MSc PHYSICS

1588/MP1 **MAY 2008**

Paper I — MATHEMATICAL PHYSICS

Time : Three hours Maximum : 100 marks

Answer ALL questions.

All questions carry equal marks.

$(5 \times 20 = 100)$

1. (a) (i) State and prove Stoke's theorem.

(ii) Verify Stoke's theorem for the vector F = (z,x,y) taken over the half of the sphere $x^2+y^2+z^2=a^2$ lying above the xy plane.

Or

(b) (i) Define subgroups, classes, cosets and invariant subgroups of a group.

(ii) Define conjugate classes of a group. Show that all the elements of a class have the same order and the same character.

2. (a) (i) Explain convergence and divergence of a series.

(ii) Explain Cauchy's integral test.

(iii) Check whether the following series in convergent or not. $f(x)=\sum_{n=1}^{\infty}\frac{n^2}{3^n}$

Or

(b) (i) Deduce the value of $\Gamma\left(\frac{1}{2}\right)$.

(ii) Find the relation between Beta and Gamma functions.

(iii) Show that $\beta(m,n)=\beta(n,m)$.

3. (a) (i) State and prove Cauchy's Residue theorem.

(ii) Apply the above theorem to evaluate $\int_0^{2\pi} e^{\cos\theta} \cos(n\theta-\sin\theta)d\theta$.

Or

(b) (i) Find the Fourier sine Transform of e^{-x}.

(ii) Find the Fourier cosine Transforrm of $x^n.e^{-ax}$.

4. (a) (i) Starting from the definition of $J_n(x)$ prove that $J_{n-1}(x)+J_{n+1}(x)=\frac{2n}{x}J_n(x)$.

(ii) Obtain the series solution of the Hermite differential equation. $y''-2xy'+2ny=0$ when n=2.

Or

(b) (i) Obtain differential form of Hermite polynomials and then find $H_3(x)$.

(ii) Prove the following recurrence relations for Hermite polynomials.

$$H'_n(x)=2xH_n(x)-H_{n+1}(x)$$
$$H'_{2n+1}(x)=2(2n+1)H_{2n}(x)$$

5. (a) Derive the wave equation for a perfectly flexible stretched string and then construct Fourier series solution for it.

Or

(b) (i) Construct the Green's function for the nonhomogeneous problem $\frac{d^2u}{dx^2}=f(x)$ with boundary condition $u(0)=u(L)=0$.

(ii) Expand the Green's function in a series of eigen functions of the homogeneous equation.

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