DipIETE – ET / CS (OLD SCHEME)

Code: DE23/DC23 **Subject: MATHEMATICS - II**
Time: 3 Hours **Max. Marks: 100**

DECEMBER 2009

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: (2x10)

a. If $\left(\frac{x}{2}-\frac{y}{3}\right)+\left(\frac{2}{3}y\right)i=-2+4i$ then x, y equal to

(A) $x=0, y=6$ **(B)** $x=1, y=6$

(C) $x=6, y=0$ **(D)** $x=6, y=-1$

b. The value of $\left[2(\cos 40^\circ+\sin 40^\circ)\right]^6$ is

(A) $32\left(1-i\sqrt{3}\right)$ **(B)** $32\left(1+i\sqrt{3}\right)$

(C) $-32\left(-1-i\sqrt{3}\right)$ **(D)** $-32\left(1+i\sqrt{3}\right)$

c. If $\vec{a}$ & $\vec{b}$ be two vectors indefined at an angle θ, then $\vec{a}.\vec{b}$ is:

(A) $-\left|\vec{a}\right|\left|\vec{b}\right|\cos\theta$ **(B)** $\left|\vec{a}\right|\left|\vec{b}\right|\cos\theta$

(C) $\left|\vec{a}\right|\left|\vec{b}\right|\sin\theta$ **(D) -** $\left|\vec{a}\right|\left|\vec{b}\right|\sin\theta$

d. If $\vec{a}=2\hat{i}+3\hat{j}+4\hat{k}$ and $\vec{b}=\hat{i}+\hat{j}+\hat{k}$ then $\vec{a}\times\vec{b}$ is

(A) $\hat{i}+2\hat{j}+\hat{k}$ **(B)** $-\hat{i}-2\hat{j}-\hat{k}$

(C) $-\hat{i}+2\hat{j}-\hat{k}$ **(D)** $-\hat{i}+2\hat{j}+\hat{k}$

e. The values of x, y, z if

$$\begin{bmatrix} x & 3 \\ 4 & 2 \end{bmatrix}+\begin{bmatrix} 2 & y \\ 1 & -2 \end{bmatrix}-\begin{bmatrix} 1 & 1 \\ -2 & z \end{bmatrix}=\begin{bmatrix} 5 & 0 \\ 7 & 3 \end{bmatrix}$$

(A) $x=4, y=-2, z=-3$ **(B)** $x=-4, y=2, z=3$

(C) $x=-4, y=-2, z=-3$ **(D)** $x=4, y=2, z=3$

f. $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$ is equal to

(A) -1 **(B)** 0
(C) (b-a) (c-d) **(D)** (a-b) (b-c) (c-a)

g. The characteristic equation of $\begin{bmatrix} -1 & 3 \\ 4 & 2 \end{bmatrix}$ is

(A) $\lambda^2 + \lambda + 14 = 0$ **(B)** $2\lambda^2 - \lambda + 13 = 0$

(C) $\lambda^2 - \lambda - 14 = 0$ **(D)** $\lambda^2 + \lambda - 13 = 0$

h. The period of $\sin^2 x$ is

(A) 2π **(B)** $-\pi$

(C) -2π **(D)** π

i. The laplace transform of the function $t^3 e^{-2t}$ is

(A) $\frac{6}{(s-2)^4}$ **(B)** $\frac{6}{(s+2)^3}$

(C) $\frac{6}{(s+2)^4}$ **(D)** $\frac{-6}{(s+2)^3}$

j. The solution of differential equation $\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = \cos 2x$ is

(A) $y = e^{-x/2}\left(c_1 \cos\frac{\sqrt{3}x}{2} + c_2 \sin\frac{\sqrt{3}x}{2}\right) - \frac{1}{13}(-2\sin 2x + 3\cos 2x)$

(B) $y = e^{x/2}\left(c_1 \cos\frac{\sqrt{3}x}{2} + c_2 \sin\frac{\sqrt{3}x}{2}\right) + \frac{1}{13}(-2\sin 2x + 3\cos 2x)$

(C) $y = e^{-x/2}\left(c_1 \cos\frac{x}{2} + c_2 \sin\frac{x}{2}\right) - \frac{1}{13}(-2\sin 2x + 3\cos 2x)$

(D) $y = e^{x/2}\left(c_1 \cos\frac{\sqrt{3}x}{2} + c_2 \sin\frac{\sqrt{3}x}{2}\right) + \frac{1}{13}(-2\sin 2x + 3\cos 2x)$

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. If $\frac{c+i}{c-i} = a + ib$, where c is real, prove that $a^2 + b^2 = 1$ and $\frac{b}{a} = \frac{2c}{c^2 - 1}$. **(8)**

b. If n is a positive integer, prove that $\left(\sqrt{3}+i\right)^n + \left(\sqrt{3}-i\right)^n = 2^{n+1}\cos\frac{n\pi}{6}$. **(8)**

Q.3 a. For what value of x and y are the numbers $-3+ix^2y$ and x^2+y+4i conjugate complex? **(8)**

b. The adjacent sides of a parallelogram are represented by the vectors $2\hat{i}+4\hat{j}-5\hat{k}$ and $\hat{i}+2\hat{j}+3\hat{k}$. Find unit vectors parallel to the diagonals of a parallelogram. **(8)**

Q.4 a. Prove that the points having position vectors $\vec{a}=2\hat{i}-\hat{j}+\hat{k}, \vec{b}=\hat{i}-3\hat{j}-5\hat{k}$, $\vec{c}=3\hat{i}-4\hat{j}-4\hat{k}$ form a right angled triangle. **(8)**

b. Find the area of the triangle formed by the points whose position vectors are $3\hat{i}+\hat{j}$, $5\hat{i}+2\hat{j}+\hat{k}$, $\hat{i}-2\hat{j}+3\hat{k}$. **(8)**

Q.5 a. Let $f(x)=x^2-5x+6$, find f(A) if $A=\begin{bmatrix}2&0&1\\2&1&3\\1&-1&0\end{bmatrix}$. **(8)**

b. Prove that $\begin{vmatrix}1+a&1&1\\1&1+b&1\\1&1&1+c\end{vmatrix}=abc\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+1\right)=abc+ab+bc+ca$. **(8)**

Q.6 a. Solve the system of equations by matrix method.

$$2x-2y+z=2$$
$$3x+y-z=0$$
$$x+3y+2z=2$$ **(8)**

b. Verify Cayley-Hamilton theorem for the matrix A and find its inverse.

$$A=\begin{bmatrix}2&-1&1\\-1&2&-1\\1&-2&2\end{bmatrix}$$ **(8)**

Q.7 a. Find the Laplace transform of $t^2\sin 2t$. **(8)**

b. Find the inverse Laplace transform of $\frac{s}{(s+1)^5}$. **(8)**

Q.8 a. Solve $\frac{d^2y}{dx^2}+4y=e^x+\sin 2x$. **(8)**

b. Solve the differential equation $\frac{d^2x}{dt^2} - 3\frac{dx}{dt} + 2x = 4t + e^{3t}$, given that x=1 & $\frac{dx}{dt} = -1$ when t =0. **(8)**

Q.9 a. Determine the period of the following functions:

(i) $\cos 2\pi x$ (ii) $\cos \frac{nx}{2\pi}$

(iii) $\cos^2 \frac{x}{2}$ (iv) $\cos\left(\frac{x}{2}+5\right)$ **(8)**

b. Obtain the fourier series for

$$f(x)=\begin{cases} 0, & -\pi < x < 0 \\ \frac{\pi x}{4}, & 0 < x < \pi \end{cases}$$

Hence, prove that $\frac{\pi^2}{8}=\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}+..................$ **(8)**