Code: D-01 / DC-01 **Subject: MATHEMATICS - I Time: 3 Hours** Max. Marks: 100

NOTE: There are 11 Questions in all.

- Question 1 is compulsory and carries 16 marks. Answer to Q. 1. must be written in the space provided for it in the answer book supplied and nowhere else.
- Answer any THREE Questions each from Part I and Part II. Each of these questions carries 14 marks.
- Any required data not explicitly given, may be suitably assumed and stated.

Q.1 Choose the correct or best alternative in the following:

a.
$$1^2 + 2^2 + 3^2 + \dots + n^2$$
 is equal to

(A)
$$\frac{n(n+1)}{2}$$
.
(C) $\frac{n(n+1)(2n+1)}{6}$.

(B)
$$\frac{(n+1)(2n+1)}{2}$$
.

- b. If $\log_9 x = 1.5$ then x is equal to
 - **(A)** 3

(B) 27

(C) 9

- **(D)** 15
- c. The value of $\sin 75^{\circ} \cos 15^{\circ} + \cos 75^{\circ} \sin 15^{\circ}$ is equal to
 - **(A)** 1.

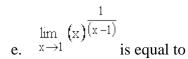
(B) 0.

(C) -1.

- **(D)** $\frac{1}{2}$
- d. If α , β are the roots of $ax^2 + bx + c = 0$ then $\alpha^2 + \beta^2$ is

(B) $\frac{b^2 - 2ac}{a^2}$.

(C) b² - 2ac.



(A) 1.

(B) 0.

(**C**) e.

(**D**) =

f. $\int \log x \, dx$ is equal to

(A) $x \log x - x + c$.

(B) ^{x log x}.

(C) log x.

(**D**) $\frac{1}{x} \log x$.

g. The maximum value of $y = 2 \cos 2x - \cos 4x$, $0 \le x \le \frac{\pi}{2}$ is

(A) -1.

(B) $\frac{1}{2}$.

(c) $\frac{3}{2}$.

(D) 1.

h. The equation of the line which is perpendicular to the line 3x - 4y + 7 = 0 and passes through the point (-3, 2) is

(A) 4x + 3y + 5 = 0.

(B) 4x + 3y - 3 = 0.

(C) 4x + 3y + 6 = 0.

(D) 3x - 4y + 6 = 0.

PART I

Answer any THREE Questions. Each question carries 14 marks.

Q.2 a. Prove that 7 divides $2^{3n} - 1$ for all positive integers n.

(7)

b. Find the condition that the roots of equation $ax^2 + bx + c = 0$ are equal. (7)

Q.3 a. Evaluate $\tan\left(\frac{5\pi}{12}\right)$.

(6)

b. If $x + \frac{1}{x} = 2\cos\theta$, prove that $x^3 + \frac{1}{x^3} = 2\cos 3\theta$. (8)

- Q.4 a. If a, b, c are lengths of sides opposite to angles A, B, C in a triangle ABC, then show that $a^2 = b^2 + c^2 2bc \cos A$. (7)
 - b. Show that in a triangle ABC,

 a Sin (B C) + b Sin (C A) + c Sin (A B) =
 0,
 where a, b, c are lengths of sides opposite to angles A, B, C. (7)
- Q.5 a. Find the condition that the points (1, 1), (3, 5) and (a, b) are collinear. (7)
 - b. Find equations of lines which pass through the point (4, 5) and make an angle 45° with the line 2x + y + 1 = 0. (7)
- Q.6 a. Find the equation of the circle concentric with the circle $x^2 + y^2 4x 6y 9 = 0$ and which passes through (-4, 5). (7)
 - b. Show that $y^2 8y x + 19 = 0$ represents a parabola. Find its focus, vertex and directrix. (7)

PART II

Answer any THREE Questions. Each question carries 14 marks.

Q.7 a. Find
$$\lim_{x\to 0} \frac{\sin 3x}{x}$$
. (6)

- b. Examine the continuity of the function f(x) = [x], where [x] is greatest integer $\leq x$, x being any real number. (8)
- **Q.8** a. Show that the semi verticle angle of a cone of maximum volume and a given slant height is $\tan^{-1} \sqrt{2}$. (7)
 - b. Find the equation of tangent and normal to the curve $y = x^2 9$ at the point where it intersects the positive x-axis. (7)
- Q.9 a. Find a reduction formula for the integral $\int \sin^{n} x dx$. (7)

b. Evaluate
$$\int_{0}^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$$

- Q.10 a. Find the area bounded by $y^2 = 4 ax$ and its latus rectum. (7)
 - b. Find the volume of the solid obtained by revolving the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, about its major axis. (7)
- Q.11 a. Solve the equation $\frac{dy}{dx} = \frac{x y}{x + y}$
 - b. Solve the equation $x = \frac{dy}{dx} 3y = x^2$. (8)