SATHYABAMA UNIVERSITY

(Established under section 3 of UGC Act,1956)

Course & Branch: B. E/B.Tech - CSE/ECE/EEE/MECH/M&P/E&C/EIE/CIVIL/IT/CHEM/ETCE/AERO
Title of the paper: Engineering Mathematics - IV
Semester: IV — Max. Marks: 80
Sub.Code: 401(2002/2003/2004) — Time: 3 Hours
Date: 30-11-2006 — Session: AN

PART – A (10 x 2 = 20)

Answer ALL the Questions

1. Find a Fourier sine series for the function $f(x) = 1, 0 < x < \pi$.

2. State Dirichlet's conditions for a given function to expand in Fourier series.

3. Find the partial differential equation of the family of spheres having their centres on the line x = y = z.

4. Find the completer integral of q = 2px.

5. State the assumptions involved in obtaining the PDE of vibration of string.

6. How many conditions are required to $\frac{\partial^2 y}{\partial t^2} = \propto^2 \frac{\partial^2 y}{\partial x^2}$.

7. What are the solutions of $\nabla^2 u = 0$ in polar form?

8. What are the possible solution of $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$.

9. If F[f(x)] = F(s) then, F[f(x – a)] = ……

10. Find the Fourier cosine transform of e^{-x}, $x \geq 0$.

PART – B (5 x 12 = 60)

Answer ALL the Questions

11. (a) Find the Fourier series of $\left(\frac{\pi - x}{2}\right)^2$ in $(0, 2\pi)$ and hence find the sum of the series $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots\ldots\ldots$

(b) Find the sine series of $f(x) = \begin{cases} x & \text{in } 0 < x < 1 \\ 2 - x & \text{in } 1 < x < 2 \end{cases}$

(or)

12. (a) Obtain the Fourier series of $f(x) = x \sin x$, in $-\pi < x < \pi$.

(b) Expand $f(x) = x$ as a cosine series in $0 < x < 1$ and deduce the values of $\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \ldots\ldots\ldots\ldots\ldots$

13. (a) Obtain the partial differential equation by eliminating f from $f(x^2 + y^2 + z^2, x + y + z) = 0$

(b) Solve $z^2(P^2 + q^2) = x^2 + y^2$.

(or)

14. (a) Solve $(x^2 = yz)p + (y^2 - zx)q = z^2 - xy$

(b) Solve $(D^2 + 2DD' + 2D'^2)z = x^2y$

15. A string of length $2l$ is fastened at both ends. The mid point of the string is taken to a height h and then released from rest in that position. Find the displacement y(x,t)

(or)

16. Solve $\frac{\partial u}{\partial t} = \propto^2 \frac{\partial^2 u}{\partial x^2}$ subject to the conditions.

(a) u is finite as $t \to \propto$

(b) $\frac{\partial u}{\partial x} = 0$ for $x = 0$ and $x = l$.

(c) $u = \begin{cases} x & \text{for } 0 < x < \frac{l}{2} \\ l - x & \text{for } \frac{l}{2} \le x < l. \end{cases}$

17. Find the steady state temperature in a circular plate of radius 'a' which has one half of its circumference at 0°C and the other half at 100°C.

(or)

18. A semi-Circular plate of radius a has its circumference maintained at $u(a,\theta) = K\theta(\pi - \theta)$ for $0 < \theta < \pi$ while the bounding diameter is maintained at 0°C. Assuming the lateral surface of the plate is insulated, find the Temperature distribution $u(r, \theta)$ in the steady state.

19. (a) Find the Fourier transform of $f(x) = \begin{cases} a - |x| & \text{if } |x| \le a \\ 0 & \text{if } |x| > a > 0 \end{cases}$

(b) Hence deduce that $\int_0^{\frac{\pi}{2}} \left(\frac{\sin t}{t}\right)^2 dt = \frac{\pi}{2}$.

(or)

20. (a) State and prove parseval's identify.

(b) Using parseval's identify evaluate $\int_0^{\infty} \frac{dx}{(a^2 + x^2)^2}$.