lim\limits_{x \to 0} \frac{\sqrt{(1+x+x^2)} - 1}{\text{Sin } 4x}$. **(8)**

Q.7 a. Find the local maximum and minimum values of the function y = sin 3x – 3 sin x, $0 \leq x < 2\pi$. **(8)**

b. Evaluate $\int \frac{x\,dx}{\sqrt{x^2 + a^2} + \sqrt{x^2 - a^2}}$. **(8)**

Q.8 a. Find the area bounded by the curve $\sqrt{y} + \sqrt{x} = \sqrt{a}$ and the coordinate axes. **(8)**

b. Evaluate $\int_{0}^{\pi/2} \frac{\cos x \, dx}{(1+\sin x)(2+\sin x)}$. **(8)**

Q.9 Solve any **TWO** of the following differential equations:-

(i) $xy \frac{dy}{dx} = 1 + x + y + xy$.

(ii) $(x^2 - y^2) dx = 2xy \, dy$.

(iii) $(1 - x^2) \frac{dy}{dx} - xy = 1$. **(2 x 8 = 16)**