Geometry

Based on Maharashtra State Board Syllabus

Std. X

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As Per New Syllabus

Std. X

Geometry

Mr. Biju Babu
(B.E Prod.)

Mr. Kannan G. Iyer
(B.Sc. Maths)

Mr. Pankaj Dubey
(B.E. Mech.)

Salient Features:

- ✓ Written as per the new textbook.
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PREFACE

Geometry is the mathematics of properties, measurement and relationships of points, lines, angles, surfaces and solids. It is widely used in the fields of science, engineering, computers, architecture etc. It is a vast subject dealing with the study of properties, definitions, theorems, areas, perimeter, angles, triangles, mensuration, co-ordinates, constructions etc.

The study of Geometry requires a deep and intrinsic understanding of concepts. Hence to ease this task we bring to you **"Std. X: Geometry"** a complete and thorough guide critically analysed and extensively drafted to boost the students confidence. The question answer format of this book helps the student to understand and grasp each and every concept thoroughly. The book is based on the new text book and covers the entire syllabus. It contains answers to textual exercises, problems sets and Question bank. It also includes additional questions for practice. All the diagrams are neat and have proper labelling. The book has a unique feature that all the constructions are as per the scale. Another feature of the book is its layout which is attractive and inspires the student to read.

There is always room for improvement and hence we welcome all suggestions and regret any errors that may have occurred in the making of this book.

A book affects eternity; one can never tell where its influence stops.

Best of luck to all the aspirants!

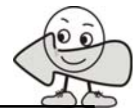
Your's faithfully

Publisher

Contents

01 SIMILARITY

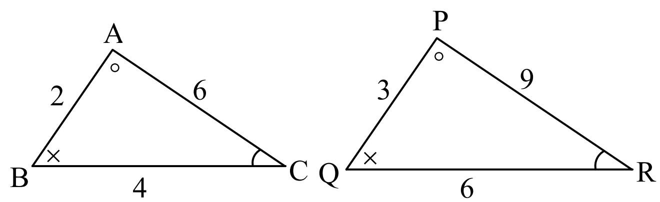
Concepts of Std. IX



Similarity of triangles

For a given one-to-one correspondence between the vertices of two triangles if,

i. their corresponding angles are congruent and
ii. their corresponding sides are in proportion then the correspondence is known as similarity and the two triangles are said to be similar.



In the figure, for correspondence ABC ↔ PQR.

i. $\angle A \cong \angle P$, $\angle B \cong \angle Q$, $\angle C \cong \angle R$

ii. $\frac{AB}{PQ} = \frac{2}{3}$, $\frac{BC}{QR} = \frac{4}{6} = \frac{2}{3}$, $\frac{AC}{PR} = \frac{6}{9} = \frac{2}{3}$

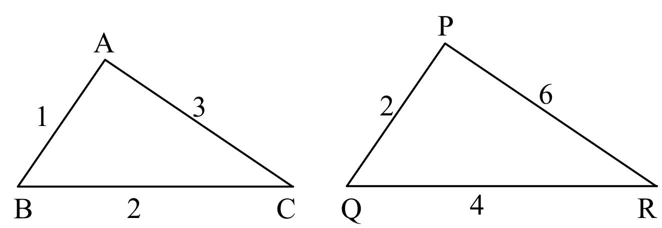
i.e. $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$

Hence, ΔABC and ΔPQR are similar triangles and are symbolically written as ΔABC ~ ΔPQR.

Test of similarity of triangles

1. **S–S–S test of similarity:**

For a given one-to-one correspondence between the vertices of two triangles, the two triangles are similar, if the sides of one triangle are proportional to the corresponding sides of the other triangle.



In the figure,

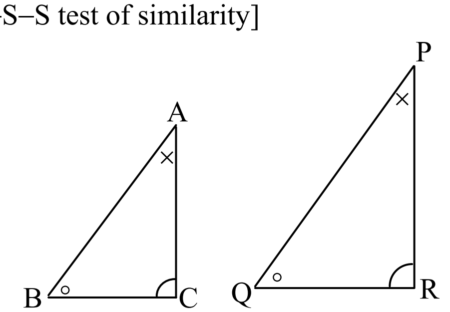
$\frac{AB}{PQ} = \frac{1}{2}$, $\frac{BC}{QR} = \frac{2}{4} = \frac{1}{2}$, $\frac{AC}{PR} = \frac{3}{6} = \frac{1}{2}$

$\therefore \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$

then ΔABC ↔ ΔPQR ----- [By S–S–S test of similarity]

2. **A–A–A test of similarity [A–A test]:**

For a given one to one correspondence between the vertices of two triangles, the two triangles are similar if the angles of one triangle are congruent to the corresponding angles of the other triangle.



In the figure,

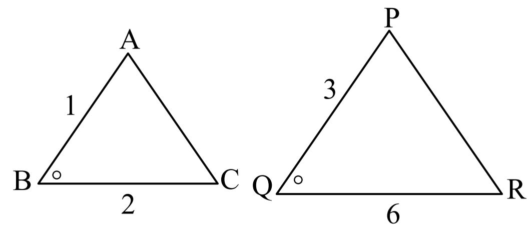
if $\angle A \cong \angle P$, $\angle B \cong \angle Q$, $\angle C \cong \angle R$

then ΔABC ~ ΔPQR ----- [By A–A–A test of similarity]

Note: A–A–A test is verified same as A–A test of similarity.

3. **S–A–S test of similarity:**

For a given one to one correspondence between the vertices of two triangles, the two triangles are similar if two sides of a triangle are proportional to the two corresponding sides of the other triangle and the corresponding included angles are also congruent.



In the figure,

$\frac{AB}{PQ} = \frac{1}{3}, \frac{BC}{QR} = \frac{2}{6} = \frac{1}{3}$

$\therefore$ $\frac{AB}{PQ} = \frac{BC}{QR}$ and $\angle B \cong \angle Q$

then $\Delta ABC \sim \Delta PQR$ ----- [By S–A–S test of similarity]

Converse of the test for similarity:

i. **Converse of S–S–S test:**

If two triangles are similar, then the corresponding sides are in proportion.

If $\Delta ABC \sim \Delta PQR$ then,

$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$ ----- [Corresponding sides of similar triangles]

ii. **Converse of A–A–A test:**

If two triangles are similar, then the corresponding angles are congruent.

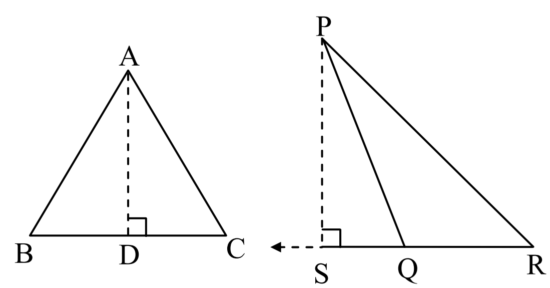
If $\Delta ABC \sim \Delta PQR$

then $\angle A \cong \angle P$, $\angle B \cong \angle Q$, and $\angle C \cong \angle R$ ----- [Corresponding angles of similar triangles]

Properties of ratios of areas of two triangles

Property – I

The ratio of areas of two triangles is equal to the ratio of the product of their base and corresponding height.



Given: In ΔABC and ΔPQR, seg $AD \perp$ seg BC, B–D–C, seg $PS \perp$ ray QR, S–Q–R

To prove that: $\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{BC \times AD}{QR \times PS}$

Proof:

$A(\Delta ABC) = \frac{1}{2} \times BC \times AD$ ----- (i) [Area of a triangle = $\frac{1}{2}$ × base × height]

$A(\Delta PQR) = \frac{1}{2} \times QR \times PS$ ----- (ii)

Dividing (i) by (ii), we get

$$\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{\frac{1}{2} \times BC \times AD}{\frac{1}{2} \times QR \times PS}$$

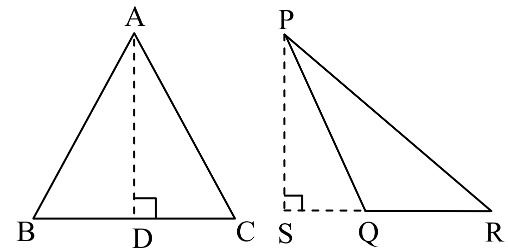
$\therefore$ $\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{BC \times AD}{QR \times PS}$

For Understanding

When do you say the triangles have equal heights?

We discuss this in three cases.

Case – I

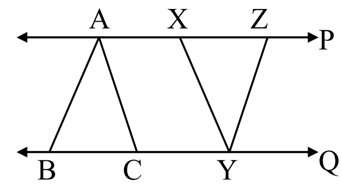


In the adjoining figure,

segments AD and PS are the corresponding heights of ΔABC and ΔPQR respectively.

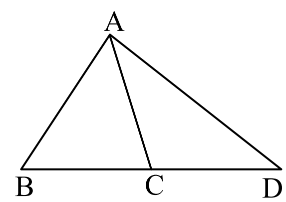
If AD = PS, then ΔABC and ΔPQR are said to have equal height.

Case – II



In the adjoining figure, ΔABC and ΔXYZ have their one vertex on one of the parallel lines and the other two vertices lie on the other parallel line, hence the two triangles are said to lie between the same parallel lines and are said to have equal heights.

Case – III

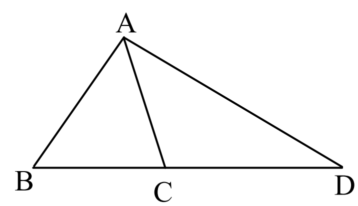


In the adjoining figure, ΔABC, ΔACD and ΔABD have a common vertex A and the opposite sides BC, CD and BD of the triangles lie on the same line, hence ΔABC, ΔACD and ΔABD are said to have equal heights and BC, CD and BD are their respective bases.

Property – II

The ratio of areas of two triangles having equal height is equal to the ratio of their corresponding bases.

Example:



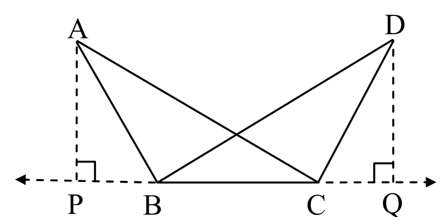
ΔABC, ΔACD and ΔABD have a common vertex A and their opposite sides BC, CD, BD lie on the same line, hence they have equal heights.

$$\therefore \quad \frac{A(\Delta ABC)}{A(\Delta ACD)} = \frac{BC}{CD}, \frac{A(\Delta ABC)}{A(\Delta ABD)} = \frac{BC}{BD}, \frac{A(\Delta ACD)}{A(\Delta ABD)} = \frac{CD}{BD}$$

Property – III

The ratio of areas of two triangles having equal base is equal to the ratio of their corresponding heights.

Example:



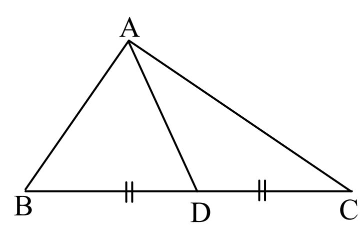
ΔABC and ΔDCB have a common base BC.

$$\therefore \quad \frac{A(\Delta ABC)}{A(\Delta DCB)} = \frac{AP}{DQ}$$

Property IV

Areas of two triangles having equal bases and equal heights are equal.

Example:

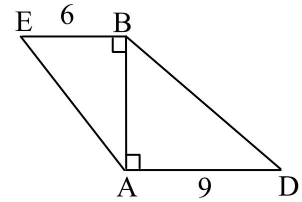


ΔABD and ΔACD have a common vertex A and their opposite sides BD and DC lie on the same line, hence the triangles have equal heights. Also their bases BD and DC are equal.

$\therefore \quad A(\Delta ABD) = A(\Delta ACD)$

Exercise 1.1

1. In the adjoining figure, seg BE ⊥ seg AB and seg BA ⊥ seg AD.
If BE = 6 and AD = 9, find $\frac{A(\Delta ABE)}{A(\Delta BAD)}$.



Solution:

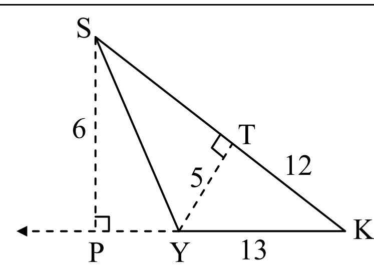
$$\frac{A(\Delta ABE)}{A(\Delta BAD)} = \frac{BE}{AD}$$ ----- [Ratio of areas of two triangles having equal base is equal to the ratio of their corresponding heights.]

$$\therefore \frac{A(\Delta ABE)}{A(\Delta BAD)} = \frac{6}{9}$$

$$\therefore \boxed{\frac{\mathbf{A(\Delta ABE)}}{\mathbf{A(\Delta BAD)}} = \frac{\mathbf{2}}{\mathbf{3}}}$$

---

2. In the adjoining figure, seg SP ⊥ side YK and seg YT ⊥ seg SK.
If SP = 6, YK = 13, YT = 5 and TK = 12,
then find A(ΔSYK):A(ΔYTK).



Solution:

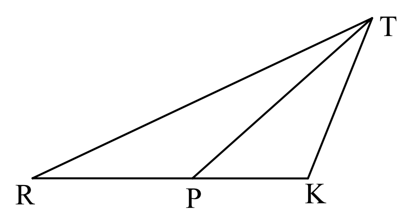
$$\frac{A(\Delta SYK)}{A(\Delta YTK)} = \frac{YK \times SP}{TK \times YT}$$ ----- [Ratio of areas of two triangles is equal to the ratio of the product of their bases and corresponding heights.]

$$\therefore \frac{A(\Delta SYK)}{A(\Delta YTK)} = \frac{13 \times 6}{12 \times 5}$$

$$\boxed{\therefore \frac{\mathbf{A(\Delta SYK)}}{\mathbf{A(\Delta YTK)}} = \frac{\mathbf{13}}{\mathbf{10}}}$$

---

3. In the adjoining figure, RP:PK = 3:2, then find the values of:
i. A(ΔTRP):A(ΔTPK)
ii. A(ΔTRK):A(ΔTPK)
iii. A(ΔTRP):A(ΔTRK)



Solution:

RP:PK = 3:2 ----- [Given]

Let the common multiple be x.

∴ RP = $3x$, PK = $2x$

RK = RP + PK ----- [R–P–K]

∴ RK = $3x + 2x$

∴ RK = $5x$

i. $$\frac{A(\Delta TRP)}{A(\Delta TPK)} = \frac{RP}{PK}$$ ----- [Ratio of the areas of two triangles having equal heights is equal to the ratio of their corresponding bases.]

$$\therefore \frac{A(\Delta TRP)}{A(\Delta TPK)} = \frac{3x}{2x}$$

$$\boxed{\therefore \frac{\mathbf{A(\Delta TRP)}}{\mathbf{A(\Delta TPK)}} = \frac{\mathbf{3}}{\mathbf{2}}}$$

ii. $\frac{A(\Delta TRK)}{A(\Delta TPK)} = \frac{RK}{PK}$ ----- [Ratio of the areas of two triangles having equal heights is equal to the ratio of their corresponding bases.]

$\therefore \frac{A(\Delta TRK)}{A(\Delta TPK)} = \frac{5x}{2x}$

$\therefore \boxed{\frac{\mathbf{A(\Delta TRK)}}{\mathbf{A(\Delta TPK)}} = \frac{\mathbf{5}}{\mathbf{2}}}$

iii. $\frac{A(\Delta TRP)}{A(\Delta TRK)} = \frac{RP}{RK}$ ----- [Ratio of the areas of two triangles having equal heights is equal to the ratio of their corresponding bases.]

$\therefore \frac{A(\Delta TRP)}{A(\Delta TRK)} = \frac{3x}{5x}$

$\therefore \boxed{\frac{\mathbf{A(\Delta TRP)}}{\mathbf{A(\Delta TRK)}} = \frac{\mathbf{3}}{\mathbf{5}}}$

---

4. The ratio of the areas of two triangles with the common base is 6:5. Height of the larger triangle is 9 cm. Then find the corresponding height of the smaller triangle.

Solution:

Let A_1 and A_2 be the areas of larger triangle and smaller triangle respectively.

$\frac{A_1}{A_2} = \frac{6}{5}$ ----- [Given]

Let the corresponding heights be h_1 and h_2 respectively.

$\therefore \frac{A_1}{A_2} = \frac{h_1}{h_2}$ ----- [Ratio of the areas of two triangles having equal base is equal to the ratio of their corresponding heights.]

$\therefore \frac{6}{5} = \frac{9}{h_2}$ ----- [$\because h_1 = 9$, given]

$\therefore h_2 = \frac{5 \times 9}{6}$

$\therefore h_2 = \frac{15}{2}$

$\therefore h_2 = 7.5$ cm

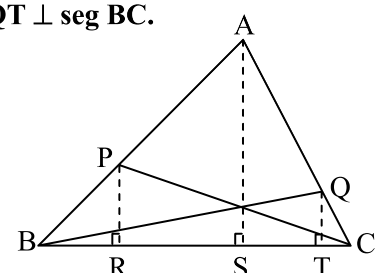
$\therefore$ **Height of smaller triangle is 7.5 cm.**

---

5. In the adjoining figure, seg PR $\perp$ seg BC, seg AS $\perp$ seg BC and seg QT $\perp$ seg BC. Find the following ratios:

i. $\frac{\mathbf{A(\Delta ABC)}}{\mathbf{A(\Delta PBC)}}$ **ii.** $\frac{\mathbf{A(\Delta ABS)}}{\mathbf{A(\Delta ASC)}}$

iii. $\frac{\mathbf{A(\Delta PRC)}}{\mathbf{A(\Delta BQT)}}$ **iv.** $\frac{\mathbf{A(\Delta BPR)}}{\mathbf{A(\Delta CQT)}}$



Solution:

i. $\frac{A(\Delta ABC)}{A(\Delta PBC)} = \frac{AS}{PR}$ ----- [Ratio of the areas of two triangles having equal bases is equal to the ratio of their corresponding heights.]

ii. $\frac{A(\Delta ABS)}{A(\Delta ASC)} = \frac{BS}{SC}$ ----- [Ratio of the areas of two triangles having equal heights is equal to the ratio of their corresponding bases.]

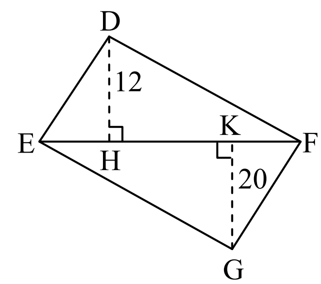
iii. $\frac{A(\Delta PRC)}{A(\Delta BQT)} = \frac{RC \times PR}{BT \times QT}$ ----- [Ratio of the areas of two triangles is equal to the ratio of product of their bases and corresponding heights.]

iv. $\frac{A(\Delta BPR)}{A(\Delta CQT)} = \frac{BR \times PR}{CT \times QT}$ ----- [Ratio of the areas of two triangles is equal to the ratio of product of their bases and corresponding heights.]

---

6. In the adjoining figure, seg DH ⊥ seg EF and seg GK ⊥ seg EF. If DH = 12 cm, GK = 20 cm and A(ΔDEF) = 300 cm², then find

i. length of EF ii. A(ΔGEF) iii. A(□DFGE)



Solution:

i. Area of triangle = $\frac{1}{2} \times$ base $\times$ height

$\therefore$ $A(\Delta DEF) = \frac{1}{2} \times EF \times DH$

$\therefore$ $300 = \frac{1}{2} \times EF \times 12$ ----- [$\because$ $A(\Delta DEF) = 300\ cm^2$]

$\therefore$ $300 = EF \times 6$

$\therefore$ $\frac{300}{6} = EF$

$\boxed{\therefore \quad \textbf{EF} = \textbf{50 cm.}}$

ii. $\frac{A(\Delta DEF)}{A(\Delta GEF)} = \frac{DH}{GK}$ ----- [Ratio of the areas of two triangles having equal bases is equal to the ratio of their corresponding heights.]

$\therefore$ $\frac{300}{A(\Delta GEF)} = \frac{12}{20}$

$\therefore$ $300 \times 20 = 12 \times A(\Delta GEF)$

$\therefore$ $\frac{300 \times 20}{12} = A(\Delta GEF)$

$\therefore$ $A(\Delta GEF) = \frac{300 \times 20}{12}$

$\boxed{\therefore \quad \mathbf{A(\Delta GEF) = 500\ cm^2}}$

iii. A(□DFGE) = A(ΔDEF) + A(ΔGEF) ----- [Area addition property]

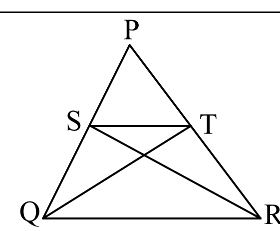
$\therefore$ A(□DFGE) = 300 + 500 = 800

$\boxed{\therefore \quad \mathbf{A(\square DFGE) = 800\ cm^2}}$

---

7. In the adjoining figure, seg ST || side QR. Find the following ratios.

i. $\frac{\mathbf{A(\Delta PST)}}{\mathbf{A(\Delta QST)}}$ **ii.** $\frac{\mathbf{A(\Delta PST)}}{\mathbf{A(\Delta RST)}}$ **iii.** $\frac{\mathbf{A(\Delta QST)}}{\mathbf{A(\Delta RST)}}$



Solution:

i. $\dfrac{A(\Delta PST)}{A(\Delta QST)} = \dfrac{PS}{QS}$

ii. $\dfrac{A(\Delta PST)}{A(\Delta RST)} = \dfrac{PT}{TR}$

[Ratio of the areas of two triangles having equal heights is equal to the ratio of their corresponding bases.]

iii. ΔQST and ΔRST lie between the same parallels ST and QR

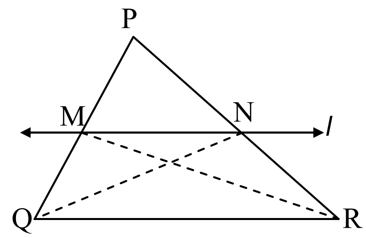
$\therefore$ Their heights are equal.

$\therefore$ $A(\Delta QST) = A(\Delta RST)$ ----- [Areas of two triangles having common base i.e. ST and equal heights are equal.]

$\therefore$ $\dfrac{A(\Delta QST)}{A(\Delta RST)} = 1$

Basic Proportionality Theorem

If a line parallel to a side of a triangle intersects the other sides in two distinct points, then the line divides these sides in proportion.



Given: In ΔPQR, line l || side QR.
Line l intersects side PQ and side PR in points M and N respectively such that P–M–Q and P–N–R.

To Prove that: $\dfrac{PM}{MQ} = \dfrac{PN}{NR}$

Construction: Draw seg QN and seg RM.

Proof:

In ΔPMN and ΔQMN, where P–M–Q

$\dfrac{A(\Delta PMN)}{A(\Delta QMN)} = \dfrac{PM}{MQ}$ ----- (i) [Ratio of the areas of two triangles having equal heights is equal to the ratio of their corresponding bases.]

In ΔPMN and ΔRMN, where P–N–R

$\dfrac{A(\Delta PMN)}{A(\Delta RMN)} = \dfrac{PN}{NR}$ ----- (ii) [Ratio of the areas of two triangles having equal heights is equal to the ratio of their corresponding bases.]

$A(\Delta QMN) = A(\Delta RMN)$ ----- (iii) [Areas of two triangles having equal bases and equal heights are equal.]

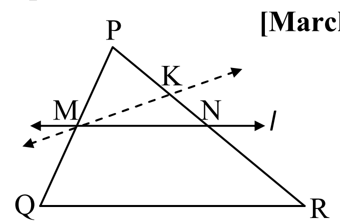
$\therefore$ $\dfrac{A(\Delta PMN)}{A(\Delta QMN)} = \dfrac{A(\Delta PMN)}{A(\Delta RMN)}$ ----- (iv) [From (i), (ii) and (iii)]

$\therefore$ $\dfrac{PM}{MQ} = \dfrac{PN}{NR}$ ----- [From (i), (ii) and (iv)]

Converse of Basic Proportionality Theorem

If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.

[March 2013]



Given: Line l intersects the side PQ and side PR of ΔPQR in the points M and N respectively such that $\dfrac{PM}{MQ} = \dfrac{PN}{NR}$.

To Prove that: Line l || side QR.

Proof:

Let us consider that line *l* is not parallel to the side QR.

Then, there must be another line passing through M which is parallel to the side QR

Let line MK be that line.

Line MK intersects the side PR at K, ----- [P–K–R]

In ΔPQR,

line MK || side QR.

$\therefore \quad \frac{PM}{MQ} = \frac{PK}{KR}$ ----- (i) [By B.P.T.]

But, $\frac{PM}{MQ} = \frac{PN}{NR}$ ----- (ii) [Given]

$\therefore \quad \frac{PK}{KR} = \frac{PN}{NR}$ ----- [From (i) and (ii)]

$\therefore \quad \frac{PK + KR}{KR} = \frac{PN + NR}{NR}$ ----- [By componendo]

$\therefore \quad \frac{PR}{KR} = \frac{PR}{NR}$ ----- [P–K–R, P–N–R]

$\therefore \quad KR = NR$

∴ Points K and N are not different.

∴ Line MK and line MN coincide.

∴ line MN || side QR

∴ line *l* || side QR

Applications of Basic Proportionality Theorem

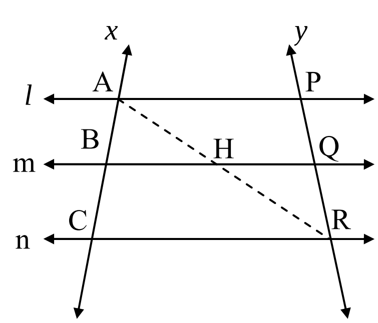
i. Property of intercepts made by three parallel lines on a transversal:
The ratio of the intercepts made on a transversal by three parallel lines is equal to the ratio of the corresponding intercepts made on any other transversal by the same three parallel lines.

Given: Line *l* || line m || line n

The transversals x and y intersect these parallel lines at points A, B, C and P, Q, R respectively.

To Prove that: $\frac{AB}{BC} = \frac{PQ}{QR}$

Construction: Draw seg AR to intersect line m at point H.



Proof:

In ΔACR,

seg BH || side CR

$\therefore \quad \frac{AB}{BC} = \frac{AH}{HR}$ ----- (i) [By B.P.T.]

In ΔARP,

seg HQ || side AP

$\therefore \quad \frac{PQ}{QR} = \frac{AH}{HR}$ ----- (ii) [By B.P.T.]

$\therefore \quad \frac{AB}{BC} = \frac{PQ}{QR}$ ----- [From (i) and (ii)]

$AC^2 = AD^2 + DC^2$ ----- [By Pythagoras theorem]

$\therefore$ $AC^2 = AD^2 + (BD + BC)^2$ ----- [D–B–C]

$\therefore$ $AC^2 = AD^2 + BD^2 + 2BD \cdot BC + BC^2$ ----- (i)

In ΔADB,

$\angle ADB = 90°$ ----- [Given]

$\therefore$ $AB^2 = AD^2 + BD^2$ ----- (ii) [By Pythagoras theorem]

$\therefore$ $AC^2 = AB^2 + 2BD \cdot BC + BC^2$ ----- [From (i) and (ii)]

$\therefore$ $\mathbf{AC^2 = AB^2 + BC^2 + 2BD \cdot BC}$

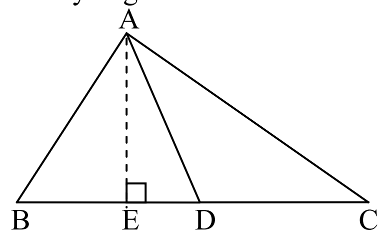
Appollonius theorem

It is a theorem relating to the length of the median of a triangle to the length of the sides.
Here, we shall prove this theorem for an acute angled triangle using application of Pythagoras theorem.

Given: In ΔABC, seg AD is the median.

To prove that: $AB^2 + AC^2 = 2AD^2 + 2CD^2$

Construction: Draw seg AE $\perp$ seg BC such that B–E–D.



Proof:

In ΔABD,

$\angle ADB < 90°$ ----- [Given]

$\therefore$ $AB^2 = AD^2 + BD^2 - 2BD \cdot DE$ ----- (i) [By application of Pythagoras theorem]

In ΔADC,

$\angle ADC > 90°$ ----- [Given]

$\therefore$ $AC^2 = AD^2 + CD^2 + 2CD \cdot DE$ ----- (ii) [By application of Pythagoras theorem]

Adding (i) and (ii)

$AB^2 + AC^2 = 2AD^2 - 2BD \cdot DE + 2CD \cdot DE + BD^2 + CD^2$ ----- (iii)

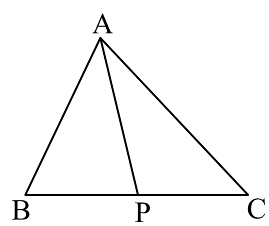
But, BD = CD ----- (iv) [D is the midpoint of seg BC.]

$AB^2 + AC^2 = 2AD^2 - 2CD \cdot DE + 2CD \cdot DE + CD^2 + CD^2$ ----- [From (iii) and (iv)]

$\therefore$ $\mathbf{AB^2 + AC^2 = 2AD^2 + 2CD^2}$

Exercise 1.7

1. In ΔABC, AP is a median. If AP = 7, $AB^2 + AC^2 = 260$, then find BC.



Solution:

In ΔABC,

seg AP is the median ----- [Given]

$AB^2 + AC^2 = 2AP^2 + 2PC^2$ ----- [By Appollonius Principle]

$\therefore$ $260 = 2(7)^2 + 2PC^2$

$\therefore$ $260 = 2 \times 49 + 2PC^2$

$\therefore$ $260 = 98 + 2PC^2$

$\therefore$ $260 - 98 = 2PC^2$

$\therefore$ $162 = 2PC^2$

$\therefore$ $\dfrac{162}{2} = PC^2$

$\therefore$ $81 = PC^2$

$\therefore$ PC = 9 units ----- [Taking square roots]

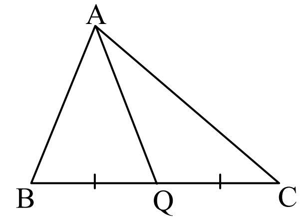
$BC = 2 \times PC$ ----- [P is the midpoint of seg BC.]

$\therefore$ $BC = 2 \times 9$

$\boxed{\therefore \ \mathbf{BC = 18\ units}}$

---

2. In the adjoining figure, $AB^2 + AC^2 = 122$, $BC = 10$.
Find the length of the median on side BC.



Solution:

seg AQ is the median on side BC.

$\therefore$ $BQ = \frac{1}{2}BC$ ----- [Q is the midpoint of side BC.]

$\therefore$ $BQ = \frac{1}{2} \times 10$

$\therefore$ $BQ = 5$ units

In ΔABC,

seg AQ is the median

$\therefore$ $AB^2 + AC^2 = 2AQ^2 + 2BQ^2$ ----- [By Appollonius theorem]

$\therefore$ $122 = 2AQ^2 + 2(5)^2$

$\therefore$ $122 = 2AQ^2 + 50$

$\therefore$ $122 - 50 = 2AQ^2$

$\therefore$ $72 = 2AQ^2$

$\therefore$ $AQ^2 = \frac{72}{2}$

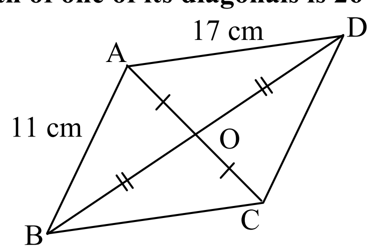
$\therefore$ $AQ^2 = 36$

$\therefore$ $AQ = 6$ units ----- [Taking square roots]

$\boxed{\therefore \ \text{Length of the median on side BC = 6 units.}}$

---

3. Adjacent sides of a parallelogram are 11 cm and 17 cm. If the length of one of its diagonals is 26 cm, find the length of the other diagonal.



Solution:

Let □ABCD be the parallelogram and its diagonals AC and BD intersect each other at O.

$AB = 11$ cm, $AD = 17$ cm, $BD = 26$cm

$\therefore$ $BO = \frac{1}{2}BD$ ----- [Diagonals of a parallelogram bisect each other.]

$\therefore$ $BO = \frac{1}{2} \times 26$

$\therefore$ $BO = 13$cm

In ΔABD,

O is the midpoint of seg BD ----- [Diagonals of a parallelogram bisect each other.]

$\therefore$ seg AO is the median ----- [By definition]

$\therefore$ $AB^2 + AD^2 = 2AO^2 + 2BO^2$ ----- [By Appollonius theorem]

$\therefore$ $11^2 + 17^2 = 2AO^2 + 2(13)^2$

$\therefore$ $121 + 289 = 2AO^2 + 2 \times 169$

$\therefore$ $410 = 2(AO)^2 + 338$

$\therefore$ $410 - 338 = 2AO^2$

$\therefore$ $72 = 2AO^2$

$\therefore$ $\frac{72}{2} = AO^2$

$\therefore$ $AO^2 = 36$

$\therefore$ AO = 6 units ----- [Taking square roots]

$AO = \frac{1}{2}AC$ ----- [Diagonals of a parallelogram bisect each other.]

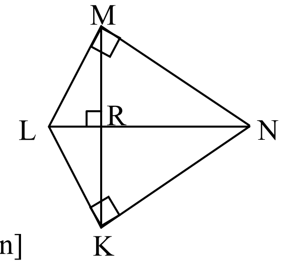
$\therefore$ $6 = \frac{1}{2}AC$

$\therefore$ AC = 12 units

$\therefore$ **Length of the other diagonal = 12 units.**

---

4. In the adjoining figure, ∠LMN = 90° and LKN = 90°, seg MK ⊥ LN. Prove that R is the midpoint of seg MK.



Proof:

In ΔLMN,

m∠LMN = 90° ----- [Given]

seg MR ⊥ hypotenuse LN ----- [Given]

$\therefore$ $MR^2 = LR \times RN$ ----- (i) [By property of geometric mean]

In ΔLKN,

∠LKN = 90° ----- [Given]

seg KR ⊥ hypotenuse LN ----- [Given]

$\therefore$ $KR^2 = LR \times RN$ ----- (ii) [By property of geometric mean]

$\therefore$ $MR^2 = KR^2$ ----- [From (i) and (ii)]

$\therefore$ MR = KR

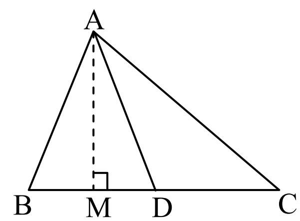
$\therefore$ **R is midpoint of seg MK.**

---

5. seg AD is the median of ΔABC and AM ⊥ BC.

Prove that: i. $AC^2 = AD^2 + BC \times DM + \left(\frac{BC}{2}\right)^2$

ii. $AB^2 = AD^2 - BC \times DM + \left(\frac{BC}{2}\right)^2$



Proof:

i. In ΔADC,

∠ADC > 90° ----- [Given]

$\therefore$ $AC^2 = AD^2 + CD^2 + 2\,CD \cdot DM$ ----- (i) [By application of theorem of Pythagoras]

$CD = \frac{1}{2}BC$ ----- (ii) [D is the midpoint of seg BC.]

$\therefore$ $AC^2 = AD^2 + \left(\frac{1}{2}BC\right)^2 + 2\left(\frac{1}{2}BC\right) \cdot DM$ ----- [From (i) and (ii)]

$\therefore$ $AC^2 = AD^2 + \left(\frac{BC}{2}\right)^2 + BC \cdot DM$

$\therefore$ $\mathbf{AC^2 = AD^2 + BC \times DM + \left(\frac{BC}{2}\right)^2}$

ii. In acute angled ΔABD,
seg AM $\perp$ side BD

$\therefore$ $AB^2 = AD^2 + BD^2 - 2\ BD \cdot DM$ ----- (i) [By application of theorem of Pythagoras]

$BD = \frac{1}{2}BC$ ----- (ii) [D is the midpoint of seg BC.]

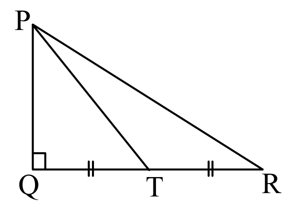
$\therefore$ $AB^2 = AD^2 + \left(\frac{1}{2}BC\right)^2 - 2\left(\frac{1}{2}BC\right) \cdot DM$ ----- [From (i) and (ii)]

$\therefore$ $AB^2 = AD^2 + \left(\frac{BC}{2}\right)^2 - BC \cdot DM$

$\therefore$ $\mathbf{AB^2 = AD^2 - BC \cdot DM + \left(\frac{BC}{2}\right)^2}$

---

6. In the adjoining figure, $\angle PQR = 90°$, T is the mid point of side QR.
Prove that: $(PR)^2 = 4(PT)^2 - 3(PQ)^2$



Proof :

In ΔPQR,
seg PT is the median ----- [By definition]

$\therefore$ $PQ^2 + PR^2 = 2PT^2 + 2QT^2$ ----- [By Appollonius theorem]

$\therefore$ $PR^2 = 2PT^2 + 2QT^2 - PQ^2$ ----- (i)

In ΔPQT,
$\angle PQT = 90°$ ----- [Given]

$\therefore$ $PT^2 = PQ^2 + QT^2$ ----- [By Pythagoras theorem]

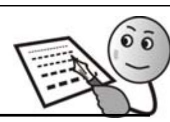
$\therefore$ $QT^2 = PT^2 - PQ^2$ ----- (ii)

$\therefore$ $PR^2 = 2PT^2 + 2[PT^2 - PQ^2] - PQ^2$ ----- [From (i) and (ii)]

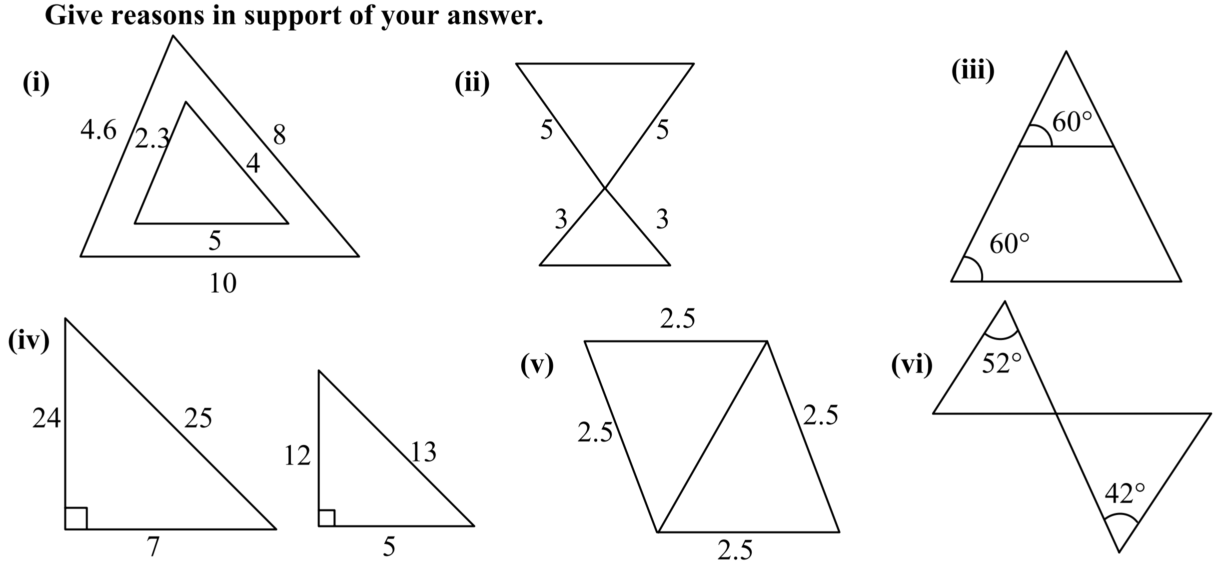
$\therefore$ $PR^2 = 2PT^2 + 2PT^2 - 2PQ^2 - PQ^2$

$\therefore$ $\mathbf{PR^2 = 4PT^2 - 3PQ^2}$

Problem Set - I



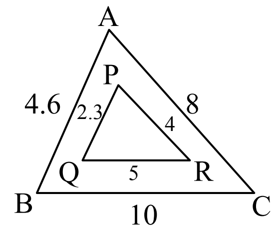
1. In each of the following figures, you find two triangles. Indicate whether the triangles are similar. Give reasons in support of your answer.



Solution:

i. The given two triangles are similar.

Reason:



$\frac{AB}{PQ} = \frac{4.6}{2.3} = \frac{2}{1}$ ----- (i)

$\frac{BC}{QR} = \frac{10}{5} = \frac{2}{1}$ ----- (ii)

$\frac{AC}{PR} = \frac{8}{4} = \frac{2}{1}$ ----- (iii)

In ΔABC and ΔPQR,

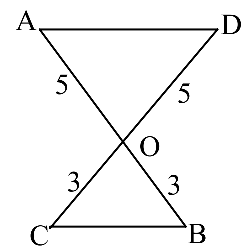
$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$ ----- [From (i), (ii) and (iii)]

$\therefore$ $\Delta ABC \sim \Delta PQR$ ----- [By S–S–S test of similarity]

Hence, the given two triangles are similar.

ii. The given two triangles are similar.

Reason:



$\frac{OA}{OB} = \frac{5}{3}$ ----- (i)

$\frac{OD}{OC} = \frac{5}{3}$ ----- (ii)

In ΔAOD and ΔBOC,

$\frac{OA}{OB} = \frac{OD}{OC}$ ----- [From (i) and (ii)]

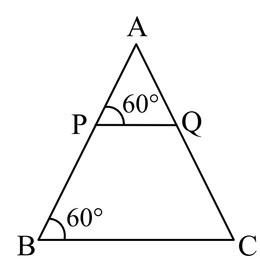
$\angle AOD \cong \angle BOC$ ----- [Vertically opposite angles]

$\therefore$ $\Delta AOD \sim \Delta BOC$ ----- [By S–A–S test of similarity]

Hence, the given two triangles are similar.

iii. The given two triangles are similar.

Reason:



In ΔAPQ and ΔABC,

$\angle APQ \cong \angle ABC$ ----- [Each is 60°]

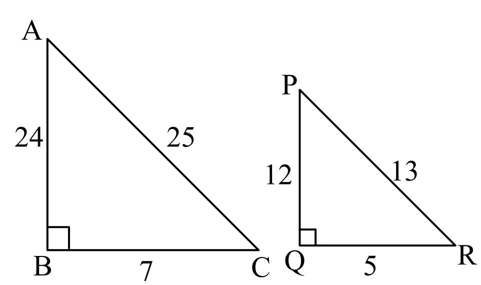
$\angle PAQ \cong \angle BAC$ ----- [Common angle]

$\therefore$ $\Delta APQ \sim \Delta ABC$ ----- [By A–A test of similarity]

Hence, given two triangles are similar.

iv. The given two triangles are not similar.

Reason:



$\frac{AB}{PQ} = \frac{24}{12} = \frac{2}{1}$ ----- (i)

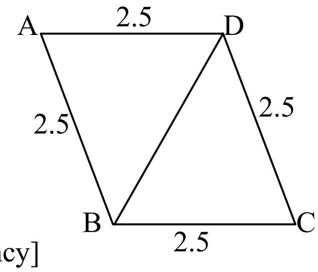
$\frac{BC}{QR} = \frac{7}{5}$ ----- (ii)

$\frac{AC}{PR} = \frac{25}{13}$ ----- (iii)

$\therefore$ $\frac{AB}{PQ} \neq \frac{BC}{QR} \neq \frac{AC}{PR}$ ----- [From (i), (ii) and (iii)]

$\therefore$ ΔABC is not similar to ΔPQR.

Hence, the given two triangles are not similar.



v. The given two triangles are similar.

Reason:

In ΔABD and ΔCBD,

seg AB ≅ seg BC ----- [Each is 2.5 units]

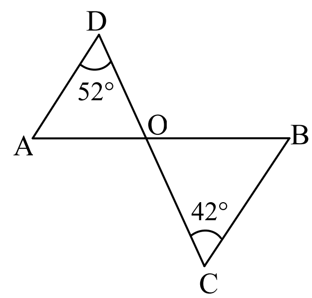
seg AD ≅ seg CD ----- [Each is 2.5 units]

seg BD ≅ seg BD ----- [Common side]

∴ $\Delta ABD \cong \Delta CBD$ ----- [By S–S–S test of congruency]

∴ $\Delta ABD \sim \Delta CBD$ ----- [Two congruent triangles are similar to each other]

Hence, the given two triangles are similar.



vi. The given two triangles are not similar.

Reason:

$m\angle ADO = 52°$

$m\angle BCO = 42°$

$\angle ADO \neq \angle BCO$

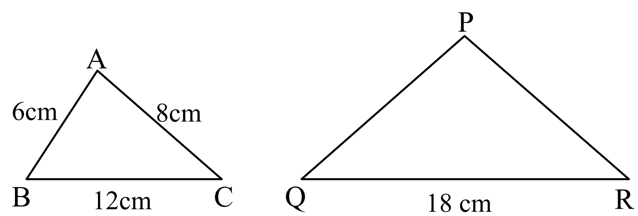
∴ Corresponding angles of ΔADO and ΔBCO are not congruent.

∴ **ΔADO is not similar to ΔBCO.**

---

2. A triangle ABC with sides AB = 6 cm, BC = 12 cm and AC = 8 cm is enlarged to ΔPQR such that its largest side is 18 cm. Find the ratio and hence, find the lengths of the remaining sides of ΔPQR.

Solution:



$\Delta ABC \sim \Delta PQR$ ----- [A figure and its enlarged figure are similar]

∴ $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$ ----- [Corresponding sides of similar triangles]

$\frac{6}{PQ} = \frac{12}{18} = \frac{8}{PR}$

∴ $\frac{6}{PQ} = \frac{8}{PR} = \frac{2}{3}$ ----- (i)

∴ **Ratio of the sides is 2:3**

$\frac{6}{PQ} = \frac{2}{3}$ ----- [From (i)]

∴ $PQ = \frac{3 \times 6}{2}$

∴ **PQ = 9 cm**

$\frac{8}{PR} = \frac{2}{3}$ ----- [From (i)]

∴ $PR = \frac{8 \times 3}{2}$

∴ **PR = 12 cm**

In ΔABC and ΔDEF,

$\therefore$ $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$ ----- [From (i), (ii) and (iii)]

$\therefore$ $\Delta ABC \sim \Delta DEF$ ----- [By S–S–S test of similarity]

$\angle C \cong \angle F$ ----- (iv) [Corresponding angles of similar triangles]

In ΔABC,

$\angle A + \angle B + \angle C = 180°$ ----- [Sum of the measures of all angles of a triangle is 180°.]

$\therefore$ $80° + 60° + \angle C = 180°$

$\therefore$ $\angle C = 180° - 140°$

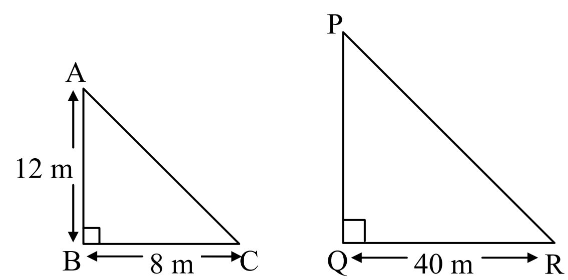
$\therefore$ $\angle C = 40°$ ----- (v)

$\boxed{\therefore \quad \angle \mathbf{F = 40°}}$ ----- [From (iv) and (v)]

---

10. A vertical stick 12 m long casts a shadow 8 m long on the ground. At the same time a tower casts the shadow of length 40 m on the ground. Determine the height of the tower.

Solution:



Let AB represent the vertical stick, AB = 12 m.

BC represents the shadow of the stick, BC = 8 m.

PQ represents the height of the tower.

QR represents the shadow of the tower.

QR = 40 m

$\Delta ABC \sim \Delta PQR$

$\therefore$ $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$ ----- [c.s.s.t.]

$\therefore$ $\frac{12}{PQ} = \frac{8}{40}$

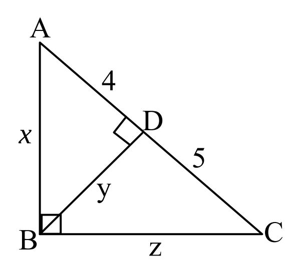
$\therefore$ $PQ = 12 \times 5 = 60$

$\boxed{\therefore \quad \textbf{Height of the tower = 60 m}}$

---

11. In each of the figure an altitude is drawn to the hypotenuse. The lengths of different segments are marked in each figure. Determine the value of x, y, z in each case.

Solution:



i. In ΔABC, $m\angle ABC = 90°$ ----- [Given]

seg $BD \perp$ hypotenuse AC ----- [Given]

$\therefore$ $BD^2 = AD \times DC$ ----- [By property of geometric mean]

$\therefore$ $y^2 = 4 \times 5$

$\therefore$ $y = \sqrt{4 \times 5}$ ----- [Taking square roots]

$\boxed{\therefore \quad \boldsymbol{y = 2\sqrt{5}}}$

In ΔADB,

$m\angle ADB = 90°$ ----- [Given]

$AB^2 = AD^2 + BD^2$ ----- [By Pythagoras theorem]

$\therefore$ $AB^2 = 4^2 + y^2$

$\therefore \quad x^2 = 4^2 + (2\sqrt{5})^2$

$\therefore \quad x^2 = 16 + 20$

$\therefore \quad x^2 = 36$

$\boxed{\therefore \quad x = 6}$ ----- [Taking square roots]

In ΔBDC,

m$\angle$BDC = 90°

$\therefore \quad BC^2 = BD^2 + CD^2$ ----- [By Pythagoras theorem]

$\therefore \quad z^2 = y^2 + 5^2$

$\therefore \quad z^2 = (2\sqrt{5})^2 + 5^2$

$\therefore \quad z^2 = 20 + 25$

$\therefore \quad z^2 = 45$

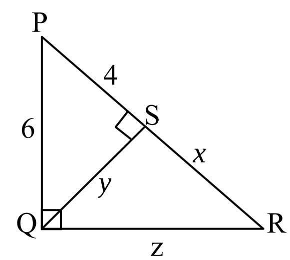
$\therefore \quad z = \sqrt{9 \times 5}$ ----- [Taking square roots]

$\boxed{\therefore \quad z = 3\sqrt{5}}$

ii. In ΔPSQ,

m $\angle$PSQ = 90° ----- [Given]

$\therefore \quad PQ^2 = PS^2 + QS^2$ ----- [By Pythagoras theorem]



$\therefore \quad 6^2 = 4^2 + y^2$

$\therefore \quad 36 = 16 + y^2$

$\therefore \quad y^2 = 36 - 16$

$\therefore \quad y^2 = 20$

$\therefore \quad y = \sqrt{4 \times 5}$ ----- [Taking square roots]

$\boxed{\therefore \quad y = 2\sqrt{5}}$

In ΔPSQ,

m $\angle$PSQ = 90° ----- [Given]

seg QS $\perp$ hypotenuse PR ----- [Given]

$\therefore \quad QS^2 = PS \times SR$

$\therefore \quad y^2 = 4 \times x$

$\therefore \quad \left(2\sqrt{5}\right)^2 = 4x$

$\therefore \quad 20 = 4x$

$\therefore \quad x = \frac{20}{4}$

$\boxed{\therefore \quad x = 5}$

In ΔQSR,

m $\angle$QSR = 90° ----- [Given]

$\therefore \quad QR^2 = QS^2 + SR^2$ ----- [By Pythagoras theorem]

$\therefore \quad z^2 = y^2 + x^2$

$\therefore \quad z^2 = (2\sqrt{5})^2 + (5)^2$

$\therefore \quad z^2 = 20 + 25$

$\therefore \quad z^2 = 45$

$\therefore \quad z = \sqrt{9 \times 5}$ ----- [Taking square roots]

$\boxed{\therefore \quad z = 3\sqrt{5}}$

$\therefore$ $MR^2 = \frac{128}{2}$ $\therefore$ $MR^2 = 64$

$\therefore$ MR = 8 units ----- [Taking square roots]

QR = 2MR ----- [M is the midpoint of seg QR.]

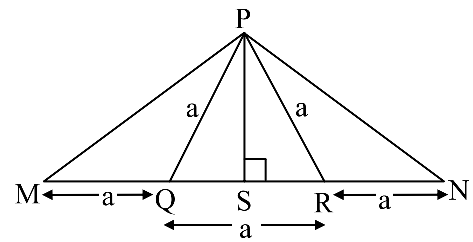
$\therefore$ QR = 2×8 = 16 units

$\boxed{\therefore \ \textbf{QR = 16 units}}$

---

14. From the information given in the adjoining figure,

Prove that: PM = PN = $\sqrt{3} \times a$, where QR = a.



Proof:

In ΔPQR,

QM = QR = a ----- [Given]

$\therefore$ Q is midpoint of seg MR. ----- [By definition]

$\therefore$ seg PQ is the median ----- [By definition]

$\therefore$ $PM^2 + PR^2 = 2PQ^2 + 2QM^2$ ----- [By Appollonius theorem]

$\therefore$ $PM^2 + a^2 = 2a^2 + 2a^2$

$\therefore$ $PM^2 + a^2 = 4a^2$ $\therefore$ $PM^2 = 4a^2 - a^2$

$\therefore$ $PM^2 = 3a^2$ $\therefore$ PM = $\sqrt{3}$ a

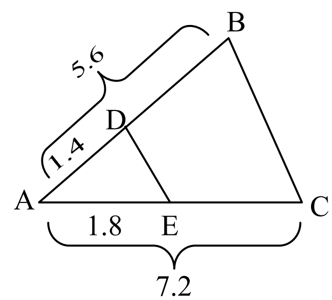
Similarly, we can prove

PN = $\sqrt{3}$ a

$\therefore$ **PM = PN = $\sqrt{3}$ a.**

---

15. D and E are the points on sides AB and AC such that AB = 5.6, AD = 1.4, AC = 7.2 and AE = 1.8 Show that DE || BC.



Proof:

DB = AB – AD ----- [A–D–B]

$\therefore$ DB = 5.6 – 1.4

$\therefore$ DB = 4.2 units

$\therefore$ $\frac{AD}{DB} = \frac{1.4}{4.2} = \frac{1}{3}$ ----- (i)

EC = AC – AE ----- [A–E–C]

$\therefore$ EC = 7.2 – 1.8

$\therefore$ EC = 5.4 units

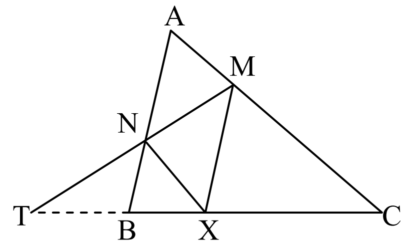
$\therefore$ $\frac{AE}{EC} = \frac{1.8}{5.4} = \frac{1}{3}$ ----- (ii)

In ΔABC,

$\frac{AD}{DB} = \frac{AE}{EC}$ ----- [From (i) and (ii)]

$\therefore$ **seg DE || seg BC** ----- [By converse of B.P.T.]

18. Let X be any point on side BC of ΔABC, XM and XN are drawn parallel to BA and CA. MN meets produced BC in T. Prove that TX^2 = TB·TC.



Proof:

In ΔTXM,

seg BN || seg XM ----- [Given]

∴ $\dfrac{TN}{NM} = \dfrac{TB}{BX}$ ----- (i) [By B.P.T.]

In ΔTMC,

seg XN || seg CM ----- [Given]

∴ $\dfrac{TN}{NM} = \dfrac{TX}{CX}$ ----- (ii)

∴ $\dfrac{TB}{BX} = \dfrac{TX}{CX}$ ----- [From (i) and (ii)]

∴ $\dfrac{BX}{TB} = \dfrac{CX}{TX}$ ----- [By invertendo]

∴ $\dfrac{BX+TB}{TB} = \dfrac{CX+TX}{TX}$ ----- [By componendo]

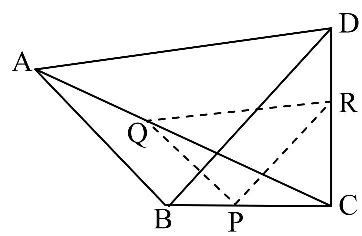
∴ $\dfrac{TX}{TB} = \dfrac{TC}{TX}$ ----- [T–B–X, T–X–C]

∴ **TX^2 = TB·TC**

---

19. Two triangles, ΔABC and ΔDBC, lie on the same side of the base BC. From a point P on BC, PQ || AB and PR || BD are drawn. They intersect AC at Q and DC at R.

Prove that QR || AD.



Proof:

In ΔABC,

seg PQ || side AB ----- [Given]

∴ $\dfrac{CP}{PB} = \dfrac{CQ}{AQ}$ ----- (i) [By B.P.T.]

In ΔBCD,

seg PR || side BD ----- [Given]

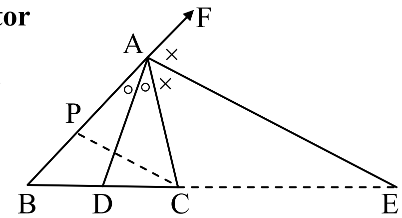
$\dfrac{CP}{PB} = \dfrac{CR}{RD}$ ----- (ii) [By B.P.T.]

In ΔACD,

$\dfrac{CQ}{AQ} = \dfrac{CR}{RD}$ ----- [From (i) and (ii)]

∴ **seg QR || seg AD** ----- [By converse of B.P.T.]

22. The bisector of interior $\angle$A of ΔABC meets BC in D. The bisector of exterior $\angle$A meets BC produced in E. Prove that $\frac{BD}{BE} = \frac{CD}{CE}$.



Construction: Draw seg CP || seg AE meeting AB at P.

Proof:

In ΔABC,

Ray AD is bisector of $\angle$BAC ----- [Given]

$\therefore \frac{AB}{AC} = \frac{BD}{CD}$ ----- (i) [By property of angle bisector of triangle]

In ΔABE

seg CP || seg AE ----- [Given]

$\therefore \frac{BC}{CE} = \frac{BP}{AP}$ ----- [B. P. T]

$\frac{BC+CE}{CE} = \frac{BP+AP}{AP}$ ----- [componendo]

$\therefore \frac{BE}{CE} = \frac{AB}{AP}$ ----- (ii)

seg CP || seg AE, on transversal BF.

$\angle FAE \cong \angle APC$ ----- (iii) [corresponding angles]

seg CP || seg AE on transversal AC.

$\angle CAE \cong \angle ACP$ ----- (iv) [alternate angles]

Also, $\angle FAE \cong \angle CAE$ ----- (v) [seg AE bisects $\angle$FAC]

$\therefore \angle APC \cong \angle ACP$ ----- (vi) [From (iii), (iv) and (v)]

In ΔAPC,

$\angle APC \cong \angle ACP$ ----- [From (vi)]

$\therefore$ AP = AC ----- (vii) [By converse of isosecles triangle theorem]

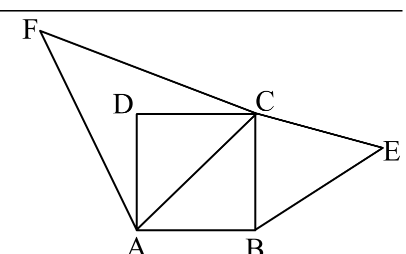
$\therefore \frac{BE}{CE} = \frac{AB}{AC}$ ----- (viii) [from (ii) and (vii)]

$\therefore \frac{BD}{CD} = \frac{BE}{CE}$ ----- [from (i) and (viii)

$\therefore \frac{BD}{BE} = \frac{CD}{CE}$ ----- [alternendo]

---

23. In the adjoining figure, □ABCD is a square. The ΔBCE on side BC and ΔACF on the diagonal AC are similar to each other. Then, show that A(ΔBCE) = $\frac{1}{2}$A(ΔACF).



Proof:

□ABCD is a square ----- [Given]

$\therefore$ AC = $\sqrt{2}$ BC ----- (i) [$\because$ Diagonal of a square = $\sqrt{2}$ × side of square]

$\Delta BCE \sim \Delta ACF$ ----- [Given]

$\therefore \frac{A(\Delta BCE)}{A(\Delta ACF)} = \frac{(BC)^2}{(AC)^2}$ ----- (ii) [By theorem on areas of similar triangles]

$\therefore \frac{A(\Delta BCE)}{A(\Delta ACF)} = \frac{(BC)^2}{(\sqrt{2}.BC)^2}$ ----- [From (i) and (ii)]

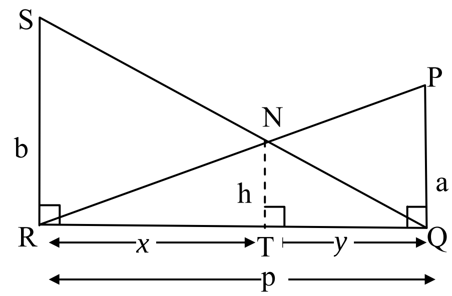
$\therefore \quad \dfrac{A(\Delta BCE)}{A(\Delta ACF)} = \dfrac{BC^2}{2BC^2}$

$\therefore \quad \dfrac{A(\Delta BCE)}{A(\Delta ACF)} = \dfrac{1}{2}$

$\therefore \quad \mathbf{A(\Delta BCE) = \dfrac{1}{2} A(\Delta ACF)}$

---

24. Two poles of height 'a' meters and 'b' metres are 'p' meters apart. Prove that the height 'h' drawn from the point of intersection N of the lines joining the top of each pole to the foot of the opposite pole is $\dfrac{ab}{a+b}$ metres.



Proof:

In ΔPQR and ΔNTR,

$\angle PQR \cong \angle NTR$ ----- [Each is 90°]

$\angle PRQ \cong \angle NRT$ ----- [Common angle]

$\therefore \quad \Delta PQR \sim \Delta NTR$ ----- [By A.A. test of similarity]

$\therefore \quad \dfrac{PQ}{NT} = \dfrac{QR}{TR}$ ----- [c.s.s.t.]

$\therefore \quad \dfrac{a}{h} = \dfrac{p}{x}$

$\therefore \quad a = \dfrac{ph}{x}$ ----- (i)

In ΔSRQ and ΔNTQ,

$\angle SRQ \cong \angle NTQ$ ----- [Each is 90°]

$\angle SQR \cong \angle NQT$ ----- [Common angle]

$\Delta SRQ \sim \Delta NTQ$ ----- [By A–A test of similarity]

$\therefore \quad \dfrac{SR}{NT} = \dfrac{QR}{QT}$

$\therefore \quad \dfrac{b}{h} = \dfrac{p}{y}$

$\therefore \quad b = \dfrac{ph}{y}$ ----- (ii)

Consider,

$ab = \dfrac{ph}{x} \times \dfrac{ph}{y}$

$\therefore \quad ab = \dfrac{p^2h^2}{xy}$ ----- (iii)

Consider,

$a + b = \dfrac{ph}{x} + \dfrac{ph}{y}$

$= ph\left[\dfrac{1}{x} + \dfrac{1}{y}\right]$

$\therefore \quad a + b = ph\dfrac{(y+x)}{xy}$ ----- (iv)

Dividing (iii) by (iv)

$$\frac{ab}{a+b} = \frac{\dfrac{p^2h^2}{xy}}{\dfrac{ph(x+y)}{xy}}$$

$\therefore \quad \dfrac{ab}{a+b} = \dfrac{p^2h^2}{xy} \times \dfrac{xy}{ph(x+y)}$

$\therefore \quad \dfrac{ab}{a+b} = \dfrac{ph}{x+y}$

$\therefore \quad \dfrac{ab}{a+b} = \dfrac{ph}{p}$ ----- [$\because x + y = p$, R–T–Q]

$\therefore \quad \mathbf{h = \dfrac{ab}{a+b}}$

---

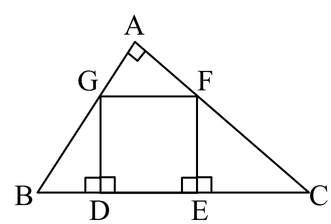
25. In the adjoining figure, □DEFG is a square and ∠BAC = 90°.

Prove that: i. ΔAGF ~ ΔDBG

ii. ΔAGF ~ ΔEFC

iii. ΔDBG ~ ΔEFC

iv. DE^2 = BD·EC



Proof:

i □DEFG is a square ----- [Given]

seg GF || seg DE ----- [Opposite sides of a square]

∴ seg GF || seg BC ----- (i) [B–D–E–C]

In ΔAGF and ΔDBG,

∠GAF ≅ ∠BDG ----- [Each is 90°]

∠AGF ≅ ∠DBG ----- [Corresponding angles of parallel lines GF and BC]

∴ **ΔAGF ~ ΔDBG** ----- (ii) [By A–A test of similarity]

ii In ΔAGF and ΔEFC,

∠GAF ≅ ∠FEC ----- [Each is 90°]

∠AFG ≅ ∠ECF ----- [Corresponding angles on parallel lines GF and BC]

iii. Since ΔAGF ~ ΔEFC

∴ **ΔAGF ~ ΔEFC** ----- (iii) [By A–A test of similarity]

∴ **ΔDBG ~ ΔEFC** ----- [From (ii) and (iii)]

$\therefore \quad \dfrac{BD}{FE} = \dfrac{DG}{EC}$ ----- [c.s.s.t.]

∴ DG × FE = BD × EC ----- (iv)

But, DG = EF = DE ----- (v) [Sides of a square]

∴ DE × DE = DB × EC ----- [From (iv) and (v)]

∴ $\mathbf{DE^2 = DB \cdot EC}$

One-Mark Questions

1. **In ΔABC and ΔXYZ , $\frac{AB}{YZ} = \frac{BC}{ZX} = \frac{AC}{XY}$,**

then state by which correspondence are ΔABC and ΔXYZ similar.

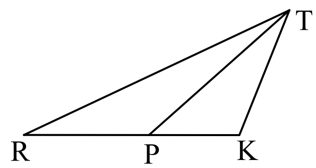
Solution:

Δ ABC ~ ΔXYZ by ABC ↔ YZX.

---

2. **In the figure, RP : PK = 3:2.**

Find $\frac{A(\Delta TRP)}{A(\Delta TPK)}$.



Solution:

$\frac{A(\Delta TRP)}{A(\Delta TPK)} = \frac{3}{2}$ ----- [Triangles with equal heights]

---

3. **Write the statement of Basic Proportionality Theorem.**

Solution:

If a line parallel to a side of a triangle intersects the other sides in two districnt points, then the line divides those sides in proportion.

---

4. **What is the ratio among the length of the sides of any triangle of angles 30°– 60°– 90°?**

Solution:

The ratio is 1: $\sqrt{3}$:2.

---

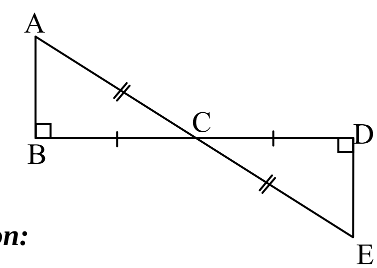
5. **What is the ratio among the length of the sides of any triangle of angles 45°– 45°– 90°?**

Solution:

The ratio is 1:1: $\sqrt{2}$.

---

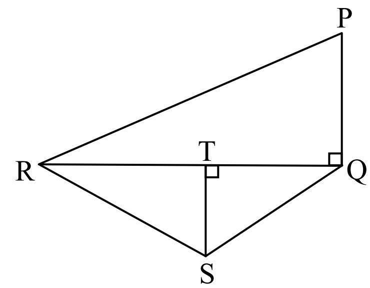
6. **State the test by which the given triangles are similar.**



Solution:

ΔABC ~ ΔEDC by SAS test.

7. In the adjoining figure, find $\frac{A(\Delta PQR)}{A(\Delta RSQ)}$.



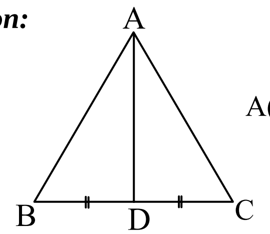
Solution:

$$\frac{A(\Delta PQR)}{A(\Delta RSQ)} = \frac{PQ}{ST} \quad \text{----- [Triangles with common base]}$$

---

8. Draw a pair of triangles which are equal in areas.

Solution:



A(ΔABD) = A(ΔADC)

---

9. State the relation between diagonal of a square and its side.

Solution:

Diagonal of a square = $\sqrt{2}$ × side.

---

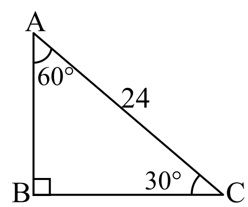
10. Adjacent sides of parallelogram are 11 cm and 17 cm respectively. If length of one diagonal is 26 cm, then using which theorem/property can we find the length of the other diagonal?

Solution:

We can find the length of the other diagonal by using Appollonius' Theorem.

---

11. In the adjoining figure, using given information, find BC.



Solution:

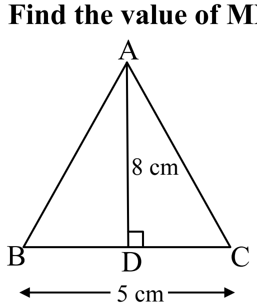
$$BC = \frac{\sqrt{3}}{2} \times AC \quad \text{----- [side opposite to 60°]}$$

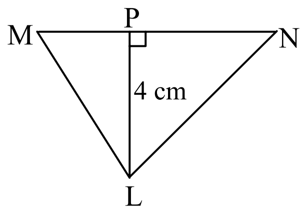
$$= \frac{\sqrt{3}}{2} \times 24$$

∴ **BC = 12 $\sqrt{3}$ units**

---

12. Find the value of MN, so that A(ΔABC) = A(ΔLMN).





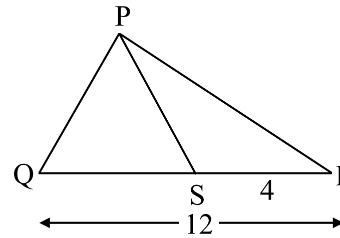
Solution:

MN = 10 cm

Additional Problems for Practice

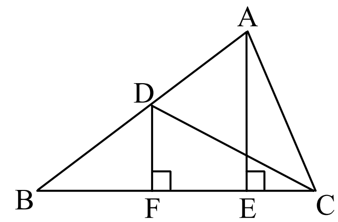
Based on Exercise 1.1

1. In the adjoining figure, QR = 12 and SR = 4.
Find values of
i. $\frac{A(\Delta PSR)}{A(\Delta PQR)}$ ii. $\frac{A(\Delta PQS)}{A(\Delta PQR)}$ iii. $\frac{A(\Delta PQS)}{A(\Delta PSR)}$



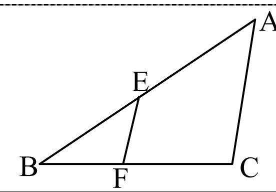
2. The ratio of the areas of two triangles with the equal heights is 3 : 4. Base of the smaller triangle is 15 cm. Find the corresponding base of the larger triangle.

3. In the adjoining figure, seg AE ⊥ seg BC and seg DF ⊥ seg BC.
Find
i. $\frac{A(\Delta ABC)}{A(\Delta DBC)}$ ii. $\frac{A(\Delta DBF)}{A(\Delta DFC)}$ iii. $\frac{A(\Delta AEC)}{A(\Delta DBF)}$

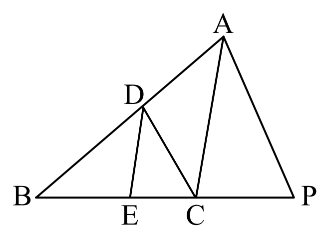


Based on Exercise 1.2

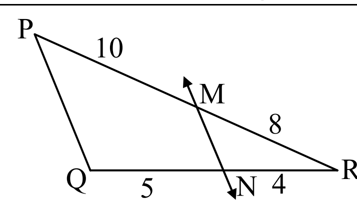
4. In the adjoining figure,
seg EF | | side AC,
AB = 18, AE = 10, BF = 4.
Find BC.



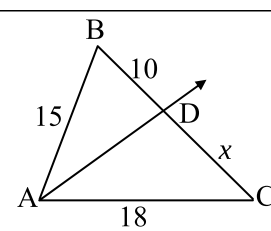
5. In the adjoining figure,
seg DE | | side AC and
seg DC | | side AP.
Prove that $\frac{BE}{EC} = \frac{BC}{CP}$



6. In the adjoining figure,
PM = 10, MR = 8,
QN = 5, NR = 4.
State with reason whether
line MN is parallel to side PQ or not ?



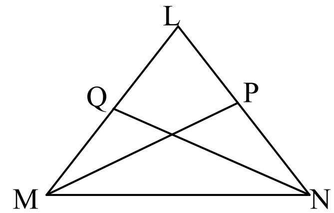
7. In the adjoining figure,
Ray AD is the angle bisector of ∠BAC of ΔABC.
From the given information find value of x.



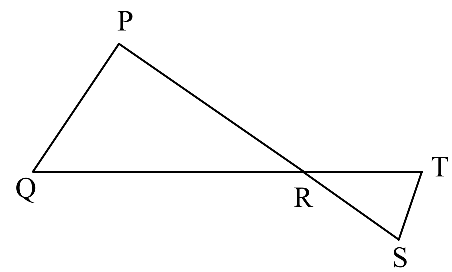
8. Bisectors of ∠B and ∠C in ΔABC meet each other at P. Line AP cuts the side BC at Q.
Then prove that $\frac{AP}{PQ} = \frac{AB + AC}{BC}$.

Based on Exercise 1.3

9. In the adjoining figure,
$\Delta MPL \sim \Delta NQL$,
MP = 21, ML = 35,
NQ = 18, QL = 24.
Find PL and NL.



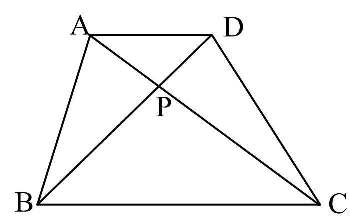
10. In the adjoining figure,
ΔPQR and ΔRST are similar
under PQR $\leftrightarrow$ STR,
PQ = 12, PR = 15,
$\frac{QR}{TR} = \frac{3}{2}$.
Find ST and SR.



11. In the map of a triangular field, sides are shown by 8 cm, 7 cm and 6 cm. If the largest side of the triangular field is 400 m, find the remaining sides of the field.

12. $\Delta EFG \sim \Delta RST$ and EF = 8, FG = 10, EG = 6, RS = 4. Find ST and RT.

13. In □ABCD, **[Oct 09]**
side BC | | side AD.
Diagonals AC and BD intersect each other at P.
If AP = $\frac{1}{3}$AC, then prove that DP = $\frac{1}{2}$BP.



Based on Exercise 1.4

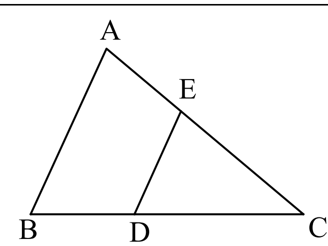
14. If $\Delta PQR \sim \Delta PMN$ and $9A(\Delta PQR) = 16A(\Delta PMN)$, then find $\frac{QR}{MN}$.

15. $\Delta LMN \sim \Delta RST$ and $A(\Delta LMN) = 100$ sq. cm, $A(\Delta RST) = 144$ sq. cm, LM = 5 cm. Find RS.

16. ΔABC and ΔDEF are equilateral triangles. $A(\Delta ABC):A(\Delta DEF) = 1:2$ and AB = 4 cm. Find DE.

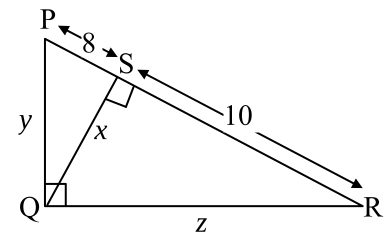
17. If the areas of two similar triangles are equal, then prove that they are congruent.

18. In the adjoining figure,
seg DE | | side AB,
DC = 2BD,
$A(\Delta CDE) = 20$ cm^2.
Find A(□ABDE).

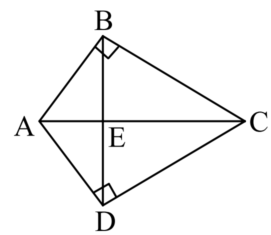


Based on Exercise 1.5

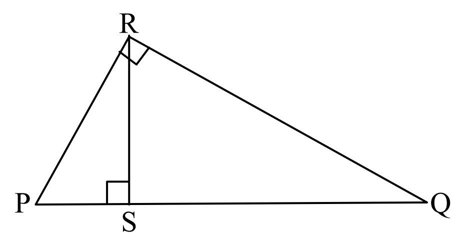
19. In the adjoining figure,
$\angle PQR = 90°$,
seg QS $\perp$ side PR.
Find values of x, y and z.



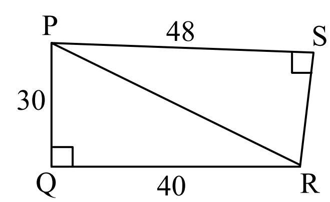
20. In the adjoining figure,
$\angle ABC = \angle ADC = 90°$,
seg BD $\perp$ seg AC.
Prove that:
E is the midpoint of seg BD.



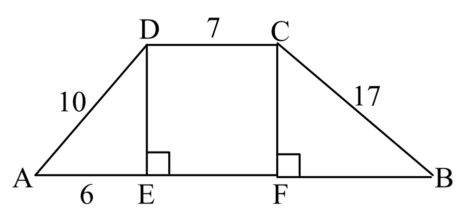
21. In the adjoining figure,
$\angle PRQ = 90°$,
seg RS $\perp$ seg PQ.
Prove that : $\dfrac{PR^2}{QR^2} = \dfrac{PS}{QS}$



22. In the adjoining figure,
$\angle PQR = 90°$, $\angle PSR = 90°$.
Find:
i. PR and
ii. RS



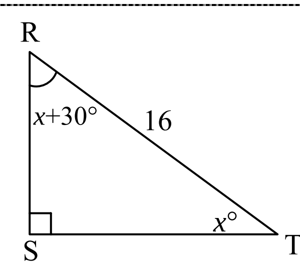
23. In the adjoining figure,
□ ABCD is a trapezium,
seg AB $\parallel$ seg DC,
seg DE $\perp$ side AB,
seg CF $\perp$ side AB.
Find:
i. DE and CF ii. BF iii. AB.



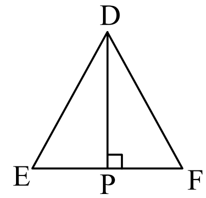
24. Starting from Anil's house, Peter first goes 50m to south, then 75m to west, then 62 m to North and finally 40 m to east and reaches Salim's house. Then find the distance between Anil's house and Salim's house.

Based on Exercise 1.6

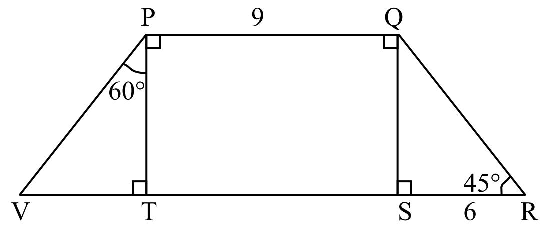
25. In the adjoining figure,
$\angle S = 90°$, $\angle T = x°$,
$\angle R = (x + 30)°$,
RT = 16.
Find: i. RS ii. ST



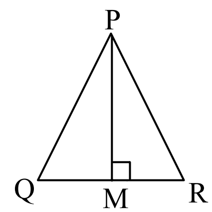
26. ΔDEF is an equilateral triangle. **[Sept 08]**
seg DP $\perp$ side EF,
and E–P–F.
Prove that : $DP^2 = 3\ EP^2$



27. In the adjoining figure, **[March 2013]**
□PQRV is a trapezium,
seg PQ || seg VR.
SR = 6, PQ= 9,
Find VR.



28. In the adjoining figure,
ΔPQR is an equilateral triangle,
seg PM $\perp$ side QR.
Prove that: $PQ^2 = 4QM^2$



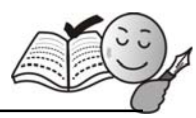
Based on Exercise 1.7

29. In ΔPQR, seg PM is a median. PM = 9 and $PQ^2 + PR^2 = 290$. Find QR.

30. Adjacent sides of a parallelogram are 11 cm and 17 cm. Its one diagonal is 26 cm. Find its other diagonal.

31. In ΔABC, $\angle$ABC = 90°, AB = 12, BC = 16 and seg BP is a median. Find BP.

Answers to additional problems for practice



1. i. $\frac{1}{3}$ ii. $\frac{2}{3}$
   iii. $\frac{2}{1}$
2. 20 cm
3. i. $\frac{AE}{DF}$ ii. $\frac{BF}{FC}$
   iii. $\frac{EC \times AE}{BF \times DF}$
4. 9 units
7. 12 units
9. PL = 28 units and NL = 30 units
10. ST = 8 units and SR = 10 units
11. Remaining sides of field are 350 m and 300 m resp.
12. ST = 5 units and RT = 3 units
14. $\frac{4}{3}$
15. 6 cm
16. $4\sqrt{2}$ cm
18. 25 cm^2
19. $x = 4\sqrt{\mathbf{5}}$ units, y = 12units and $z = 6\sqrt{\mathbf{5}}$ units
22. i. 50 units
    ii. 14 units
23. i. DE = 8 units and CF = 8 units
    ii. BF = 15 units
    iii. AB = 28 units
24. 37 m
25. i. 8 units ii. $8\sqrt{3}$ units
27. $(15 + 6\sqrt{3})$ units
29. 16 units
30. 12 cm
31. 10 units