Code: D-01 / DC-01 **Subject: MATHEMATICS - I**
Time: 3 Hours **June 2006**
Max. Marks: 100

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q. 1. must be written in the space provided for it in the answer book supplied and nowhere else.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or best alternative in the following: (2x10)

a. Sum of the series $S = 1^2 - 2^2 + 3^2 - 4^2 + \ldots - 100^2 + 101^2$ is equal to

(A) 348551 **(B)** -1000
(C) 5151 **(D)** None of the above

b. The value of tan 15° is

(A) $2-\sqrt{3}$ **(B)** $-2+\sqrt{3}$
(C) $2+\sqrt{3}$ **(D)** $-2-\sqrt{3}$

c. In a triangle ABC, let a = BC, b = CA and c = AB. If $\angle B = 60^\circ$, then

(A) $(a-b)^2 = c^2 - ab$ **(B)** $(b-c)^2 = a^2 - bc$
(C) $(c-a)^2 = b^2 - ac$ **(D)** None of the above

d. The circles $x^2 + y^2 + px + py - 7 = 0$ and $x^2 + y^2 - 10x + 2py + 1 = 0$ cut orthogonally if the value of p is

(A) 3 **(B)** -2
(C) -3 **(D)** 1

e. The eccentricity of the ellipse $16x^2 + 25y^2 = 400$ is

(A) 3 **(B)** $5/3$

(C) 5 **(D)** $3/5$

f. The derivative of $-\cos(\log x)$ is

(A) $\sin(\log x)$ **(B)** $\frac{\sin(\log x)}{x}$

(C) $-\sin(\log x)$ **(D)** $-\cos\left(\frac{1}{x}\right)$

g. The value of the $\lim_{x\to 0} \frac{\sin xe^{-x}}{x}$ is

(A) 0 **(B)** 1

(C) e **(D)** Does not exist

h. The integral $\int_0^1 xe^x$ is equal to

(A) $e-1$ **(B)** $e+1$

(C) 0 **(D)** 1

i. The area under the curve $y = x^2$ between x = 0 and x = 1 is

(A) 1 **(B)** $1/2$

(C) $1/3$ **(D)** $1/4$

j. The solution of $\frac{dy}{dx} = y^2, y(1) = -1$ is

(A) $y = -\frac{1}{x}$ **(B)** $y = \frac{1}{x}$

(C) $y = x + 1$ **(D)** $y = x^2 + 1$

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. Show that the sum to n terms of the series 1.3.5 + 3.5.7 + 5.7.9 + … is $n(2n^3 + 8n^2 + 7n - 2)$. **(8)**

b. If α, β are the roots of the quadratic equation $x^2 + px + 1 = 0$ and γ, δ are the roots of the quadratic equation $x^2 + qx + 1 = 0,$ then show that $(\alpha - \gamma)(\beta - \gamma)(\alpha + \delta)(\beta + \delta) = q^2 - p^2$. **(8)**

Q.3 a. If A + B + C = $180°$, prove that $\sin(B + C - A) + \sin(C + A - B) + \sin\ (A + B - C)$ $= 4 \sin A \sin B \sin C$. **(8)**

b. Show that sin $\frac{\pi}{14}$ is a root of the equation $8x^3 - 4x^2 - 4x + 1 = 0$. **(8)**

Q.4 a. Find the value of c_1 such that the circles $x^2 + y^2 + 2x + 2y + 1 = 0$ and $x^2 + y^2 + 2x + 2y + c_1 = 0$ touch each other. **(8)**

b. For what values of k the points $(-1,4), (2,-2)$ and $(-4-k, 6-2k)$ are collinear? **(8)**

Q.5 a. Find the equation of the circle for which $x - y - 1 = 0$ is a tangent and $x + y = 0,$ $x - y + 4 = 0$ are normals. **(8)**

b. Find the values of a, b such that the line ax + by + 1 = 0 is tangent to the hyperbola $3x^2 - y^2 = 3$ and is parallel to the line y = 2x + 4. **(8)**

Q.6 a. Evaluate the limit $\lim_{x \to 0} \frac{\sin^{-1} x - \tan^{-1} x}{x^3}$. **(8)**

b. Consider the function $f(x) = \begin{cases} x^2 \sin \frac{1}{x}, & \text{for } x \neq 0, \\ 0, & \text{for } x = 0. \end{cases}$. Find $f'(0)$ using first principle. Is $f'(x)$ continuous at x = 0? **(8)**

Q.7 a. Find the local maximum and minimum values of $f(x) = e^{\sin x}$ in $(0, 2\pi)$. **(8)**

b. Find the area of the region bounded by $y = x^2 + 2$, $y = -x, x = 0$ and x = 1. **(8)**

Q.8 a. Evaluate the following integral $\int \frac{dx}{\cos^6 x + \sin^6 x}$. **(8)**

b. Evaluate the following definite integral $\int_0^1 \frac{x^3 dx}{x^2 + 2x + 1}$. **(8)**

Q.9 a. Solve the differential equation

$$\frac{dy}{dx} + \left(\frac{2x+1}{x}\right)y = e^{-2x}.$$ **(8)**

b. Solve the differential equation

$x \sin y dx + (x^2 + 1) \cos y dy = 0$. **(8)**