SECTION - A

$10 \times 2 = 20$

VERY SHORT ANSWER TYPE QUESTIONS

Answer All questions. Each question carries 2 marks.

1. If $f(x) = \dfrac{x+1}{x-1}$ $(x \neq 1)$ then find the values of $(f \circ f \circ f)(x)$.
2. Find the domain of $\dfrac{\sqrt{3+x} + \sqrt{3-x}}{x}$.
3. Find p if $4\mathbf{i} + \dfrac{2p}{3}\mathbf{j} + p\mathbf{k}$ is parallel to $\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$.
4. If D, E, F are the midpoints of the sides BC, CA, AB of ΔABC, then show that $\vec{AD} + \vec{BE} + \vec{CF} = \mathbf{0}$.
5. Find the angle between $\mathbf{a}$ and $\mathbf{b}$ if $|\mathbf{a} + \mathbf{b}| = |\mathbf{a} - \mathbf{b}|$.
6. Find the period of the function $\cos\left(\dfrac{4x+9}{5}\right)$.
7. Prove that $\cot\dfrac{\pi}{20}\cot\dfrac{3\pi}{20}\cot\dfrac{5\pi}{20}\cot\dfrac{7\pi}{20}\cot\dfrac{9\pi}{20} = 1$
8. Prove that $Sinh^{-1}x = \log_e(x + \sqrt{x^2+1})$.
9. Prove that $2(bc\cos A + ca\cos B + ab\cos C) = a^2 + b^2 + c^2$.
10. If the point P denotes the complex number $z = x + iy$ in the Argand plane and if $\dfrac{z-i}{z-1}$ is a purely imaginary number, find the locus of P.

SECTION - B

$5 \times 4 = 20$

SHORT ANSWER TYPE QUESTIONS

Attempt any 5 questions. Each question carries 4 marks.

11. If $ABCDEF$ is a regular hexagon, and G is its centre, then prove that $\vec{AB} + \vec{AC} + \vec{AD} + \vec{AE} + \vec{AF} = 3\vec{AD} = 6\vec{AG}$.
12. Find the unit vector perpendicular to the plane passing through the points $(1, 2, 3)$, $(2, -1, 1)$ and $(1, 2, -4)$.

13. If $A + B = 225^\circ$, prove that $\frac{\cot A}{1 + \cot A} \cdot \frac{\cot B}{1 + \cot B} = \frac{1}{2}$.

14. Solve $\sin 2\theta - \cos 2\theta - \sin \theta + \cos \theta = 0$.

15. Show that $Tan^{-1}\left(\frac{1}{7}\right) + Tan^{-1}\left(\frac{1}{8}\right) = Cot^{-1}\left(\frac{201}{43}\right) + Cot^{-1}(18)$.

16. Show that $\cot \frac{A}{2}, \cot \frac{B}{2}, \cot \frac{C}{2}$ are in A.P. iff a, b, c are in A.P.

17. Show that $2^5 \sin^6 \theta = 10 - 15 \cos 2\theta + 6 \cos 4\theta - \cos 6\theta$.

SECTION - C

5 × 7 = 35

LONG ANSWER TYPE QUESTIONS

Attempt any 5 questions. Each question carries 7 marks.

18. Let $f : A \to B$ and I_A and I_B be the identity functions on A and B respectively. Then prove that $f \circ I_A = I_B \circ f = f$.

19. Prove by induction that $\frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} + \ldots n$ terms $= \frac{n}{2n+1}$.

20. Find the shortest distance between the straight line passing through the point $(6, 2, 2)$ and parallel to the vector $(1, -2, 2)$ and the straight line passing through $(-4, 0, -1)$ and parallel to the vector $(3, -2, -2)$.

21. If $A + B + C = 180^\circ$ then show that

$$\cos \frac{A}{2} + \cos \frac{B}{2} - \cos \frac{C}{2} = 4 \cos\left(\frac{\pi + A}{4}\right) \cos\left(\frac{\pi + B}{4}\right) \cos\left(\frac{\pi - C}{4}\right).$$

22. Show that $\cos^2\left(\frac{A}{2}\right) + \cos^2\left(\frac{B}{2}\right) + \cos^2\left(\frac{C}{2}\right) = 2 + \frac{r}{2R}$.

23. From a point A on the level ground away from the foot of a spire, the angle of elevation of the top of the spire is 30°. From a point at a height of h metres vertically above A, the angle of depression of the foot of the spire is 60°. Find the height of the spire in terms of h.

24. If n is a positive integer, prove that $(1 + i)^{2n} + (1 - i)^{2n} = 2^{(n+1)} \cos(n\pi/2)$.