SATHYABAMA UNIVERSITY
(Established under section 3 of UGC Act, 1956)

Course & Branch: B.E/B.Tech – Common to ALL Branches (Except to Bio Groups & EEE 2005 Batch)
Title of the paper: Engineering Mathematics - IV
Semester: IV Max. Marks: 80
Sub.Code: 401(2003/2004/2005)6C0054 Time: 3 Hours
Date: 10-11-2008 Session: AN

PART – A (10 x 2 = 20)
Answer All the Questions

1. State the conditions for which a function f(x) to be expaded as a Fourier series.

2. State the Parseval's identity corresponding to a complex fourier series.

3. Form the partial differential equation of all the planes cutting equal intercepts on u X and Y axes.

4. Solve $\left(D^2 + DD' + D'^2\right)Z = 0$. .

5. Write the possible solutions of the one-dimensional wave equation $y_{tt} = c^2 y_{xx}$.

6. State the empirical assumed in deriving one-dimensional heat flow equation (un steady state).

7. Define steady state, unsteady state.

8. Write the periodic solutions in y and x of laplace equation $\nabla^2 u = 0$.

9. State the change of scale property on Fourier transforms.

10. Find the infinite fourier sine transform of $\frac{1}{x}$.

PART – B (5 x 12 = 60)

Answer ALL the Questions

11. (a) Expand f(x) = |x| as a full range Fourier series and hence deduce the value of $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots \infty$.

(b) Expand $f(x) = 2x - x^2$, $0 < x < 3$ as a half range sine series.

(or)

12. (a) Expand the function f(x) = sinx, $0 < x < \pi$ as a series of cosines.

(b) Find the constant term and the co-efficient of the first sine and cosine terms in the fourier expansion of y as given in the following table

X	0	1	2	3	4	5
Y	9	18	24	28	26	20

13. (a) Form the PDE by eliminating the arbitrary constants a, b from the relation: $z = \frac{1}{2}\left(\sqrt{x+a} + \sqrt{y-a} + b.\right)$.

(b) Solve $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6\frac{\partial^2 z}{\partial y^2} = y \cos x$.

(or)

14. (a) Find the general solution of
$x(z^2 - y^2)\, p + y\,(x^2 - z^2)q = z(y^2 - x^2)$

(b) Solve $z\,(p^2 + q^2) = x^2 + y^2$.

15. If a string of length l is initially at rest in equilibrium position and each point of it is given the velocity $\left(\dfrac{\partial y}{\partial t}\right)_{(x,0)} = V_0 \sin^3\left(\dfrac{\pi x}{l}\right), 0 < x < l.$ determine the transverse displacement y = (x, t).

(or)

16. Solve $\dfrac{\partial u}{\partial t} = \alpha^2 \dfrac{\partial^2 u}{\partial x^2}$ subject to the conditions

(i) u is not infinite as $t \to \infty$
(ii) $u = 0$ for $x = 0$ and $x = \pi$ for all t
(iii) $u = \pi x - x^2$ for $t = 0$ in $0 < x < \pi$.

17. A square plate is bounded by the lines x = 0, y = 0, x = 20, y = 20. Its faces are insulated The temperature along the upper horizontal edge is given by u(x, 20) = x(20-x) where 0 < x < 20, which the other three edges are kept at 0°C. Find the steady state temperature u(x, y).

(or)

18. A semi circular plate of radius 'a' cm has insulated faces and heat flows in plane curves. The bounding diameter is kept at 0°C and the semi-circumference is maintained at temperature given by

$$u(a,\theta) = \begin{cases} \dfrac{k\theta}{\pi} & 0 \le \theta \le \dfrac{\pi}{2} \\ \dfrac{k}{\pi}(\pi - \theta) & \dfrac{\pi}{2} \le \theta \le \pi \end{cases}$$

Find the steady-state temperature distribution.

19. (a) Show that the Fourier transform of $f(x) = \begin{cases} a - |x| & \text{for } |x| < a \\ 0 & \text{for } |x| > a > 0 \end{cases}$

is $\sqrt{\frac{2}{\pi}\left(\frac{1-\cos as}{s^2}\right)}$ Hence deduce the value of $\int_0^\infty \left(\frac{\sin t}{t}\right)^2 dt$.

(b) Find the finite Fourier sine, Cosine transforms of $f(x) = \left(1 - \frac{x}{\pi}\right)^2, 0 < x < \pi$

(or)

20. (a) Find the Fourier cosine transform of $e^{-a^2x^2}$ and hence evaluate the fourier sine transform of $xe^{-a^2x^2}$.

(b) Solve the integral equation

$$\int_0^\infty f(x)\sin \lambda x dx = \begin{cases} 1 & \text{for} & 0 \le \lambda < 1 \\ 2 & \text{for} & 1 \le \lambda \le 2 \\ 0 & \text{for} & \lambda \ge 2 \end{cases}$$