Algebra

Based on Maharashtra State Board Syllabus

Std. X

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Std. X

Algebra

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PREFACE

Algebra is the branch of mathematics which deals with the study of rules of operations and relations, and the concepts arising from them. It has wide applications in different fields of science and technology. It deals with concepts like linear equations, quadratic equations, Arithmetic and Geometric progressions etc. Its application in statistics deals with measures of central tendency, representation of statistical data etc.

The study of Algebra requires a deep and intrinsic understanding of concepts, terms and formulas. Hence to ease this task we bring to you **"Std. X: Algebra"** a complete and thorough guide extensively drafted to boost the students confidence. The topicwise question and answer format of this book helps the student to understand each and every concept thoroughly. It covers all the textual as well as higher order thinking problems which are completely solved with accurate answers. The book also includes precise theory, important definitions, formulas and procedures to solve the problems. Graphs are drawn with proper scale and pie diagram are with correct measures.

And lastly, I would like to thank all those who have helped me in preparing this book. There is always room for improvement and hence I welcome all suggestions and regret any errors that may have occurred in the making of this book.

A book affects eternity; one can never tell where its influence stops.

Best of luck to all the aspirants!

Your's faithfully

Publisher

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01 Arithmetic Progression and Geometric Progression

1.0 Introduction

We have observed different relations or specific patterns in some numbers while studying the operations on numbers like addition, subtraction, multiplication and division.

Examples:

i. 1, 2, 3, …

This is a collection of all the positive integers in which the difference between two consecutive numbers is 1.

ii. 1, 3, 5, 7, 9, …

This is a collection of all the odd natural numbers in which the difference between two consecutive numbers is 2.

Such patterns are also observed in our day-to-day life.

1.1 Sequence

a. Sequence:

A sequence is a collection of numbers arranged in a definite order according to some definite rule.

Examples:

i. 1, 4, 9, 16, … (Collection of perfect squares of natural numbers)

ii. 2, 4, 6, 8, 10, … (Collection of positive even integers)

iii. 1, 3, 5, 7, … (Collection of positive odd integers)

iv. –2, –4, –6, … (Collection of negative even integers)

v. 5, 10, 15, 20, … 100 (Collection of first 20 integral multiples of 5)

b. Term:

Each number in the sequence is called a term of the sequence.

The number in the first position is called the first term and is denoted by t_1.

The number in the second position is called the second term and is denoted by t_2.

Similarly, the number in the 'n^{th}' position of the sequence is called the n^{th} term and is denoted by t_n.

If t_n is given then a sequence can be formed.

Example: If $t_n = 2n + 1$, then

by putting n = 1, 2, 3, …, we get

$t_1 = 2 \times 1 + 1 = 3$,

$t_2 = 2 \times 2 + 1 = 5$,

$t_3 = 2 \times 3 + 1 = 7$ and so on

$\therefore$ The sequence can be written as 3, 5, 7,...

c. Sum of the first n terms of a sequence:

If a sequence consists of n terms then its sum can be represented as

$S_n = t_1 + t_2 + t_3 + \ldots + t_n$

Putting n = 1, 2, 3, … , we get

$S_1 = t_1$

$S_2 = t_1 + t_2$

$S_3 = t_1 + t_2 + t_3$

.

.

.

$S_n = t_1 + t_2 + t_3 + \ldots + t_n$

d. n^{th} term from S_n:

If S_n is given then t_n can also be found out.

Since $S_n = t_1 + t_2 + t_3 + \ldots + t_n$

$S_1 = t_1$

$S_2 = t_1 + t_2$

$S_3 = t_1 + t_2 + t_3$

$\therefore$ $S_2 - S_1 = (t_1 + t_2) - t_1 = t_2$

$S_3 - S_2 = (t_1 + t_2 + t_3) - (t_1 + t_2) = t_3$

Similarly,

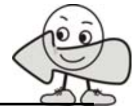
$S_n - S_{n-1}$

$= (t_1 + t_2 + t_3 + .. + t_n) - (t_1 + t_2 + t_3 + .. + t_{n-1})$

$= t_n$

$\therefore$ $t_n = S_n - S_{n-1}$, for $n > 1$

1.2 Types of Sequences



There are two types of sequences:

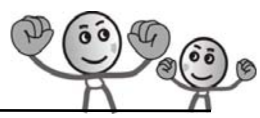
a. **Finite sequence:** If the number of terms in a sequence is finite (countable) i.e. if there is an end of terms in the sequence then it is called a finite sequence.

Examples:
i. 1, 2, 3, … 20.
ii. 4, 6, 8, … 50.
iii. 1, 4, 9, 16, … 100.

b. **Infinite sequence:** If the number of terms in a sequence is infinite (uncountable) i.e. there is no end of terms in the sequence then it is called an infinite sequence.

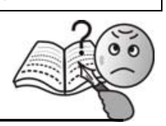
Examples:
i. 1, 3, 5, 7, …
ii. 5, 10, 15, …
iii. 2, 4, 6, 8, …

Differences between a sequence and a set



	Sequence	*Set*
1.	The elements of a sequence are in a specific order, so they cannot be rearranged.	Elements are at random, so they can be rearranged.
2.	The same value can appear many times.	Any value can appear only once.

Exercise 1.1



1. For each of the following sequences, find the next four terms.

i. 1, 2, 4, 7, 11, …
ii. 3, 9, 27, 81, …
iii. 1, 3, 7, 15, 31, … [March 2013]
iv. 192, –96, 48, – 24, …
v. 2, 6, 12, 20, 30, …
vi. 0.1, 0.01, 0.001, 0.0001, …
vii. 2, 5, 8, 11, …
viii. –25, –23, –21, –19, …
ix. 2, 4, 8, 16, …
x. $\mathbf{\frac{1}{2}, \frac{1}{6}, \frac{1}{18}, \frac{1}{54}, \ldots}$

Solution:

i. The given sequence is 1, 2, 4, 7, 11, ...
Here, $t_1 = 1$, $t_2 = 2$, $t_3 = 4$, $t_4 = 7$, $t_5 = 11$
The differences between two consecutive terms are 1, 2, 3, 4, ...
$\therefore$ $t_6 = 11 + 5 = 16$
$t_7 = 16 + 6 = 22$
$t_8 = 22 + 7 = 29$
$t_9 = 29 + 8 = 37$
$\therefore$ **The next four terms are 16, 22, 29 and 37.**

ii. The given sequence is 3, 9, 27, 81, …
Here, $t_1 = 3$, $t_2 = 9$, $t_3 = 27$, $t_4 = 81$
This sequence is in the form $3^1, 3^2, 3^3, 3^4$
$\therefore$ $t_5 = 3^5 = 243$
$t_6 = 3^6 = 729$
$t_7 = 3^7 = 2187$
$t_8 = 3^8 = 6561$
$\therefore$ **The next four terms are 243, 729, 2187 and 6561.**

iii. The given sequence is 1, 3, 7, 15, 31, …
Here, $t_1 = 1$, $t_2 = 3$, $t_3 = 7$, $t_4 = 15$, $t_5 = 31$
The differences between two consecutive terms are 2, 4, 8, 16, … i.e. $2^1, 2^2, 2^3, 2^4, \ldots$
$\therefore$ $t_6 = 31 + 2^5 = 31 + 32 = 63$
$t_7 = 63 + 2^6 = 63 + 64 = 127$
$t_8 = 127 + 2^7 = 127 + 128 = 255$
$t_9 = 255 + 2^8 = 255 + 256 = 511$
$\therefore$ **The next four terms are 63, 127, 255 and 511.**

iv. The given sequence is 192, –96, 48, – 24, …
Here, $t_1 = 192$, $t_2 = -96$, $t_3 = 48$, $t_4 = -24$
The common ratio of two consecutive terms is $-\frac{1}{2}$
$\therefore$ $t_5 = -24 \times -\frac{1}{2} = 12$
$t_6 = 12 \times -\frac{1}{2} = -6$
$t_7 = -6 \times -\frac{1}{2} = 3$
$t_8 = 3 \times -\frac{1}{2} = -\frac{3}{2}$
$\therefore$ **The next four terms are 12, –6, 3 &** $\mathbf{-\frac{3}{2}}$**.**

v. The given sequence is 2, 6, 12, 20, 30, …

Here, $t_1 = 2$, $t_2 = 6$, $t_3 = 12$, $t_4 = 20$, $t_5 = 30$

The differences between two consecutive terms are 4, 6, 8, 10...

$\therefore$ $t_6 = 30 + 12 = 42$

$t_7 = 42 + 14 = 56$

$t_8 = 56 + 16 = 72$

$t_9 = 72 + 18 = 90$

$\therefore$ **The next four terms are 42, 56, 72 and 90.**

vi. The given sequence is 0.1, 0.01, 0.001, 0.0001, …

Here, $t_1 = 0.1$, $t_2 = 0.01$, $t_3 = 0.001$, $t_4 = 0.0001$.

The common ratio of two consecutive terms is 0.1

$\therefore$ $t_5 = 0.0001 \times 0.1 = 0.00001$

$t_6 = 0.00001 \times 0.1 = 0.000001$

$t_7 = 0.000001 \times 0.1 = 0.0000001$

$t_8 = 0.0000001 \times 0.1 = 0.00000001$

$\therefore$ **The next four terms are 0.00001, 0.000001, 0.0000001 and 0.00000001.**

vii. The given sequence is 2, 5, 8, 11, …

Here, $t_1 = 2$, $t_2 = 5$, $t_3 = 8$, $t_4 = 11$

The common difference between two consecutive terms is 3

$\therefore$ $t_5 = 11 + 3 = 14$

$t_6 = 14 + 3 = 17$

$t_7 = 17 + 3 = 20$

$t_8 = 20 + 3 = 23$

$\therefore$ **The next four terms are 14, 17, 20 and 23.**

viii. The given sequence is –25, –23, –21, –19, …

Here, $t_1 = -25$, $t_2 = -23$, $t_3 = -21$, $t_4 = -19$

The common difference between two consecutive terms is 2

$\therefore$ $t_5 = -19 + 2 = -17$

$t_6 = -17 + 2 = -15$

$t_7 = -15 + 2 = -13$

$t_8 = -13 + 2 = -11$

$\therefore$ **The next four terms are –17, –15, –13 and –11.**

ix. The given sequence is 2, 4, 8, 16, …

Here, $t_1 = 2$, $t_2 = 4$, $t_3 = 8$, $t_4 = 16$

The common ratio of two consecutive terms is 2

$\therefore$ $t_5 = 16 \times 2 = 32$

$t_6 = 32 \times 2 = 64$

$t_7 = 64 \times 2 = 128$

$t_8 = 128 \times 2 = 256$

$\therefore$ **The next four terms are 32, 64, 128 and 256.**

x. The given sequence is $\frac{1}{2}, \frac{1}{6}, \frac{1}{18}, \frac{1}{54}, \ldots$

Here, $t_1 = \frac{1}{2}$, $t_2 = \frac{1}{6}$, $t_3 = \frac{1}{18}$, $t_4 = \frac{1}{54}$

The common ratio of two consecutive terms is $\frac{1}{3}$

$\therefore$ $t_5 = \frac{1}{54} \times \frac{1}{3} = \frac{1}{162}$

$t_6 = \frac{1}{162} \times \frac{1}{3} = \frac{1}{486}$

$t_7 = \frac{1}{486} \times \frac{1}{3} = \frac{1}{1458}$

$t_8 = \frac{1}{1458} \times \frac{1}{3} = \frac{1}{4374}$

$\therefore$ **The next four terms are $\frac{1}{162}, \frac{1}{486}, \frac{1}{1458}$ and $\frac{1}{4374}$.**

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2. Find the first five terms of the following sequences, whose ‘n[th]’ terms are given.

i. $t_n = 4n - 3$ [March 2013]

ii. $t_n = 2n - 5$ [March 2013]

iii. $t_n = n + 2$ [March 2013]

iv. $t_n = n^2 - 2n$ [March 2013]

v. $t_n = n^3$

vi. $t_n = \frac{1}{n+1}$

Solution:

i. Given, $t_n = 4n - 3$

For n = 1, $t_1 = 4(1) - 3 = 1$

For n = 2, $t_2 = 4(2) - 3 = 5$

For n = 3, $t_3 = 4(3) - 3 = 9$

For n = 4, $t_4 = 4(4) - 3 = 13$

For n = 5, $t_5 = 4(5) - 3 = 17$

$\therefore$ **The first five terms are 1, 5, 9, 13 and 17.**

ii. Given, $t_n = 2n - 5$

For n = 1, $t_1 = 2(1) - 5 = -3$

For n = 2, $t_2 = 2(2) - 5 = -1$

For n = 3, $t_3 = 2(3) - 5 = 1$

For n = 4, $t_4 = 2(4) - 5 = 3$

For n = 5, $t_5 = 2(5) - 5 = 5$

$\therefore$ **The first five terms are –3, –1, 1, 3 and 5.**

iii. Given, $t_n = n + 2$

For n = 1, $t_1 = 1 + 2 = 3$

For n = 2, $t_2 = 2 + 2 = 4$

For n = 3, $t_3 = 3 + 2 = 5$

For n = 4, $t_4 = 4 + 2 = 6$

For n = 5, $t_5 = 5 + 2 = 7$

$\therefore$ **The first five terms are 3, 4, 5, 6 and 7.**

iv. Given, $t_n = n^2 - 2n$

For n = 1, $t_1 = (1)^2 - 2(1) = -1$

For n = 2, $t_2 = (2)^2 - 2(2) = 0$

For n = 3, $t_3 = (3)^2 - 2(3) = 3$

For n = 4, $t_4 = (4)^2 - 2(4) = 8$

For n = 5, $t_5 = (5)^2 - 2(5) = 15$

$\therefore$ **The first five terms are –1, 0, 3, 8 and 15.**

v. Given, $t_n = n^3$

For n = 1, $t_1 = (1)^3 = 1$

For n = 2, $t_2 = (2)^3 = 8$

For n = 3, $t_3 = (3)^3 = 27$

For n = 4, $t_4 = (4)^3 = 64$

For n = 5, $t_5 = (5)^3 = 125$

$\therefore$ **The first five terms are 1, 8, 27, 64 and 125.**

vi. Given, $t_n = \frac{1}{n+1}$

For n = 1, $t_1 = \frac{1}{1+1} = \frac{1}{2}$

For n = 2, $t_2 = \frac{1}{2+1} = \frac{1}{3}$

For n = 3, $t_3 = \frac{1}{3+1} = \frac{1}{4}$

For n = 4, $t_4 = \frac{1}{4+1} = \frac{1}{5}$

For n = 5, $t_5 = \frac{1}{5+1} = \frac{1}{6}$

$\therefore$ **The first five terms are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$ and $\frac{1}{6}$.**

3. Find the first three terms of the sequences for which S_n is given below:

i. $S_n = n^2(n + 1)$

ii. $S_n = \frac{n^2(n+1)^2}{4}$

iii. $S_n = \frac{n(n+1)(2n+1)}{6}$

Solution:

i. Given, $S_n = n^2(n + 1)$

For n = 1, $S_1 = (1)^2(1 + 1) = 1 \times 2 = 2$

For n = 2, $S_2 = (2)^2(2 + 1) = 4 \times 3 = 12$

For n = 3, $S_3 = (3)^2(3 + 1) = 9 \times 4 = 36$

Now, $t_1 = S_1$ and $t_n = S_n - S_{n-1}$, for n > 1

$\therefore$ $t_1 = 2$

$t_2 = S_2 - S_1 = 12 - 2 = 10$

$t_3 = S_3 - S_2 = 36 - 12 = 24$

$\therefore$ **The first three terms are 2, 10 and 24.**

ii. Given, $S_n = \frac{n^2(n+1)^2}{4}$

For n = 1, $S_1 = \frac{(1)^2(1+1)^2}{4} = \frac{1\times4}{4} = 1$

For n = 2, $S_2 = \frac{(2)^2(2+1)^2}{4} = \frac{4\times9}{4} = 9$

For n = 3, $S_3 = \frac{(3)^2(3+1)^2}{4} = \frac{9\times16}{4} = 36$

Now, $t_1 = S_1$ and $t_n = S_n - S_{n-1}$, for n > 1

$\therefore$ $t_1 = 1$

$t_2 = S_2 - S_1 = 9 - 1 = 8$

$t_3 = S_3 - S_2 = 36 - 9 = 27$

$\therefore$ **The first three terms are 1, 8 and 27.**

iii. Given, $S_n = \frac{n(n+1)(2n+1)}{6}$

For n = 1, $S_1 = \frac{1(1+1)(2\times1+1)}{6} = \frac{1\times2\times3}{6} = 1$

For n = 2, $S_2 = \frac{2(2+1)(2\times2+1)}{6} = \frac{2\times3\times5}{6} = 5$

For n = 3, $S_3 = \frac{3(3+1)(2\times3+1)}{6} = \frac{3\times4\times7}{6} = 14$

Now, $t_1 = S_1$ and $t_n = S_n - S_{n-1}$, for n > 1

$\therefore$ $t_1 = 1$

$t_2 = S_2 - S_1 = 5 - 1 = 4$

$t_3 = S_3 - S_2 = 14 - 5 = 9$

$\therefore$ **The first three terms are 1, 4 and 9.**

1.3 Progressions

a. Definition:

A progression is a special type of sequence in which the relationship between any two consecutive terms is the same.

Examples:

i. 3, 6, 9, 12, … 27

ii. 2, 4, 8, 16, …

iii. $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \ldots$

In example i.,

$t_2 - t_1 = t_3 - t_2 = \ldots = 3 = \text{constant}$

In example ii.,

$\frac{t_2}{t_1} = \frac{t_3}{t_2} = \frac{t_4}{t_3} = \ldots = 2 = \text{constant}$

In example iii.,

$\frac{1}{t_2} - \frac{1}{t_1} = \frac{1}{t_3} - \frac{1}{t_2} = \ldots = 2 = \text{constant}$

Hence, each example represents a progression.

Think it over

The following are not progressions. Explain why?

i. 1, 4, 9, 16, …

ii. 3, 5, 8, 13, …

Solution:

i. The given sequence is 1, 4, 9, 16, …

Here, 4 – 1 = 3

9 – 4 = 5

16 – 9 = 7

Since, there is no fixed (same) relationship between any two consecutive terms, the given sequence is not a progression.

ii. The given sequence is 3, 5, 8, 13, …

Here, 5 – 3 = 2

8 – 5 = 3

13 – 8 = 5

Since, there is no fixed (same) relationship between any two consecutive terms, the given sequence is not a progression.

b. Types of progressions:

There are three types of progressions:

i. Arithmetic progression (A.P.)

ii. Geometric progression (G.P.)

iii. Harmonic progression (H.P.)

We shall study only A.P. and G.P. in this chapter.

1.4 Arithmetic Progression (A.P.)

Definition: An Arithmetic progression is a sequence in which the difference between any two consecutive terms is constant.

Examples:

i. 10, 20, 30, 40, ...

ii. 18, 16, 14, ...

iii. $\frac{2}{5}, \frac{3}{5}, \frac{4}{5}, \ldots$

In example i., $t_2 - t_1 = t_3 - t_2 = \ldots = 10 = \text{constant}$

In example ii., $t_2 - t_1 = t_3 - t_2 = \ldots = -2 = \text{constant}$

In example iii., $t_2 - t_1 = t_3 - t_2 = \ldots = \frac{1}{5} = \text{constant}$

Note:

i. If $t_{n+1} - t_n$ is constant, for all $n \in N$, then the sequence is an A.P.

ii. In an A.P., the first term is denoted by 'a' and the common difference is denoted by 'd'.

Now, t_1 is the first term

$\therefore$ $t_1 = a$

t_2 is the second term

$\therefore$ $t_2 = t_1 + d = a + d = a + (2-1)d = a + (n-1)d$

where n = 2

t_3 is the third term

$\therefore$ $t_3 = t_2 + d = a + d + d = a + 2d = a + (3-1)d$

$= a + (n-1)d$

where n = 3

Continuing in this manner,

$t_n = t_{n-1} + d = a + (n-1)d.$

$\therefore$ $t_1 = a$

$t_2 = a + d$

$t_3 = a + 2d$

$t_4 = a + 3d$ and so on.

Think it over

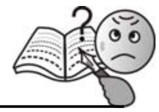
The triplets 1, 25, 49 form an A.P

Can you find some more such triplets?

Solution:

Triplets like 2, 4, 6 and 17, 14, 11 form an A.P.

Exercise 1.2



1. Which of the following lists of numbers are Arithmetic Progressions? Justify.

i. 1, 3, 6, 10, … ii. 3, 5, 7, 9, 11, …

iii. 1, 4, 7, 10, … iv. 3, 6, 12, 24, …

v. 22, 26, 28, 31, … vi. 0.5, 2, 3.5, 5, …

vii. 4, 3, 2, 1, …

viii. –10, –13, –16, –19, …

Solution:

i. The given list of numbers is 1, 3, 6, 10, …
Here, $t_1 = 1$, $t_2 = 3$, $t_3 = 6$, $t_4 = 10$

$\therefore$ $t_2 - t_1 = 3 - 1 = 2$
$t_3 - t_2 = 6 - 3 = 3$
$t_4 - t_3 = 10 - 6 = 4$

$\therefore$ $t_2 - t_1 \neq t_3 - t_2 \neq t_4 - t_3$
Since, the difference between two consecutive terms is not constant.

$\therefore$ **The given list of numbers is not an A.P.**

ii. The given list of numbers is 3, 5, 7, 9, 11, ...
Here, $t_1 = 3$, $t_2 = 5$, $t_3 = 7$, $t_4 = 9$, $t_5 = 11$

$\therefore$ $t_2 - t_1 = 5 - 3 = 2$
$t_3 - t_2 = 7 - 5 = 2$
$t_4 - t_3 = 9 - 7 = 2$
$t_5 - t_4 = 11 - 9 = 2$

$\therefore$ $t_2 - t_1 = t_3 - t_2 = ... = 2 =$ constant

$\therefore$ **The given list of numbers is an A.P.**

iii. The given list of numbers is 1, 4, 7, 10, …
Here, $t_1 = 1$, $t_2 = 4$, $t_3 = 7$, $t_4 = 10$

$\therefore$ $t_2 - t_1 = 4 - 1 = 3$
$t_3 - t_2 = 7 - 4 = 3$
$t_4 - t_3 = 10 - 7 = 3$

$\therefore$ $t_2 - t_1 = t_3 - t_2 = ... = 3 =$ constant

$\therefore$ **The given list of numbers is an A.P.**

iv. The given list of numbers is 3, 6, 12, 24, …
Here, $t_1 = 3$, $t_2 = 6$, $t_3 = 12$, $t_4 = 24$

$\therefore$ $t_2 - t_1 = 6 - 3 = 3$
$t_3 - t_2 = 12 - 6 = 6$
$t_4 - t_3 = 24 - 12 = 12$

$\therefore$ $t_2 - t_1 \neq t_3 - t_2 \neq t_4 - t_3$
Since, the difference between two consecutive terms is not constant.

$\therefore$ **The given list of numbers is not an A.P.**

v. The given list of numbers is 22, 26, 28, 31, …
Here, $t_1 = 22$, $t_2 = 26$, $t_3 = 28$, $t_4 = 31$

$\therefore$ $t_2 - t_1 = 26 - 22 = 4$
$t_3 - t_2 = 28 - 26 = 2$
$t_4 - t_3 = 31 - 28 = 3$

$\therefore$ $t_2 - t_1 \neq t_3 - t_2 \neq t_4 - t_3$
Since, the difference between two consecutive terms is not constant.

$\therefore$ **The given list of numbers is not an A.P.**

vi. The given list of numbers is 0.5, 2, 3.5, 5, …
Here, $t_1 = 0.5$, $t_2 = 2$, $t_3 = 3.5$, $t_4 = 5$

$\therefore$ $t_2 - t_1 = 2 - 0.5 = 1.5$
$t_3 - t_2 = 3.5 - 2 = 1.5$
$t_4 - t_3 = 5 - 3.5 = 1.5$

$\therefore$ $t_2 - t_1 = t_3 - t_2 = ... = 1.5 =$ constant

$\therefore$ **The given list of numbers is an A.P.**

vii. The given list of numbers is 4, 3, 2, 1, …
Here, $t_1 = 4$, $t_2 = 3$, $t_3 = 2$, $t_4 = 1$

$\therefore$ $t_2 - t_1 = 3 - 4 = -1$
$t_3 - t_2 = 2 - 3 = -1$
$t_4 - t_3 = 1 - 2 = -1$

$\therefore$ $t_2 - t_1 = t_3 - t_2 = ... = -1 =$ constant

$\therefore$ **The given list of numbers is an A.P.**

viii. The given list of numbers is
–10, –13, –16, –19, …
Here, $t_1 = -10$, $t_2 = -13$, $t_3 = -16$, $t_4 = -19$

$\therefore$ $t_2 - t_1 = -13 - (-10) = -3$
$t_3 - t_2 = -16 - (-13) = -3$
$t_4 - t_3 = -19 - (-16) = -3$

$\therefore$ $t_2 - t_1 = t_3 - t_2 = ... = -3 =$ constant

$\therefore$ **The given list of numbers is an A.P.**

---

2. Write the first five terms of the following Arithmetic Progressions where the common difference ‘d’ and the first term ‘a’ are given.

i. a = 2, d = 2.5 ii. a = 10, d = – 3

iii. a = 4, d = 0 iv. a = 5, d = 2

v. a = 3, d = 4 vi. a = 6, d = 6

Solution :

i. Given, a = 2, d = 2.5

$\therefore$ $t_1 = a = 2$
$t_2 = t_1 + d = 2 + 2.5 = 4.5$
$t_3 = t_2 + d = 4.5 + 2.5 = 7$
$t_4 = t_3 + d = 7 + 2.5 = 9.5$
$t_5 = t_4 + d = 9.5 + 2.5 = 12$

$\therefore$ **The first five terms of the A.P. are 2, 4.5, 7, 9.5 and 12.**

ii. Given, $a = 10, d = -3$

$\therefore$ $t_1 = a = 10$

$t_2 = t_1 + d = 10 + (-3) = 7$

$t_3 = t_2 + d = 7 + (-3) = 4$

$t_4 = t_3 + d = 4 + (-3) = 1$

$t_5 = t_4 + d = 1 + (-3) = -2$

$\therefore$ **The first five terms of the A.P. are 10, 7, 4, 1 and –2.**

iii. Given, $a = 4, d = 0$

$\therefore$ $t_1 = a = 4$

$t_2 = t_1 + d = 4 + 0 = 4$

$t_3 = t_2 + d = 4 + 0 = 4$

$t_4 = t_3 + d = 4 + 0 = 4$

$t_5 = t_4 + d = 4 + 0 = 4$

$\therefore$ **The first five terms of the A.P. are 4, 4, 4, 4 and 4.**

iv. Given, $a = 5, d = 2$

$\therefore$ $t_1 = a = 5$

$t_2 = t_1 + d = 5 + 2 = 7$

$t_3 = t_2 + d = 7 + 2 = 9$

$t_4 = t_3 + d = 9 + 2 = 11$

$t_5 = t_4 + d = 11 + 2 = 13$

$\therefore$ **The first five terms of the A.P. are 5, 7, 9, 11 and 13.**

v. Given, $a = 3, d = 4$

$\therefore$ $t_1 = a = 3$

$t_2 = t_1 + d = 3 + 4 = 7$

$t_3 = t_2 + d = 7 + 4 = 11$

$t_4 = t_3 + d = 11 + 4 = 15$

$t_5 = t_4 + d = 15 + 4 = 19$

$\therefore$ **The first five terms of the A.P. are 3, 7, 11, 15 and 19.**

vi. Given, $a = 6, d = 6$

$\therefore$ $t_1 = a = 6$

$t_2 = t_1 + d = 6 + 6 = 12$

$t_3 = t_2 + d = 12 + 6 = 18$

$t_4 = t_3 + d = 18 + 6 = 24$

$t_5 = t_4 + d = 24 + 6 = 30$

$\therefore$ **The first five terms of the A.P. are 6, 12, 18, 24 and 30.**

The n^{th} term of an A.P.

Consider the A.P. a, a + d, a + 2d, a + 3d, ...

Here, $t_1 = a$ … (i)

$t_2 - t_1 = d$ … (ii)

$t_3 - t_2 = d$ … (iii)

$t_4 - t_3 = d$ … (iv)

⋮

$t_{n-1} - t_{n-2} = d$ … (n – 1)

$t_n - t_{n-1} = d$ … (n)

Adding all the above equations, we get

$t_1 + (t_2 - t_1) + (t_3 - t_2) + \ldots + (t_n - t_{n-1})$

$= a + d + d + \ldots + d$ [d is added (n – 1) times]

$\therefore$ $t_n = a + (n - 1)\,d.$

This is the n^{th} term of an A.P. with first term ‘a’ and common difference ‘d’.

Exercise 1.3

1. Find the twenty fifth term of the A.P. : 12, 16, 20, 24, …

Solution:

The given A.P. is 12, 16, 20, 24, ...

Here, $a = 12$, $d = 16 - 12 = 4$

Since, $t_n = a + (n - 1)d$

$\therefore$ $t_{25} = 12 + (25 - 1)\,4$

$= 12 + 24 \times 4$

$= 12 + 96$

$= 108$

$\therefore$ **The twenty fifth term of the given A.P. = 108.**

---

2. Find the eighteenth term of the A.P.: 1, 7, 13, 19, …

Solution:

The given A.P. is 1, 7, 13, 19, ...

Here, $a = 1$, $d = 7 - 1 = 6$

Since, $t_n = a + (n - 1)d$

$\therefore$ $t_{18} = 1 + (18 - 1)\,6$

$= 1 + 17 \times 6$

$= 1 + 102$

$= 103$

$\therefore$ **The eighteenth term of the given A.P. = 103.**

3. Find t_n for an A.P. where $t_3 = 22$, $t_{17} = -20$.

Solution:

Given, $t_3 = 22$, $t_{17} = -20$

Since, $t_n = a + (n - 1)d$

$\therefore \quad a + (3 - 1)d = t_3$

$\therefore \quad a + 2d = 22 \quad$ ...(i)

Also, $a + (17 - 1)d = t_{17}$

$\therefore \quad a + 16d = -20 \quad$ ...(ii)

Subtracting (i) from (ii), we get

$$\begin{array}{rcrcr} a & + & 16d & = & -20 \\ a & + & 2d & = & 22 \\ (-) & & (-) & & (-) \\ \hline & & 14d & = & -42 \end{array}$$

$\therefore \quad d = \dfrac{-42}{14}$

$\therefore \quad d = -3 \quad$ ... (iii)

Substituting $d = -3$ in equation (i), we get

$a + 2(-3) = 22$

$\therefore \quad a - 6 = 22$

$\therefore \quad a = 28 \quad$ ... (iv)

$\therefore \quad t_n = a + (n - 1)d$

$= 28 + (n - 1)(-3) \quad$ ... from (iii) and (iv)

$= 28 - 3n + 3$

$\therefore \quad$ **$t_n = -3n + 31$.**

---

4. For an A.P. if $t_4 = 12$ and $d = -10$, then find its general term.

Solution:

Given, $t_4 = 12$, $d = -10$

Since, $t_n = a + (n - 1)d$

$\therefore \quad a + (4 - 1)(-10) = t_4$

$\therefore \quad a + 3 \times (-10) = 12$

$\therefore \quad a - 30 = 12$

$\therefore \quad a = 12 + 30$

$\therefore \quad a = 42$

$\therefore \quad t_n = 42 + (n - 1)(-10)$

$\therefore \quad t_n = 42 - 10n + 10$

$\therefore \quad$ **The general term $t_n = -10n + 52$.**

---

5. Given the following sequence, determine whether it is arithmetic or not. If it is an arithmetic progression, find its general term.

$-5, 2, 9, 16, 23, 30, \ldots$

Solution:

The given sequence is $-5, 2, 9, 16, 23, 30, \ldots$

Here, $t_1 = -5$, $t_2 = 2$, $t_3 = 9$, $t_4 = 16$, $t_5 = 23$, $t_6 = 30$

$\therefore \quad t_2 - t_1 = 2 - (-5) = 7$

$t_3 - t_2 = 9 - 2 = 7$

$t_4 - t_3 = 16 - 9 = 7$

$t_5 - t_4 = 23 - 16 = 7$

$t_6 - t_5 = 30 - 23 = 7$

Since the common difference (7) is a constant, the given sequence is an A.P.

Here, $a = -5$, $d = 7$

$\therefore \quad t_n = a + (n - 1)d$

$= -5 + (n - 1)7$

$= -5 + 7n - 7$

$\therefore \quad$ **The general term $t_n = 7n - 12$.**

---

6. Given the following sequence determine if it is arithmetic progression or not. If it is an arithmetic progression, find its general term.

$5, 2, -2, -6, -11, \ldots$

Solution:

The given sequence is $5, 2, -2, -6, -11, \ldots$

Here, $t_1 = 5$, $t_2 = 2$, $t_3 = -2$, $t_4 = -6$, $t_5 = -11$

$\therefore \quad t_2 - t_1 = 2 - 5 = -3$

$t_3 - t_2 = -2 - 2 = -4$

$\therefore \quad t_2 - t_1 \neq t_3 - t_2$

$\therefore \quad$ **The given sequence is not an A.P.**

---

7. How many three digit natural numbers are divisible by 4?

Solution:

Let n be the number of 3 digit natural numbers divisible by 4.

The three digit natural numbers which are divisible by 4 are 100, 104, 108, ..., 996.

This sequence is an A.P. with $a = 100$, $d = 4$, $t_n = 996$

But, $t_n = a + (n - 1)d$

$\therefore \quad 996 = 100 + (n - 1)4$

$\therefore \quad 996 - 100 = (n - 1)4$

$\therefore \quad \dfrac{896}{4} = n - 1$

$\therefore \quad 224 = n - 1$

$\therefore \quad n = 224 + 1 = 225$

$\therefore \quad$ **The number of three digit natural numbers divisible by 4 is 225.**

8. **The 11^{th} term and the 21^{st} term of an A.P. are 16 and 29 respectively then find**
   i. the 1^{st} term and the common difference
   ii. the 34^{th} term
   iii. 'n' such that $t_n = 55$.

Solution:

Given, $t_{11} = 16$, $t_{21} = 29$

i. Since, $t_n = a + (n - 1)d$

$\therefore$ $a + (11 - 1)d = t_{11}$

$\therefore$ $a + 10d = 16$ ... (i)

Also, $a + (21 - 1) = t_{21}$

$\therefore$ $a + 20d = 29$ ... (ii)

Subtracting (i) from (ii), we get

$$\begin{array}{rcl} a + 20d &=& 29 \\ a + 10d &=& 16 \\ (-)\ \ (-) & & (-) \\ \hline 10d &=& 13 \end{array}$$

$\therefore$ $d = \dfrac{13}{10}$

Substituting $d = \dfrac{13}{10}$ in (i), we get

$a + 10 \times \dfrac{13}{10} = 16$

$\therefore$ $a + 13 = 16$

$\therefore$ $a = 16 - 13 = 3$

$\therefore$ $a = 3$ and $d = \dfrac{13}{10} = 1.3$

ii. Now, $t_n = a + (n - 1) d$

$\therefore$ $t_{34} = 3 + (34 - 1) 1.3$ ... [From i.]

$= 3 + 33 \times 1.3$

$= 3 + 42.9$

$= 45.9$

iii. Given, $t_n = 55$

Since, $a + (n - 1)d = t_n$

$\therefore$ $3 + (n - 1) 1.3 = 55$ ... [From i.]

$\therefore$ $(n - 1) 1.3 = 52$

$\therefore$ $(n - 1) = 40$

$\therefore$ $n = 40 + 1 = 41$

$\therefore$ **i. The 1^{st} term = 3 and the common difference = 1.3**

ii. The 34^{th} term = 45.9

iii. $t_n = 55$ for $n = 41$

Sum of the first n terms of an A.P.

If a, a + d, a + 2d, … a + (n – 1)d is an A.P. with first term 'a' and common difference 'd', then the sum of first n terms of the A.P. is

$S_n = [a] + [a + d] + \ldots + [a + (n - 2)d]$
$\quad + [a + (n - 1)d]$ ...(i)

Reversing the terms and rewriting (i), we get

$S_n = [a + (n - 1)d] + [a + (n - 2)d] +$
$\quad \ldots + [a + d] + [a]$ ...(ii)

Now, adding equations (i) and (ii), we get

$2S_n = [a + a + (n - 1)d] + [a + d + a + (n - 2)d]$
$\quad + \ldots + [a + (n - 2)d + a + d]$
$\quad + [a + (n - 1)d + a]$

$\therefore$ $2S_n = [2a + (n - 1)d] + [2a + (n - 1)d] + \ldots$ (n times)

$\therefore$ $2S_n = n [2a + (n - 1) d]$

$\therefore$ $S_n = \dfrac{n}{2} [2a + (n - 1)d]$

Thus the sum of the first n terms of an A.P. is

$S_n = \dfrac{n}{2} [2a + (n - 1)d]$.

Think it over

i. Derive the formula for n^{th} term of the sequence of odd natural numbers and even natural numbers.

ii. Find the sum of first n odd natural numbers and first n even natural numbers.

Solution:

i. Sequence of odd natural numbers is
1, 3, 5, 7, …
This sequence is an A.P. with
$a = 1$, $d = 3 - 1 = 2$
Now, $t_n = a + (n - 1) d$
$= 1 + (n - 1) 2$
$= 1 + 2n - 2$
$= 2n - 1$

$\therefore$ **The n^{th} term of the sequence of odd natural numbers = 2n – 1.**

Sequence of even natural numbers is
2, 4, 6, 8, …
This sequence is an A.P. with
a = 2, d = 4 – 2 = 2.

$t_n = a + (n-1)d$
$= 2 + (n-1)\,2$
$= 2 + 2n - 2$
$= 2n$

∴ **The n^{th} term of the sequence of even natural numbers = 2n.**

ii. Sequence of odd natural numbers is
1, 3, 5, 7, …
This sequence is an A.P. with
a = 1, d = 3 – 1 = 2

Now, $S_n = \frac{n}{2}[2a + (n-1)\,d]$

$= \frac{n}{2}[2 \times 1 + (n-1)2]$

$= \frac{n}{2}[2 + 2n - 2] = \frac{n}{2}[2n] = n^2$

∴ **The sum of first n odd natural numbers = n^2.**

Sequence of even natural numbers is
2, 4, 6, 8, …
This sequence is an A.P. with
a = 2, d = 4 – 2 = 2

Now, $S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{n}{2}[2 \times 2 + (n-1)2]$

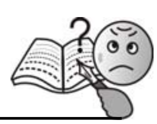
$= \frac{n}{2}[4 + 2n - 2] = \frac{n}{2}[2n + 2]$

$= \frac{2n(n+1)}{2}$

$= n(n+1)$

∴ **The sum of first n even natural numbers = n(n + 1).**

Exercise 1.4



1. Find the sum of the first 'n' natural numbers and hence find the sum of the first 20 natural numbers.

Solution:

The first 'n' natural numbers are 1, 2, 3, ... , n.
This sequence is an A.P. with a = 1, d = 2 – 1 = 1

Now, $S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{n}{2}[2 \times 1 + (n-1)1]$

$= \frac{n}{2}[2 + n - 1] = \frac{n}{2} \times (n+1)$

$= \frac{n(n+1)}{2}$

∴ $S_{20} = \frac{20(20+1)}{2} = \frac{20 \times 21}{2} = 210$

∴ **The sum of first 'n' natural numbers $= \frac{n(n+1)}{2}$ and the sum of first 20 natural numbers = 210.**

2. Find the sum of all odd natural numbers from 1 to 150.

Solution:

The odd natural numbers from 1 to 150 are 1, 3, 5, ... , 149
This sequence is an A.P. with
a = 1, d = 3–1 = 2, $t_n = 149$
But, $t_n = a + (n-1)d$

∴ $149 = 1 + (n-1)2$ ∴ $148 = (n-1)2$
∴ $n - 1 = 74$ ∴ $n = 75$

Now, $S_n = \frac{n}{2}[2a + (n-1)d]$

∴ $S_{75} = \frac{75}{2}[2 \times 1 + (75-1)2]$

$= \frac{75}{2}[2 + 148] = \frac{75}{2} \times 150$

$= 75 \times 75 = 5625$

∴ **The sum of all the odd natural numbers from 1 to 150 = 5625.**

3. Find S_{10} if a = 6 and d = 3. [March 2013]

Solution:

Given, a = 6, d = 3

Now, $S_n = \frac{n}{2}[2a + (n-1)d]$

∴ $S_{10} = \frac{10}{2}[2 \times 6 + (10-1)3]$

$= 5\,[12 + 27] = 5 \times 39$

∴ **S_{10} = 195.**

4. Find the sum of all numbers from 1 to 140 which are divisible by 4.

Solution:

The numbers 1 to 140 which are divisible by 4 are 4, 8, 12, ... 140

This sequence is an A.P. with a= 4, d = 4, t_n = 140

But, $t_n = a + (n – 1)d$

$\therefore$ $140 = 4 + (n – 1)4$

$\therefore$ $140 – 4 = (n – 1)4$

$\therefore$ $\frac{136}{4} = n – 1$ $\therefore$ $34 + 1 = n$

$\therefore$ $n = 35$

Now, $S_n = \frac{n}{2}[2a + (n – 1)d]$

$\therefore$ $S_{35} = \frac{35}{2}[2 \times 4 + (35 – 1)4]$

$= \frac{35}{2}[8 + 136] = \frac{35}{2} \times 144$

$= 35 \times 72$

$\therefore$ $S_{35} = 2520$

$\therefore$ **The sum of all numbers from 1 to 140 which are divisible by 4 = 2520.**

---

5. Find the sum of the first 'n' odd natural numbers. Hence find 1 + 3 + 5 + … + 101.

Solution:

The sequence of odd natural numbers is 1, 3, 5, …

This sequence is an A.P. with a = 1, d = 3 – 1 = 2

Now, $S_n = \frac{n}{2}[2a + (n – 1)d]$

$= \frac{n}{2}[2 \times 1 + (n – 1)2]$

$= \frac{n}{2}[2 + 2n – 2] = \frac{n}{2}[2n]$

$\therefore$ $S_n = n^2$ …(i)

For 1, 3, 5, ..., 101, $t_n = 101$

But, $t_n = a + (n – 1)d$

$\therefore$ $101 = 1 + (n – 1)2$

$\therefore$ $100 = (n – 1)2$

$\therefore$ $\frac{100}{2} = n – 1$

$\therefore$ $50 = n – 1$

$\therefore$ $n = 50 + 1 = 51$

$\therefore$ $1 + 3 + 5 + … + 101 = S_{51}$

$\therefore$ $1 + 3 + 5 + … + 101 = 51^2$ ... from (i)

$\therefore$ $1 + 3 + 5 + … + 101 = 2601$

$\therefore$ **The sum of the first 'n' odd natural numbers = n^2 and 1 + 3 + 5 + ... + 101 = 2601.**

6. Obtain the sum of the 56 terms of an A.P. whose 19^{th} and 38^{th} terms are 52 and 148 respectively.

Solution:

Given, $t_{19} = 52$ and $t_{38} = 148$

Now, $t_n = a + (n – 1)d$

$\therefore$ $t_{19} = a + (19 – 1)d$

$\therefore$ $52 = a + 18d$

$\therefore$ $a + 18d = 52$ ... (i)

Also, $a + (38 – 1)d = t_{38}$

$\therefore$ $a + 37d = 148$ ... (ii)

Subtracting (i) from (ii), we get

$$\begin{array}{rcrcl} a & + & 37d & = & 148 \\ a & + & 18d & = & 52 \\ (-) & & (-) & & (-) \\ \hline & & 19d & = & 96 \end{array}$$

$\therefore$ $d = \frac{96}{19}$ … (iii)

Substituting $d = \frac{96}{19}$ in (i), we get

$a + 18 \times \frac{96}{19} = 52$

$\therefore$ $a + \frac{1728}{19} = 52$

$\therefore$ $a = 52 - \frac{1728}{19}$

$\therefore$ $a = \frac{988 - 1728}{19}$

$\therefore$ $a = \frac{-740}{19}$ … (iv)

Also, $S_n = \frac{n}{2}[2a + (n – 1)d]$

$\therefore$ $S_{56} = \frac{56}{2}\left[2 \times \frac{-740}{19} + (56 - 1)\frac{96}{19}\right]$

$\therefore$ $S_{56} = 28\left[2 \times \frac{-740}{19} + 55 \times \frac{96}{19}\right]$ …from (iii) and (iv)

$= 28\left[-\frac{1480}{19} + \frac{5280}{19}\right]$

$= 28\left[\frac{3800}{19}\right]$

$= 28 \times 200$

$\therefore$ $\mathbf{S_{56} = 5600.}$

7. The sum of the first 55 terms of an A.P. is 3300. Find the 28th term.

Solution:

Given, $S_{55} = 3300$

But, $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$\therefore \quad S_{55} = \frac{55}{2}\left[2a + (55-1)d\right]$

$\therefore \quad 3300 = \frac{55}{2}[2a + 54d]$

$\therefore \quad 3300 = \frac{55}{2} \times 2(a + 27d)$

$\therefore \quad 3300 = 55(a + 27d)$

$\therefore \quad \frac{3300}{55} = a + 27d$

$\therefore \quad 60 = a + 27d$

$\therefore \quad a + 27d = 60 \quad \text{... (i)}$

Also, $t_n = a + (n-1)d$

$\therefore \quad t_{28} = a + (28 - 1)d$

$\therefore \quad t_{28} = a + 27d \quad \text{... (ii)}$

From (i) and (ii), we get

$\mathbf{t_{28} = 60.}$

---

8. Find the sum of the first n even natural numbers. Hence find the sum of the first 20 even natural numbers.

Solution:

The sequence of even natural numbers is 2, 4, 6, ...

This sequence is an A.P. with $a = 2$, $d = 4 - 2 = 2$

Now, $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore \quad S_n = \frac{n}{2}[2 \times 2 + (n-1)2]$

$= \frac{n}{2}[4 + 2n - 2] = \frac{n}{2}[2n + 2]$

$= \frac{n}{2} \times 2(n+1) = n(n+1)$

$\therefore \quad S_n = n(n+1)$

$\therefore \quad S_{20} = 20(20 + 1)$

$= 20 \times 21 = 420$

$\therefore$ **The sum of the first 'n' even natural numbers = n (n+1) and the sum of the first 20 even natural numbers = 420.**

1.5 Properties of an A.P.

Property I:

For an A.P. with the first term **'a'** and the common difference **'d'**, if any real number **'k'** is added to each term then the new sequence is also an A.P. with the first term**'**a + k**'** and the same common difference 'd'.

Property II:

For an A.P. with the first term 'a' and the common difference 'd', if each term of an A.P. is multiplied by any real number k, then the new sequence is also an A.P. with the first term 'ak' and the common difference 'dk'.

Note:

1. If each term of an A.P. is multiplied by 0 then the new sequence will be 0, 0, 0, …
2. If each term of an A.P. is added, subtracted multiplied or divided by a certain constant then the new sequence is also an A.P.

1.6 Particular terms in an A.P.

To solve problems, we can consider three, four or five consecutive terms of an A.P. in the following way.

i. Three consecutive terms as $a - d, a, a + d$.

ii. Four consecutive terms as $a - 3d, a - d, a + d, a + 3d$.

iii. Five consecutive terms as $a - 2d, a - d, a, a + d, a + 2d$.

Exercise 1.5

1. Find the four consecutive terms in an A.P. whose sum is 12 and the sum of the 3rd and the 4th term is 14.

Solution:

Let the four consecutive terms be $a - 3d$, $a - d$, $a + d$, $a + 3d$.

According to the first condition,

$a - 3d + a - d + a + d + a + 3d = 12$

$\therefore \quad 4a = 12 \quad \therefore \quad a = \frac{12}{4}$

$\therefore \quad a = 3 \quad \text{... (i)}$

According to the second condition,

$a + d + a + 3d = 14$

$\therefore \quad 2a + 4d = 14$

$\therefore \quad 2 \times 3 + 4d = 14$ ... from (i)

$\therefore \quad 4d = 14 - 6$

$\therefore \quad 4d = 8 \quad \therefore \quad d = 2$

Thus, $a - 3d = 3 - 3 \times 2 = -3$

$a - d = 3 - 2 = 1$

$a + d = 3 + 2 = 5$

$a + 3d = 3 + 3 \times 2 = 9$

$\therefore$ **The four consecutive terms are –3, 1, 5 and 9.**

---

2. Find the four consecutive terms in an A.P. whose sum is – 54 and the sum of the 1st and the 3rd terms is –30.

Solution:

Let the four consecutive terms be $a - 3d$, $a - d$, $a + d$, $a + 3d$.

According to the first condition,

$a - 3d + a - d + a + d + a + 3d = -54$

$\therefore \quad 4a = -54$

$\therefore \quad a = \dfrac{-54}{4} = \dfrac{-27}{2} = -13.5$ ... (i)

According to the second condition,

$a - 3d + a + d = -30$

$\therefore \quad 2a - 2d = -30$

$\therefore \quad a - d = -15$

$\therefore \quad -13.5 - d = -15$ ... from (i)

$\therefore \quad d = -13.5 + 15$

$\therefore \quad d = 1.5$

Thus, $a - 3d = -13.5 - 3 \times 1.5$

$= -13.5 - 4.5 = -18$

$a - d = -13.5 - 1.5 = -15$

$a + d = -13.5 + 1.5 = -12$

$a + 3d = -13.5 + 3 \times 1.5$

$= -13.5 + 4.5 = -9$

$\therefore$ **The four consecutive terms are –18, –15, –12, –9.**

---

3. Find the three consecutive terms in an A.P. whose sum is –3 and the product of their cubes is 512.

Solution:

Let the three consecutive terms in an A.P be $a - d$, a, $a + d$

By the first given condition,

$a - d + a + a + d = -3$

$\therefore \quad 3a = -3$

$\therefore \quad a = -1$ ... (i)

By the second given condition,

$(a - d)^3 (a)^3 (a + d)^3 = 512$

Taking cube root on both sides, we get

$(a - d)(a)(a + d) = 8$

$\therefore \quad a(a^2 - d^2) = 8$

$\therefore \quad -1[(-1)^2 - d^2] = 8$ … from (i)

$\therefore \quad -1(1 - d^2) = 8$

$\therefore \quad 1 - d^2 = -8 \quad \therefore \quad d^2 = 9$

$\therefore \quad d = \sqrt{9} = 3$ (considering the positive root)

Thus, $a - d = -1 - 3 = -4$

$a = -1$

$a + d = -1 + 3 = 2$

$\therefore$ **The three consecutive terms are –4, –1 & 2.**

---

4. In winter, the temperature at a hill station from Monday to Friday is in A.P. The sum of the temperatures of Monday, Tuesday and Wednesday is zero and the sum of the temperatures of Thursday and Friday is 15. Find temperature of each of the five days.

Solution:

Let the temperatures from Monday to Friday in A.P be $a - 2d$, $a - d$, a, $a + d$, $a + 2d$.

According to the first condition,

$a - 2d + a - d + a = 0$

$\therefore \quad 3a - 3d = 0 \quad \therefore \quad a - d = 0 \quad \therefore a = d$

According to the second condition,

$a + d + a + 2d = 15$

$\therefore \quad 2a + 3d = 15$

$\therefore \quad 2a + 3a = 15 \quad (\because d = a)$

$\therefore \quad 5a = 15$

$\therefore \quad a = 3$

$\therefore \quad d = 3 \quad (\because d = a)$

Thus, $a - 2d = 3 - 2 \times 3 = -3$

$a - d = 3 - 3 = 0$

$a = 3$

$a + d = 3 + 3 = 6$

$a + 2d = 3 + 2 \times 3 = 9$

$\therefore$ **The temperatures from Monday to Friday are –3, 0, 3, 6 and 9 respectively.**

1.7 Applications of A.P.

In this section, we will study the application of theory and formulae of A.P. to solve various word problems.

Exercise 1.6

1. Mary got a job with a starting salary of ₹ 15000/- per month. She will get an incentive of ₹ 100/- per month. What will be her salary after 20 months?

Solution:

Mary's salaries are in A.P. with the first term 15000 and common difference 100.

$\therefore$ a = 15000, d = 100, n = 20

Now, $t_n = a + (n - 1)\,d$

$\therefore$ $t_{20} = 15000 + (20 - 1)\,100$
$= 15000 + 19 \times 100$
$= 15000 + 1900$
$= 16900$

$\therefore$ **Mary's salary after 20 months = ₹16900**

2. The taxi fare is ₹ 14 for the first kilometre and ₹ 2 for each additional kilometre. What will be the fare for 10 kilometres?

Solution:

The taxi fares are in A.P. with the first term 14 and common difference 2.

$\therefore$ a = 14, d = 2, n = 10

Now, $t_n = a + (n - 1)\,d$

$\therefore$ $t_{10} = 14 + (10 - 1) \times 2$
$= 14 + 9 \times 2$
$= 32$

$\therefore$ **The taxi fare for 10 kilometres = ₹ 32.**

3. Mangala started doing physical exercise 10 minutes for the first day. She will increase the time of exercise by 5 minutes per day, till she reaches 45 minutes per day. How many days are required to reach 45 minutes?

Solution:

The daily time of exercise is an A.P. with the first term 10 and common difference 5.

$\therefore$ a = 10, d = 5, $t_n = 45$

Now, $t_n = a + (n - 1)\,d$

$\therefore$ $45 = 10 + (n - 1)\,5$

$\therefore$ $35 = (n - 1)\,5$

$\therefore$ $\frac{35}{5} = n - 1$

$\therefore$ $7 = n - 1$

$\therefore$ $n = 7 + 1 = 8$

$\therefore$ **The no. of days required to reach 45 minutes = 8.**

4. There is an auditorium with 35 rows of seats. There are 20 seats in the first row, 22 seats in the second row, 24 seats in the third row, and so on. Find the number of seats in the twenty fifth row.

Solution:

The no. of seats arranged row wise are as follows :
20, 22, 24, ...

This sequence is an A.P. with
a = 20, d = 22 – 20 = 2, n = 25

Now, $t_n = a + (n - 1)d$

$\therefore$ $t_{25} = 20 + (25 - 1)2$
$= 20 + 24 \times 2$
$= 68$

$\therefore$ **The no. of seats in the twenty fifth row = 68.**

5. A village has 4000 literate people in the year 2010 and this number increases by 400 per year. How many literate people will be there till the year 2020? Find a formula to know the number of literate people after n years?

Solution:

The number of literate people in the village is in A.P. with the first term 4000 and common difference 400.

$\therefore$ a = 4000, d = 400, n = 10

Now, $t_n = a + (n - 1)d$

$\therefore$ $t_{10} = 4000 + (10 - 1)400$
$= 4000 + 9 \times 400$
$= 4000 + 3600 = 7600$

Putting n = 10 we have

$t_n = 400 \times 10 + 3600$
$= 7600$

Since, $t_n = a + (n - 1)d$
$= 4000 + (n - 1)400$
$= 4000 + 400n - 400$
$= 400n + 3600$

$\therefore$ **The no. of literate people till the year 2020 = 7600**
and the no. of literate people after 'n' years = 400 n + 3600.

6. Neela saves in a 'Mahila Bachat Gat' ₹ 2 on the first day of February, ₹ 4 on the second day, ₹ 6 on the third day and so on. What will be her saving in the month of February 2010?

Solution:

Neela's daily savings of February 2010 are as follows :

2, 4 , 6, ...

This sequence is an A.P. with

$a = 2, d = 4 - 2 = 2, n = 28$

($\because$ Feb 2010 had 28 days as 2010 was not a leap year)

Now, $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore \quad S_{28} = \frac{28}{2}[2 \times 2 + (28-1)2]$

$= 14\,[4 + 27 \times 2]$

$= 14 \times 58$

$= 812$

$\therefore$ **Neela's saving in the month of February 2010 = ₹ 812.**

---

7. Babubhai borrows ₹ 4000 and agrees to repay with a total interest of ₹ 500 in 10 installments, each installment being less than the preceding installment by ₹ 10. What should be the first and the last installment?

Solution:

The installments are in A.P.

Here, $S_{10} = 4000 + 500 = 4500$

Also, $n = 10, d = -10$

Now, $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore \quad S_{10} = \frac{10}{2}[2a + (10-1)(-10)]$

$\therefore \quad 4500 = 5\,[2a + 9 \times (-10)]$

$\therefore \quad \frac{4500}{5} = 2a - 90$

$\therefore \quad 900 + 90 = 2a$

$\therefore \quad 990 = 2a$

$\therefore \quad a = \frac{990}{2}$

$\therefore \quad a = 495$

Also, $t_n = a + (n-1)d$

$\therefore \quad t_{10} = 495 + (10-1)(-10)$

$= 495 + 9 \times (-10)$

$= 495 - 90$

$= 405$

$\therefore$ **The first installment = ₹ 495 and the last installment = ₹ 405.**

---

8. A meeting hall has 20 seats in the first row, 24 seats in the second row, 28 seats in the third row, and so on and has in all 30 rows. How many seats are there in the meeting hall?

Solution:

The number of seats arranged row-wise are as follows:

20, 24, 28, …

This sequence is an A.P. with $a = 20, d = 4, n = 30$.

Now, $S_n = \frac{n}{2}[\,2a + (\,n-1)d]$

$\therefore \quad S_{30} = \frac{30}{2}[2 \times 20 + (30-1)4]$

$= 15\,[40 + 29 \times 4]$

$= 15\,[40 + 116]$

$= 15 \times 156 = 2340$

$\therefore$ **The number of seats in the meeting hall = 2340.**

---

9. Vijay invests some amount in the National saving certificate. For the 1st year he invests ₹ 500, for the 2nd year he invests ₹ 700, for the 3rd year he invests ₹ 900, and so on. How much amount has he invested in 12 years?

Solution:

Amount of investments year-wise are as follows:

500, 700, 900, ...

This sequence is an A.P. with

$a = 500, d = 200, n = 12$

Now, $S_n = \frac{n}{2}[\,2a + (\,n-1)d]$

$\therefore \quad S_{12} = \frac{12}{2}[2 \times 500 + (12-1)200]$

$= 6[1000 + 11 \times 200]$

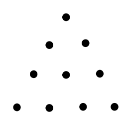
$= 6[1000 + 2200] = 6 \times 3200$

$= 19200$

$\therefore$ **Total amount invested in 12 years = ₹ 19200.**

10. In a school, a plantation programme was arranged on the occasion of world environment day, on a ground of triangular shape. The plants are to be planted as shown in the figure.

One plant in the first row, two in the second row, three in the third row and so on. If there are 25 rows then find the total number of plants to be planted.



Solution:

No. of trees planted row- wise are as follows:

1, 2, 3, …

This sequence is an A.P. with

a = 1, d =1, n = 25

Now, $S_n = \frac{n}{2}[2a + (n - 1)d]$

$\therefore \quad S_{25} = \frac{25}{2}[2 \times 1 + (25 - 1)1]$

$= \frac{25}{2}[2 + 24] = \frac{25}{2} \times 26$

$= 325$

$\therefore$ **The total number of plants to be planted = 325.**

1.8 Geometric Progression (G.P.)

a. Definition: A Geometric Progression is a sequence in which the ratio of any two consecutive terms is a constant.

i.e., In a G.P. $\frac{t_2}{t_1} = \frac{t_3}{t_2} = \dots = \frac{t_n}{t_{n-1}}$ = constant.

Note: The first term is denoted by 'a' and the common ratio is denoted by 'r'.

Examples:

i. 2, 4, 8, 16, 32, … (Infinite G.P.)

ii. $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8} \dots \frac{1}{256}$ (Finite G.P.)

iii. 10, 100, 1000, … (Infinite G.P.)

iv. $9, 3, 1, \frac{1}{3}, \frac{1}{9}, \dots \frac{1}{729}$ (Finite G.P.)

Try This

Write the next three terms for the following sequences:

i. 8, 16, 32, 64, …

ii. 6, 6, 6, 6, …

iii. –5, 10, – 20, 40, …

Solution:

i. The given sequence is 8, 16, 32, 64, …

Here, $t_1 = 8, t_2 = 16, t_3 = 32, t_4 = 64$

The common ratio of two consecutive terms is

$= \frac{t^2}{t_1} = \frac{16}{8} = 2.$

$\therefore \quad t_5 = 64 \times 2 = 128$

$t_6 = 128 \times 2 = 256$

$t_7 = 256 \times 2 = 512$

$\therefore$ **The next three terms are 128, 256 and 512.**

ii. The given sequence is 6, 6, 6, 6, …

Here, $t_1 = 6, t_2 = 6, t_3 = 6, t_4 = 6$

The common ratio of two consecutive terms is 1.

$\therefore \quad t_5 = 6 \times 1 = 6$

$t_6 = 6 \times 1 = 6$

$t_7 = 6 \times 1 = 6$

$\therefore$ **The next three terms are 6, 6 and 6.**

iii. The given sequence is –5, 10, – 20, 40, …

Here, $t_1 = -5, t_2 = 10, t_3 = -20, t_4 = 40$

The common ratio of two consecutive terms is –2.

$\therefore \quad t_5 = 40 \times (-2) = -80$

$t_6 = -80 \times (-2) = 160$

$t_7 = 160 \times (-2) = -320$

$\therefore$ **The next three terms are –80, 160 and –320.**

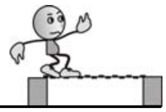
Note:

i. Since division by zero is not permissible, neither the common ratio nor any term of a G.P. can be zero.

ii. If the common ratio 'r' is a positive number then all the terms of the G.P. will have the same sign. In this case all the terms of the G.P. will be either positive or negative.

iii. If the common ratio 'r' is a negative number then any two consecutive terms of the G.P. will have the opposite sign. In this case, G.P. contains alternatively positive and negative terms.

Try This



Find the missing terms in the following table:

Common ratio	Progression
10	4, 40, **400**, 4000, **40000**, 400000
$\frac{1}{3}$	9, 3, 1, $\mathbf{\frac{1}{3}}$, $\frac{1}{9}$, $\mathbf{\frac{1}{27}}$, $\frac{1}{81}$
$\frac{1}{10}$	7, **0.7**, 0.07, **0.007**, 0.0007.
1	3, 3, 3, **3**, 3.
$-\frac{1}{2}$	1, $-\frac{1}{2}$, $\mathbf{\frac{1}{4}}$, $-\frac{1}{8}$, $\frac{1}{16}$, $\mathbf{-\frac{1}{32}}$, $\frac{1}{64}$
– 1	3, –3, **3**, – 3, 3, **– 3**, 3

b. **General representation of a G.P.:** If a is the first term and r is the common ratio of a G.P. then the sequence of G.P. can be represented as a, ar, ar^2, …

Here, $t_1 = a$, $t_2 = ar$, $t_3 = ar^2$ and so on.

The n^{th} term of a G.P.:

Now, $\frac{t_2}{t_1} = r$, $\frac{t_3}{t_2} = r$, $\frac{t_4}{t_3} = r$, … , $\frac{t_n}{t_{n-1}} = r$

Multiplying all the (n – 1) ratios, we get

$$\frac{t_2}{t_1} \times \frac{t_3}{t_2} \times \frac{t_4}{t_3} \times \ldots \times \frac{t_n}{t_{n-1}} = r \times r \times \ldots (n-1) \text{ times}$$

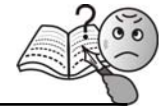
$\therefore \quad \frac{t_n}{t_1} = r^{n-1}$

But, $t_1 = a$

$\therefore \quad \frac{t_n}{a} = r^{n-1}$

$\therefore \quad t_n = ar^{n-1}$

Exercise 1.7



1. Find the ninth term of the G.P. 3, 6, 12, 24, …

Solution:

The given G.P. is 3, 6, 12, 24, ...

Here, a = 3, r = $\frac{6}{3}$ = 2

Now, $t_n = ar^{n-1}$

$\therefore \quad t_9 = 3 \times 2^{9-1} = 3 \times 2^8$

$= 3 \times 256 = 768$

$\therefore$ **The ninth term of the given G.P. is 768.**

2. Write down the first five terms of the geometric progression which has the first term 1 and the common ratio 4.

Solution:

Given, a = 1, r = 4

Now, $t_n = ar^{n-1}$

$\therefore \quad t_1 = 1 \times 4^{1-1} = 1 \times 4^0$

$= 1 \times 1 \qquad (\because 4^\circ = 1)$

$= 1$

$t_2 = 1 \times 4^{2-1} = 1 \times 4^1$

$= 4$

$t_3 = 1 \times 4^{3-1} = 1 \times 4^2$

$= 16$

$t_4 = 1 \times 4^{4-1} = 1 \times 4^3$

$= 64$

$t_5 = 1 \times 4^{5-1} = 1 \times 4^4$

$= 256$

$\therefore$ **The first five terms are 1, 4, 16, 64 and 256.**

3. Find the 4^{th} and the 9^{th} terms of the G.P. with the first term 4 and the common ratio 2.

Solution:

Given, a = 4, r = 2

Now, $t_n = ar^{n-1}$

$\therefore \quad t_4 = 4 \times 2^{4-1} = 4 \times 2^3$

$= 32$

and $\quad t_9 = 4 \times 2^{9-1} = 4 \times 2^8$

$= 1024$

$\therefore$ **The 4^{th} and 9^{th} terms of the G.P. are 32 and 1024.**

4. Find the common ratio and the 7^{th} term of the G.P. 2, –6, 18, …

Solution:

The given G.P. is 2, – 6, 18, ...

Here, $a = 2$, $r = -\frac{6}{2} = -3$

Now, $t_n = ar^{n-1}$

$\therefore$ $t_7 = 2(-3)^{7-1} = 2 \times (-3)^6$

$= 2 \times 729 = 1458$

$\therefore$ **The common ratio = –3 and the 7^{th} term = 1458.**

5. Find the 69^{th} term of the G.P. 1,–1, 1, –1, …

Solution:

The given G.P. is 1, –1, 1, –1, ...

Here, $a = 1$, $r = \frac{-1}{1} = -1$

Now, $t_n = ar^{n-1}$

$\therefore$ $t_{69} = 1(-1)^{69-1}$

$= 1(-1)^{68}$

$= 1 \times 1 = 1$

$\therefore$ **The 69^{th} term of the given G.P. is 1.**

6. Find the 15^{th} term of the G. P. 3, 12, 48, 192, …

Solution:

The given G.P. is 3, 12, 48, 192, …

Here, $a = 3$, $r = \frac{12}{3} = 4$

Now, $t_n = ar^{n-1}$

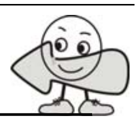
$\therefore$ $t_{15} = 3(4)^{15-1}$

$= 3(4)^{14}$

$= 3 \times 4^{14}$

$\therefore$ **The 15^{th} term of the given G. P = 3×4^{14}.**

Sum of the first n terms of a G.P.



Sequence of G.P. can be written as $a, ar, ar^2, \ldots$ where $r \neq 1$.

Let S_n be the sum of the first n terms of a G.P.

$\therefore$ $S_n = a + ar + ar^2 + \ldots + ar^{n-2} + ar^{n-1}$ …(i)

Multiplying equation (i) by ‘r’, we get

$rS_n = ar + ar^2 + ar^3 + \ldots + ar^{n-1} + ar^n$ ...(ii)

Subtracting equation (ii) from equation (i), we get

$S_n(1 - r) = a - ar^n$

$\therefore$ $S_n(1 - r) = a(1 - r^n)$

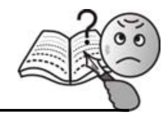
$\therefore$ $S_n = \frac{a(1-r^n)}{1-r}$ ...(iii)

Equation (iii) represents the sum of the first n terms of a G.P. when $r < 1$.

Note:

1. If the common ratio ‘r’ of the G.P. is greater than 1 i.e. $r > 1$ then $S_n = \frac{a\left(r^n - 1\right)}{r-1}$.
2. When the common ratio ‘r’ of the G.P. is equal to 1 then G.P. becomes a, a, a, …
   In this case, sum of n terms is a + a + a + … + a (upto n terms)
   $\therefore$ $S_n = na$

Exercise 1.8



1. Find the indicated sums for the following Geometric Progressions:

i. 2, 6, 18, … Find S_7

ii. 2, –4, 8, –16, … Find S_9 and S_{12}.

iii. $1, \frac{1}{2}, \frac{1}{2^2}, \frac{1}{2^3}, \ldots$ Find S_6

iv. $1, \sqrt{2}, 2, \ldots$ Find S_{10}.

Solution:

i. The given G. P. is 2, 6, 18, ...

Here, $a = 2$, $r = \frac{6}{2} = 3$

Now, $S_n = \frac{a(r^n - 1)}{r-1}$, since $r > 1$

$\therefore$ $S_7 = \frac{2(3^7 - 1)}{3-1} = \frac{2 \times (2187 - 1)}{2}$

$\therefore$ **S_7 = 2186**

ii. The given G. P. is 2, –4, 8, –16, …

Here, $a = 2$, $r = \frac{-4}{2} = -2$

Now, $S_n = \frac{a(1-r^n)}{1-r}$, since $r < 1$

$\therefore$ $S_9 = \frac{2[1-(-2)^9]}{1-(-2)} = \frac{2(1+512)}{1+2}$

$= \frac{2 \times 513}{3} = 2 \times 171$

$= 342$

$$S_{12} = \frac{2[1-(-2)^{12}]}{1-(-2)} = \frac{2(1-4096)}{1+2}$$

$$= \frac{2\times-(4095)}{3} = 2 \times (-1365) = -2730$$

$\therefore$ **$S_9 = 342$ and $S_{12} = -2730$**

iii. The given G. P. is 1, $\frac{1}{2}, \frac{1}{2^2}, \frac{1}{2^3}, \ldots$

Here, a = 1, $r = \frac{\frac{1}{2}}{1} = \frac{1}{2}$

Now, $S_n = \frac{a(1-r^n)}{(1-r)}$, since $r < 1$

$$\therefore \quad S_6 = \frac{1\left[1-\left(\frac{1}{2}\right)^6\right]}{\left(1-\frac{1}{2}\right)} = \frac{1\left(1-\frac{1}{64}\right)}{\frac{1}{2}}$$

$$\therefore \quad S_6 = \frac{\left(\frac{63}{64}\right)}{\frac{1}{2}} = \frac{63}{64} \times \frac{2}{1}$$

$\therefore$ **$S_6 = \frac{63}{32}$**

iv. The given G.P. is 1, $\sqrt{2}$, 2, …

Here, a = 1, $r = \frac{\sqrt{2}}{1} = \sqrt{2}$

Now, $S_n = \frac{a(r^n-1)}{(r-1)}$, since $r > 1$

$$\therefore \quad S_{10} = \frac{1\left[\left(\sqrt{2}\right)^{10}-1\right]}{\left(\sqrt{2}-1\right)} = \frac{2^{1/2\times10}-1}{\sqrt{2}-1}$$

$$= \frac{2^5-1}{\sqrt{2}-1} = \frac{32-1}{\sqrt{2}-1}$$

$$= \frac{31}{\sqrt{2}-1} = \frac{31\left(\sqrt{2}+1\right)}{\left(\sqrt{2}-1\right)\left(\sqrt{2}+1\right)}$$

$$= \frac{31\left(\sqrt{2}+1\right)}{\left(\sqrt{2}\right)^2-(1)^2} = \frac{31\left(\sqrt{2}+1\right)}{2-1}$$

$\therefore$ **$S_{10} = 31\left(\sqrt{2}+1\right)$**

2. If in a G.P. r = 2 and $t_8 = 64$, then find a and S_6.

Solution:

Given, r = 2, $t_8 = 64$

Now, $t_n = a.\ r^{n-1}$

$\therefore \quad t_8 = a(2)^{8-1}$

$\therefore \quad 64 = a \times 2^7$

$\therefore \quad 64 = a \times 128$

$\therefore \quad a = \frac{64}{128} = \frac{1}{2}$

Also, $S_n = \frac{a(r^n-1)}{(r-1)}$, since $r > 1$

$$\therefore \quad S_6 = \frac{\frac{1}{2}\left(2^6-1\right)}{\left(2-1\right)}$$

$$= \frac{1}{2}(64-1)$$

$$= \frac{1}{2} \times 63 = \frac{63}{2}$$

$\therefore$ **$a = \frac{1}{2}$ and $S_6 = \frac{63}{2}$**

---

3. If $S_3 = 31$ and $S_6 = 3906$ then find a and r.

Solution:

Given, $S_3 = 31$, $S_6 = 3906$

Since, $S_n = \frac{a(r^n-1)}{(r-1)}$

$$\therefore \quad S_3 = \frac{a\left(r^3-1\right)}{(r-1)} \quad \ldots \text{(i)}$$

$$\text{and} \quad S_6 = \frac{a\left(r^6-1\right)}{(r-1)} \quad \ldots \text{(ii)}$$

Dividing (ii) by (i), we get

$$\frac{S_6}{S_3} = \frac{a\left(r^6-1\right)}{(r-1)} \times \frac{(r-1)}{a(r^3-1)}$$

$$\therefore \quad \frac{3906}{31} = \frac{r^6-1}{r^3-1}$$

$$\therefore \quad 126 = \frac{\left(r^3\right)^2-(1)^2}{r^3-1}$$

$$\therefore \quad 126 = \frac{\left(r^3+1\right)\left(r^3-1\right)}{\left(r^3-1\right)}$$

$\therefore \quad 126 = r^3 + 1$

$\therefore$ 125 $= r^3$

$\therefore$ r $= \sqrt[3]{125} = 5$

Substituting r = 5 in (i), we get

$$S_3 = \frac{a\left(5^3 - 1\right)}{(5-1)}$$

$\therefore$ $31 = \dfrac{a(125-1)}{4}$

$\therefore$ $4 \times 31 = a \times 124$

$\therefore$ $a = \dfrac{4 \times 31}{124} = \dfrac{124}{124} = 1$

$\therefore$ **a = 1 and r = 5**

---

4. If $S_6 = 126$ and $S_3 = 14$ then find a and r.

Solution :

Given, $S_6 = 126$, $S_3 = 14$

Since, $S_n = \dfrac{a(r^n - 1)}{(r-1)}$

$\therefore$ $S_6 = \dfrac{a\left(r^6 - 1\right)}{(r-1)}$ ... (i)

and $S_3 = \dfrac{a\left(r^3 - 1\right)}{(r-1)}$ ...(ii)

Dividing (i) by (ii) , we get

$$\frac{S_6}{S_3} = \frac{a\left(r^6 - 1\right)}{(r-1)} \times \frac{(r-1)}{a\left(r^3 - 1\right)}$$

$\therefore$ $\dfrac{126}{14} = \dfrac{r^6 - 1}{r^3 - 1}$

$\therefore$ $9 = \dfrac{\left[\left(r^3\right)^2 - 1^2\right]}{r^3 - 1}$

$\therefore$ $9 = \dfrac{\left(r^3 + 1\right)\left(r^3 - 1\right)}{\left(r^3 - 1\right)}$

$\therefore$ 9 $= r^3 + 1$

$\therefore$ r^3 $= 8$

$\therefore$ r $= \sqrt[3]{8} = 2$

Substituting r = 2 in (ii), we get

$$S_3 = \frac{a(2^3 - 1)}{2-1}$$

$\therefore$ $14 = a(8 - 1)$

$\therefore$ $14 = a \times 7$

$\therefore$ $a = \dfrac{14}{7} = 2$

$\therefore$ **a = 2 and r = 2**

5. If the n^{th}, $(2n)^{th}$, $(3n)^{th}$ terms of a G.P. are a, b, c respectively then show that $b^2 = ac$.

Solution:

Let the first term be A and common ratio be r.

Given, $t_n = a$, $t_{2n} = b$, $t_{3n} = c$

Now, $t_n = A.r^{n-1}$

$\therefore$ $a = A.r^{n-1}$ ... (i)

Similarly, $t_{2n} = A.r^{2n-1}$ and $t_{3n} = A.r^{3n-1}$

$\therefore$ $b = A.r^{2n-1}$ ... (ii)

and $c = A.r^{3n-1}$ ... (iii)

To prove that $b^2 = ac$

L.H.S.= b^2

$= A^2(r^{2n-1})^2$ ... From (ii)

$= A^2.r^{4n-2}$

R.H.S.= ac

$= A.r^{n-1} \times A.r^{3n-1}$ ... From (i) and (iii)

$= A^2.r^{n-1+3n-1} = A^2.r^{4n-2}$

$\therefore$ L.H.S. = R.H.S.

$\therefore$ $\mathbf{b^2 = ac}$

1.9 Particular terms in a G.P.

To solve problems, we can consider three, four or five consecutive terms of a G.P. in the following way.

i. Three consecutive terms as $\dfrac{a}{r}$, a, ar.

ii. Four consecutive terms as $\dfrac{a}{r^3}$, $\dfrac{a}{r}$, ar, ar^3.

iii. Five consecutive terms as $\dfrac{a}{r^2}$, $\dfrac{a}{r}$, a, ar, ar^2.

Arithmetic Mean (A)

a. Definition:

If three numbers x, y, z are in A.P. then 'y' is called the arithmetic mean between x and z.

b. Formula:

Suppose x and y are any two numbers. Let the arithmetic mean between x and y be 'A'.Then x, A, y are in A.P.

$\therefore$ $A - x = y - A$

$\therefore$ $2A = x + y$

$\therefore \quad A = \frac{x+y}{2}$

Thus, the arithmetic mean 'A' between any two numbers x and y is given by

$$\boxed{A = \frac{x+y}{2}}$$

Example:

Find arithmetic mean between 3 and 9.

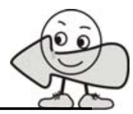
Solution:

Let A be the arithmetic mean between 3 and 9.

$\therefore \quad A = \frac{3+9}{2} = \frac{12}{2}$

$\therefore \quad A = 6$

Geometric Mean (G)



a. Definition:

If three numbers x, y, z are in G.P. then 'y' is called the geometric mean between x and z.

b. Formula:

Suppose x and y are any two numbers with the same sign. Let the geometric mean between x and y be 'G'. Then x, G, y are in G.P.

$\therefore \quad \frac{G}{x} = \frac{y}{G}$

$\therefore \quad G^2 = xy$

$\therefore \quad G = \pm\sqrt{xy}$

Thus, the geometric mean 'G' between any two numbers x and y is given by

$$\boxed{G = \pm\sqrt{xy}}$$

Example:

Find geometric mean between 2 and 8.

Solution:

Let G be the geometric mean between 2 and 8

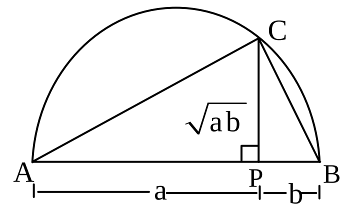
$\therefore \quad G = \pm\sqrt{2\times 8} = \pm\sqrt{16}$

$\therefore \quad G = \pm 4$

c. Euclid construction which gives geometrical interpretation of geometric mean:

Consider the line segment AP of length 'a' cm. and a line segment PB of length 'b' cm. as shown in the figure. Then the length of AB is (a + b) cm. Draw a semicircle with AB as diameter and draw a perpendicular through P which intersects the semicircle in point C. Join AC and CB. ΔACB is a triangle inscribed in a semicircle and hence it is a right angled triangle. ΔAPC and ΔCPB are also right angled triangles

Now, the triangles ΔAPC and ΔCPB are similar triangles,



$\therefore \quad \frac{AP}{CP} = \frac{CP}{PB}$

$\therefore \quad (CP)^2 = AP \times PB$

Hence $(CP)^2 = ab$

$\therefore \quad CP = \sqrt{ab}$

Thus the length of the perpendicular CP from the circumference to the diameter is exactly the geometric mean of a and b, and is denoted by G.

Note:

1. G will be a real number if and only if a and b have the same sign.
2. There exists two values of geometric mean for any two numbers, namely $+\sqrt{ab}$ and $-\sqrt{ab}$.
3. If two numbers a and b are equal i.e. if a = b then
   i. The arithmetic mean between them is $A = \frac{a+a}{2} = \frac{2a}{2} = a$.
   ii. The geometric mean between them is $G = \pm\sqrt{aa} = \pm a$.

d. Theorem: Let x and y be any two unequal positive real numbers. If A is the arithmetic mean and G is the positive geometric mean between x and y, then A > G.

Proof:

$A = \frac{x+y}{2}$ and $G = \pm\sqrt{xy}$

$$\therefore \quad A - G = \frac{x+y}{2} - \sqrt{xy} = \frac{x+y-2\sqrt{xy}}{2}$$

$$= \frac{\left(\sqrt{x}\right)^2 - 2\sqrt{x\times y} + \left(\sqrt{y}\right)^2}{2}$$

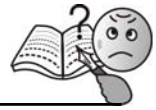
$$= \frac{(\sqrt{x}-\sqrt{y})^2}{2}$$

Since both x and y are positive, $\sqrt{x}$ and $\sqrt{y}$ are real. Since $x \neq y$, we have $(\sqrt{x}-\sqrt{y}) \neq 0$

Hence, $(\sqrt{x}-\sqrt{y})^2 > 0$

Thus, A – G > 0 i.e. A > G.

Exercise 1.9



1. Find the three consecutive terms in a G.P. such that the sum of the first two terms is 9 and the product of all the three is 216.

[March 2013]

Solution:

Let the three consecutive terms be $\frac{a}{r}$, a, ar.

Sum of first two terms is,

$$\frac{a}{r} + a = 9 \quad \text{...(i)}$$

Product of all three terms is,

$$\frac{a}{r} \times a \times ar = 216$$

$\therefore \quad a^3 = 216$

$\therefore \quad a = \sqrt[3]{216}$

$\therefore \quad a = 6$

Substituting a = 6 in (i), we get

$$\frac{6}{r} + 6 = 9$$

$\therefore \quad \frac{6}{r} = 9 - 6$

$\therefore \quad \frac{6}{r} = 3$

$\therefore \quad 3r = 6$

$\therefore \quad r = \frac{6}{3} = 2$

Thus, $\frac{a}{r} = \frac{6}{2} = 3$,

$a = 6$

$ar = 6 \times 2 = 12$

$\therefore$ **The three consecutive terms are 3, 6 and 12.**

---

2. Find the three consecutive terms in a G.P. such that the sum of the 2nd and the 3rd term is 60 and the product of all the three is 8000.

Solution:

Let the three consecutive terms be $\frac{a}{r}$, a, ar

According to the first condition,

$$a + ar = 60 \quad \text{...(i)}$$

According to the second condition,

$$\frac{a}{r} \times a \times ar = 8000$$

$\therefore \quad a^3 = 8000$

$\therefore \quad a = \sqrt[3]{8000}$

$\therefore \quad a = 20$

Substituting a = 20 in (i), we get

$20 + 20 \times r = 60$

$\therefore \quad 20r = 40$

$\therefore \quad r = 2$

Thus, $\frac{a}{r} = \frac{20}{2} = 10$,

$a = 20$,

$ar = 20 \times 2 = 40$

$\therefore$ **The three consecutive terms in G. P. are 10, 20 and 40.**

---

3. Sachin, Sehwag and Dhoni together scored 228 runs. Their individual scores are in G.P. Sehwag and Dhoni together scored 12 runs more than Sachin. Find their individual scores.

Solution:

Let the individual scores of Sachin, Sehwag and Dhoni be $\frac{a}{r}$, a and ar respectively.

According to the first condition,

$$\frac{a}{r} + a + ar = 228 \quad \text{...(i)}$$

According to the second condition,

$$a + ar - \frac{a}{r} = 12 \quad \text{...(ii)}$$

Adding (i) and (ii), we get

$$\frac{a}{r} + a + ar + a + ar - \frac{a}{r} = 228 + 12$$

$\therefore \quad 2a + 2ar = 240$

$\therefore \quad a + ar = 120$

$\therefore \quad a(1 + r) = 120 \quad \text{...(iii)}$

Subtracting (ii) from (i), we get

$$\frac{a(1+r)}{\frac{a}{r}} = \frac{120}{108}$$

$\frac{a}{r} + a + ar - a - ar + \frac{a}{r} = 228 - 12$

$\therefore \quad \frac{2a}{r} = 216$

$\therefore \quad \frac{a}{r} = 108 \qquad \text{...(iv)}$

Dividing (iii) by (iv), we get

$a(1 + r) \times \frac{r}{a} = \frac{120}{108}$

$\therefore \quad (1 + r)r = \frac{10}{9}$

$\therefore \quad (1 + r)9r = 10$

$\therefore \quad 9r^2 + 9r - 10 = 0$

$\therefore \quad 9r^2 + 15r - 6r - 10 = 0$

$\therefore \quad 3r(3r + 5) - 2(3r + 5) = 0$

$\therefore \quad (3r + 5)(3r - 2) = 0$

$\therefore \quad 3r + 5 = 0 \text{ or } 3r - 2 = 0$

$\therefore \quad 3r = -5 \text{ or } 3r = 2$

$\therefore \quad r = \frac{-5}{3} \text{ or } r = \frac{2}{3}$

Discarding the negative value of r and substituting $r = \frac{2}{3}$ in (iv), we get

$\frac{a}{\left(\frac{2}{3}\right)} = 108$

$\therefore \quad a \times \frac{3}{2} = 108$

$\therefore \quad 3a = 108 \times 2$

$\therefore \quad a = \frac{108 \times 2}{3} = 72$

Thus, $\frac{a}{r} = \frac{72}{\left(\frac{2}{3}\right)}$

$= 72 \times \frac{3}{2} = 108,$

$a = 72$

$\therefore \quad ar = 72 \times \frac{2}{3} = 48$

$\therefore$ **The individual scores of Sachin, Sehwag and Dhoni are 108, 72 and 48 respectively.**

4. If 25 is the arithmetic mean between *x* and 46, then find *x*.

Solution:

Let A = 25, $y = 46$

Now, $A = \frac{x + y}{2}$

$\therefore \quad 25 = \frac{x + 46}{2}$

$\therefore \quad 50 = x + 46$

$\therefore \quad x = 50 - 46$

$\therefore \quad \boldsymbol{x = 4}$

---

5. If $x + 3$ is the geometric mean between $x + 1$ and $x + 6$ then find *x*.

Solution:

Let $G = x + 3$, $X = x + 1$, $Y = x + 6$

Now, $G = \pm\sqrt{XY}$

$\therefore \quad x + 3 = \pm\sqrt{(x+1)(x+6)}$

Squaring both sides, we get

$(x + 3)^2 = (x + 1)(x + 6)$

$\therefore \quad x^2 + 6x + 9 = x^2 + 7x + 6$

$\therefore \quad 6x + 9 = 7x + 6$

$\therefore \quad 9 - 6 = x$

$\therefore \quad \boldsymbol{x = 3}$

---

6. Find the geometric mean of $\sqrt{82} - 1$ and $\sqrt{82} + 1$.

Solution:

Let G be the geometric mean.

Let $x = \sqrt{82} - 1$ and $y = \sqrt{82} + 1$

Now, $G = \pm\sqrt{xy}$

$= \pm\sqrt{(\sqrt{82} - 1)(\sqrt{82} + 1)}$

$G = \pm\sqrt{\left(\sqrt{82} - 1\right)\left(\sqrt{82} + 1\right)}$

$= \pm\sqrt{\left(\sqrt{82}\right)^2 - 1^2}$

$= \pm\sqrt{82 - 1} = \pm\sqrt{81}$

$\therefore \quad G = \pm 9$

$\sqrt{82} - 1$ and $\sqrt{82} + 1$ are positive.

$\therefore$ **The geometric mean of $\sqrt{82} - 1$ and $\sqrt{82} + 1$ is 9.**

7. If the arithmetic mean and the geometric mean of two numbers are in the ratio 5:4 and the sum of the two numbers is 30 then find those numbers.

Solution:

Let A be the arithmetic mean and G be the geometric mean and let the two numbers be x and y.

According to the first condition,

$A:G = 5:4$

$\therefore \quad \frac{A}{G} = \frac{5}{4}$

$\therefore \quad 4A = 5G$

$\therefore \quad 4A - 5G = 0 \quad \text{... (i)}$

According to the second condition,

$x + y = 30 \quad \text{...(ii)}$

Now, $A = \frac{x+y}{2}$

$= \frac{30}{2} \quad \text{… From (ii)}$

$= 15$

Substituting A = 15 in (i), we get

$4 \times 15 - 5G = 0$

$\therefore \quad 60 - 5G = 0$

$\therefore \quad 60 = 5G$

$\therefore \quad G = 12$

Now, $G = \pm\sqrt{xy}$

Squaring both sides, we get

$G^2 = xy$

$\therefore \quad 12^2 = xy$

$\therefore \quad xy = 144 \quad \text{... (iii)}$

$\therefore \quad (x-y)^2 = (x+y)^2 - 4xy$

$\therefore \quad (x-y)^2 = 30^2 - 4 \times 144 \quad \text{... From (ii) and (iii)}$

$= 900 - 576$

$= 324$

$\therefore \quad x - y = \sqrt{324} = 18 \quad \text{...(iv)}$

Adding (ii) and (iv), we get

$2x = 48$

$\therefore \quad x = 24$

Substituting $x = 24$ in (ii), we get

$24 + y = 30$

$\therefore \quad y = 30 - 24$

$= 6$

$\therefore$ **The two numbers are 24 and 6.**

Problem Set - 1

1. Find t_{11} from the following A.P.

4, 9, 14, …

Solution:

The given A.P. is 4, 9, 14, ...

Here, a = 4, d = 9 – 4 = 5

Now, $t_n = a + (n-1)d$

$\therefore \quad t_{11} = 4 + (11-1)5$

$= 4 + 10 \times 5$

$\mathbf{= 54}$

---

2. Find the first negative term from the following A.P.

122, 116, 110, ...

(Note: find smallest n, such that $t_n < 0$)

Solution:

The given A.P. is 122, 116, 110, ...

Here, a = 122, d = 116 – 122 = –6

Now, $t_n = a + (n-1)d$

If $t_n = 0$, then

$0 = 122 + (n-1)(-6)$

$\therefore \quad 0 = 122 - 6n + 6$

$\therefore \quad 0 = 122 + 6 - 6n$

$\therefore \quad 0 = 128 - 6n$

$\therefore \quad 6n = 128$

$\therefore \quad n = \frac{128}{6} = 21.33…$

$\therefore \quad t_n = 0$ for $n = 21.33…$

$\therefore \quad t_{22}$ will be negative and

$t_{22} = 122 + (22-1)(-6)$

$= 122 + 21 \times (-6)$

$= 122 - 126$

$= -4$

$\therefore$ **The first negative term = –4 (and n = 22).**

---

3. Find the sum of the first 11 positive numbers which are multiples of 6.

Solution:

The positive multiples of 6 are 6, 12, 18, …

This sequence is an A.P. with

$a = 6$, $d = 12 - 6 = 6$

Now, $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore \quad S_{11} = \frac{11}{2}[2 \times 6 + (11-1)6]$

$= \frac{11}{2}[12 + 10 \times 6]$

$= \frac{11}{2}[12 + 60]$

$= \frac{11}{2} \times 72$

$= 11 \times 36$

$= 396$

$\therefore$ **The sum of the first 11 positive multiples of 6 = 396.**

---

4. In the A.P. 7, 14, 21, … how many terms are to be considered for getting the sum 5740.

Solution:

The given A. P. is 7, 14, 21, …

Also, $S_n = 5740$

Here, $a = 7$, $d = 14 - 7 = 7$

Now, $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore \quad 5740 = \frac{n}{2}[2 \times 7 + (n-1)7]$

$\therefore \quad 5740 \times 2 = n(14 + 7n - 7)$

$\therefore \quad 11480 = n(7 + 7n)$

$\therefore \quad 11480 = 7n + 7n^2$

$\therefore \quad 7n^2 + 7n - 11480 = 0$

$\therefore \quad n^2 + n - 1640 = 0$

$\therefore \quad n^2 + 41n - 40n - 1640 = 0$

$\therefore \quad n(n + 41) - 40(n + 41) = 0$

$\therefore \quad (n + 41)(n - 40) = 0$

$\therefore \quad n + 41 = 0$ or $n - 40 = 0$

$\therefore \quad n = -41$ or $n = 40$

But n cannot be negative

$\therefore \quad n = 40$

$\therefore$ **The no. of terms to be considered = 40.**

5. From an A.P., the first and the last term is 13 and 216 respectively. Common difference is 7. How many terms are there in that A.P. Find the sum of all the terms.

Solution:

Let there be n no. of terms.

Given, $a = 13$, $t_n = 216$, $d = 7$

Now, $t_n = a + (n-1)d$

$\therefore \quad 216 = 13 + (n-1)7$

$\therefore \quad 216 - 13 = (n-1)7$

$\therefore \quad 203 = (n-1)7$

$\therefore \quad (n - 1) = \frac{203}{7} = 29$

$\therefore \quad n = 29 + 1 = 30$

Also, $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore \quad S_{30} = \frac{30}{2}[2 \times 13 + (30-1)7]$

$= 15[26 + 29 \times 7] = 15[26 + 203]$

$= 15 \times 229 = 3435$

$\therefore$ **The no. of terms in the A. P. = 30 and the sum of all 30 terms = 3435.**

---

6. The second and the fourth term of an A.P. is 12 and 20 respectively. Find the sum of the first 25 terms of that A.P.

Solution:

Given, $t_2 = 12$, $t_4 = 20$

Now, $t_n = a + (n-1)d$

$\therefore \quad a + (2-1)d = t_2$

$\therefore \quad a + d = 12 \quad$ ...(i)

Also, $a + (4-1)d = t_4$

$\therefore \quad a + 3d = 20 \quad$ ...(ii)

Subtracting (i) from (ii), we get

$$\begin{array}{rcl} a + 3d &=& 20 \\ a + d &=& 12 \\ (-)\ (-) && (-) \\ \hline 2d &=& 8 \end{array}$$

$\therefore \quad d = \frac{8}{2} = 4$

Substituting $d = 4$ in (i), we get

$a + 4 = 12$

$\therefore \quad a = 12 - 4 = 8$

Now, $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore \quad S_{25} = \frac{25}{2}[2\times 8 + (25-1)4]$

$= \frac{25}{2}[16 + 24 \times 4]$

$= \frac{25}{2}[16 + 96] = \frac{25}{2} \times 112$

$= 25 \times 56$

$\therefore \quad S_{25} = 1400$

$\therefore$ **The sum of first 25 terms = 1400.**

---

7. The sum of the first n terms of an A.P. is $3n + n^2$ then

i. find the first term and the sum of the first two terms

ii. find the second, third and the 15^{th} term.

Solution :

Given, $S_n = 3n + n^2$

i. For $n = 1$, $S_1 = 3(1) + (1)^2$

$= 3 + 1 = 4$

For $n = 2$, $S_2 = 3(2) + (2)^2$

$= 6 + 4 = 10$

$\therefore \quad t_1 = S_1 = 4$ and $S_2 = 10$

ii. Since, $t_n = S_n - S_{n-1}$, for $n > 1$

$\therefore \quad t_2 = S_2 - S_1$

$= 10 - 4 = 6$

$\therefore \quad a = 4, d = 6 - 4 = 2$

Now, $t_n = a + (n-1)d$

$\therefore \quad t_3 = 4 + (3-1)2$

$= 4 + 2 \times 2$

$= 4 + 4 = 8$

$t_{15} = 4 + (15-1)2$

$= 4 + 14 \times 2$

$= 4 + 28 = 32$

$\therefore$ **i. The first term = 4 and the sum of the first two terms = 10.**

ii. The second, third and the 15^{th} terms are 6, 8 and 32 respectively.

8. For an A.P. given below find t_{20} and S_{10}.

$\mathbf{\frac{1}{6}, \frac{1}{4}, \frac{1}{3}, \ldots}$

Solution:

The given A.P. is $\frac{1}{6}, \frac{1}{4}, \frac{1}{3}, \ldots$

Here, $a = \frac{1}{6}$, $d = \frac{1}{4} - \frac{1}{6} = \frac{2}{24} = \frac{1}{12}$

Now, $t_n = a + (n-1)d$

$\therefore \quad t_{20} = \frac{1}{6} + (20-1)\frac{1}{12}$

$= \frac{1}{6} + 19 \times \frac{1}{12} = \frac{1}{6} + \frac{19}{12}$

$= \frac{2+19}{12} = \frac{21}{12} = \frac{7}{4}$

Also, $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore \quad S_{10} = \frac{10}{2}\left[2 \times \frac{1}{6} + (10-1)\frac{1}{12}\right]$

$= 5\left[\frac{1}{3} + 9 \times \frac{1}{12}\right] = 5\left[\frac{1}{3} + \frac{3}{4}\right]$

$= 5\left[\frac{4+9}{12}\right] = 5 \times \frac{13}{12}$

$= \frac{65}{12}$

$\therefore \quad \mathbf{t_{20} = \frac{7}{4}}$ **and** $\mathbf{S_{10} = \frac{65}{12}}$

---

9. In a school, tree plantation on Independence day was arranged. Every student from I standard will plant 2 trees, II standard students will plant 4 trees each, III standard students will plant 8 trees each etc. If there are 5 standards, how many trees are planted by the students of that school?

Solution:

The no. of trees planted standard-wise are :

2, 4, 8, ...

This sequence is a G.P. with $a = 2$, $r = \frac{4}{2} = 2$, $n = 5$

Now, $S_n = \frac{a(r^n - 1)}{(r-1)}$, since $r > 1$

$\therefore \quad S_5 = \dfrac{2(2^5 - 1)}{(2-1)}$

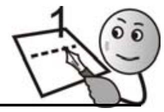
$= \dfrac{2(32-1)}{1}$

$= 2 \times 31$

$= 62$

$\therefore$ **The total trees planted by the students of that school = 62.**

One-Mark Questions



1. Write first three terms of the A.P. when the first term is 10 and common difference in zero.

Solution:

The terms are 10, 10, 10.

2. For the A.P. $\frac{3}{2}$, $\frac{1}{2}$, $\frac{-1}{2}$, … write the common difference.

Solution:

The common difference is –1.

3. For the G.P. $\frac{-1}{2}$, $\frac{1}{4}$, $\frac{-1}{8}$, … write the common ratio.

Solution:

The common ratio is $\frac{-1}{2}$.

4. Write the first 3 terms of the sequence whose n^{th} term is $\frac{1}{n^2} + 1$.

Solution:

The first 3 terms of the sequence are 2, $\frac{5}{4}, \frac{10}{9}$.

5. Write the next two terms of the following sequence: 1, –1, –3, –5, …

Solution:

The next two terms are –7 and –9.

6. For the G.P. $\frac{a}{r^2}$, $\frac{a}{r}$, a, ar, ar^2 write the common ratio.

Solution:

The common ratio is r^2.

7. Frame the AP for the following situation. The taxi fare after each km, when the fare is ₹ 17 for first km and ₹ 9 for each additional km.

Solution:

The A.P. is 17, 26, 35, …

8. In the given A.P., find the missing term: 2, __ , 26.

Solution:

The missing term is 14.

9. Find the next four terms of the G.P. 1, 2, 4, 8, …

Solution:

Here, a = 1, r = 2 and $t_1 = 1$, $t_2 = 2$, $t_3 = 4$, $t_4 = 8$

$\therefore \quad t_5 = t_4 \times 2 = 8 \times 2 = 16$

$t_6 = t_5 \times 2 = 16 \times 2 = 32$

$t_7 = t_6 \times 2 = 32 \times 2 = 64$

$t_8 = t_7 \times 2 = 64 \times 2 = 128$

$\therefore$ **Next four terms are 16, 32, 64, 128.**

10. Find the first three terms of the sequence whose n^{th} term is given by $\frac{1}{n^2} + 1$.

Solution:

$t_n = \dfrac{1}{n^2} + 1$

$\therefore \quad t_1 = \dfrac{1}{12} + 1 = 2$

$\therefore \quad t_2 = \dfrac{1}{2^2} + 1 = \dfrac{5}{4}$

$\therefore \quad t_3 = \dfrac{1}{3^2} + 1 = \dfrac{10}{9}$

$\therefore$ **First three terms are 2, $\frac{5}{4}$, $\frac{10}{9}$.**

11. For a given A.P. if $\alpha = 6$ and $d = 3$, find S_4. **[March 2013]**

Solution:

$$S_n = \frac{n}{2}[2\alpha + (n-1)d]$$

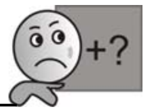
$$\therefore \quad S_4 = \frac{4}{2}[2(6) + (4-1)3]$$

$$= 2(12 + 9)$$

$$= 2(21)$$

$$\therefore \quad S_4 = 42$$

Additional Problems for Practice



Based on Exercise 1.1

1. For each sequence, find the next four terms:
   i. 2, 4, 6, 8, …
   ii. 0.2, 0.02, 0.002, 0.0002, …

2. Find the first five terms of the following:
   i. $t_n = 1 + \frac{1}{n}$ ii. $S_n = \frac{n(n+1)}{2}$

Based on Exercise 1.2

3. Check whether the sequence 7, 12, 17, 22, ….. is an A.P. If it is an A.P., find d and t_n.

4. Which of the following sequences are arithmetic progressions? Justify
   i. 2, 6, 10, 14, …
   ii. 24, 21, 18, 15, …
   iii. 4, 12, 36, 108, …
   iv. $1, \frac{3}{2}, 2, \frac{5}{2}, \ldots$
   v. – 50, – 75, – 100, …
   vi. 12, 2, – 8, – 18, …

5. Write the first four terms of the following Arithmetic Progression where the common difference ‘d’ and the first term ‘a’ are given.
   i. a = 5, d = 7
   ii. a = 8, d = 0

Based on Exercise 1.3

6. For an A.P. if $t_4 = 20$ and $t_7 = 32$, find a, d and t_n.

7. Find the
   i. 10^{th} term of the A.P. 3, 1, –1, –3, …
   ii. 7^{th} term of the A.P. 6, 10, 14, …

8. How many terms are there in the A.P. 201, 208, 215, … 369?

9. If the 5^{th} and 12^{th} terms of an A.P. are 14 and 35 respectively, find the first term and the common difference.

Based on Exercise 1.4

10. Find the sum of the first n terms of an A.P. 1, 4, 7, 10, … Also find S_{40}.

11. If for an A.P. **[March 2013]**
    i. $\alpha = 6$, d = 3, find S_8
    ii. $\alpha = 6$, d = 3, find S_6

12. If for an A.P. $t_8 = 36$, find S_{15}.

13. If for an A.P. $S_{31} = 186$, find t_{16}.

14. Find the sum of all natural numbers from 50 to 250, which are exactly divisible by 4.

Based on Exercise 1.5

15. Find four consecutive terms in an A.P. such that their sum is 26 and the product of the first and the fourth term is 40.

16. Find five consecutive terms in A.P. such that their sum is 65 and the product of the third and the fourth term exceeds the fifth by 195.

Based on Exercise 1.6

17. A man borrows ₹ 1,000 and agrees to repay without interest in 10 installments, each installment being less than the preceding installment by ₹ 8. Find his first installment.

18. A man saves ₹ 16,500 in ten years. In each year after the first he saved ₹ 100 more than he did in the preceding year. How much did he save in the first year?

19. A man borrows ₹ 2,000 and agrees to repay with a total interest of ₹ 340 in 12 monthly installments, each installment being less than the preceding one by ₹ 10. Find the amount of the first and the last installment.

20. A sum of ` 6,240 is paid off in 30 installments, such that each installment is 10 more than the preceding installment. Calculate the value of the first installment.

Based on Exercise 1.7

21. If for a G. P., a = 5, r = –2, find t_9.

22. If for a G. P., a = 7, r = $\frac{1}{3}$, find t_6.

23. If the 2nd term and 5th term of a G.P. are 24 and 81 respectively then find the first term and the common ratio.

Based on Exercise 1.8

24. Find the indicated sums for the following Geometric Progressions.
    i. 1, –3, 9, –27, ...
    ii. 1, $\sqrt{5}$, 5, $5\sqrt{5}$, ...

25. If for a G. P., S_5 = 1023, r = 4, find a.

26. If for a G. P., t_3 = 18 and t_6 = 486, find S_5.

Based on Exercise 1.9

27. Find three consecutive terms in a G.P. such that their sum is 28 and their product is 512.

28. If 33 is the arithmetic mean between x and 17, find the value of x.

29. If x – 1 is the geometric mean between x – 5 and x + 7, then find x.

Answers to additional problems for practice

1. i. 10, 12, 14, 16
   ii. 0.00002, 0.000002, 0.0000002, 0.00000002
2. i. 2, $\frac{3}{2}, \frac{4}{3}, \frac{5}{4}, \frac{6}{5}$ ii. 1, 2, 3, 4, 5
3. d = 5 and t_n = 5n + 2
4. iii. is not an A.P.
   i, ii, iv, v, vi are A.P.
5. i. 5, 12, 19, 26, …
   ii. 8, 8, 8, 8, …
6. a = 8, d = 4, t_n = 4n + 4
7. i. 10th term is –15. ii. 7th term is 30
8. There are 25 terms in the given A.P.
9. The first term is 2 and common difference is 3.
10. S_{40} = 2380
11. i. S_8 = 132 ii. S_6 = 81
12. S_{15} = 540
13. t_{16} = 6
14. The sum of all natural numbers from 50 to 250, that are divisible by 4 is 7500.
15. 5, 6, 7, 8
16. 9, 11, 13, 15, 17
17. The first installment is of ` 136.
18. The man saved ₹1200 in the first year.
19. The first installment is Rs 250 and the last installment is ₹ 140.
20. The first installment is ₹ 63.
21. 1280
22. $\frac{7}{243}$
23. t_1 = 16, r = $\frac{3}{2}$
24. i. $\frac{365}{2}$ ii. $156\left(\sqrt{5}+1\right)$
25. 3
26. 242
27. 5, 10, 20 or 20, 10, 5
28. x = 49
29. x = 9