B3.2-R3: BASIC MATHEMATICS

NOTE:

1.	**Answer question 1 and any FOUR from questions 2 to 7.**
2.	**Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) Find |z| when

$$z=\frac{(2-3i)(1+i)}{2+i}$$

b) Evaluate the determinant

$$\Delta=\begin{vmatrix}\cos x & 0 & -\sin x\\ 0 & 1 & 0\\ \sin x & 0 & \cos x\end{vmatrix}$$

c) Find

$$\lim_{x\to 0}\frac{\sin ax}{\sin bx}$$

d) Differentiate $\sin(\cos x^2)$ with respect to x.

e) Evaluate

$$\mathbf{I}=\int\frac{\mathbf{dx}}{\sqrt{\mathbf{25-16x^2}}}$$

f) Test the convergence of the following series

$$\sum_{n=1}^{\infty}\frac{1}{\sqrt{n+1}}$$

g) Find the radius of the circle

$x^2 + y^2 + 8x + 10y - 8 = 0$

(7x4)

2.

a) Find the rank of the matrix

$$\begin{bmatrix}1 & 2 & 3\\ 1 & 4 & 2\\ 2 & 6 & 5\end{bmatrix}$$

b) Solve the following system of equations by Cramer's rule:

$$\begin{aligned}3x - 2y + 3z &= 8\\ 2x + y - z &= 1\\ 4x - 3y + 2z &= 4\end{aligned}$$

c) For the matrix $A = \begin{bmatrix}1 & 1 & 1\\ 1 & 2 & -3\\ 2 & -1 & 3\end{bmatrix}$,

show that $A^3 - 6A^2 + 5A + 11\,I = O$, where I is 3x3 identity matrix and O is (3x3) zero matrix. Hence find A^{-1}.

(4+6+8)

3.

a) Find the area enclosed by the parabola $ay = 3(a^2 - x^2)$ and the *x*-axis.

b) Discuss the continuity of the function

$$f(x) = \begin{cases} 2x - 1 & \text{if} \quad x < 2 \\ \dfrac{3x}{2} & \text{if} \quad x \geq 2 \end{cases}$$

c) State Rolle's theorem. Hence verify Rolle's theorem for the function

$f(x) = x(x-1)^2$ in the interval [0, 1].

(6+6+6)

4.

a) Test the convergence of the series

$$2x + \frac{3x^2}{8} + \frac{4x^3}{27} + \ldots + \frac{(n+1)x^n}{n^3} + \ldots$$

b) Find the value of

$$I - \frac{\pi}{4},$$

where

$$I = \int_0^{\pi/2} \frac{\sin x}{(1 + \cos^2 x)} \, dx$$

c) Evaluate $\displaystyle\int \frac{dx}{(x+1)^2 (x^2 + 1)}$

(6+6+6)

5.

a) If $(x + iy)^{1/3} = a + ib$, prove that

$$\frac{x}{a} + \frac{y}{b} = 4(a^2 - b^2)$$

b) Determine the value of λ so that the vectors

$\overline{\mathbf{a}} = 2\mathbf{i} - \lambda\mathbf{j} + \mathbf{k}$ and $\overline{\mathbf{b}} = \mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$ are perpendicular to each other.

c) Find x if 17th and 18th terms of the expansion $(2 + x)^{50}$ are equal.

(6+6+6)

6.

a) If $x = a(\theta - \sin\theta),\ y = a(1-\cos\theta)$, show that

$\frac{dy}{dx} = \cot\frac{\theta}{2}$. Compute $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{2}$.

b) Determine the value of k, if

$\lim_{x\to 1}\left(\frac{x^4-1}{x-1}\right) = \lim_{x\to k}\left(\frac{x^3-k^3}{x^2-k^2}\right)$.

c) Given $y = a\ \sin x + b\cos x$, obtain the value of

$y^2 + \left(\frac{dy}{dx}\right)^2 - a^2 - b^2$.

(6+6+6)

7.

a) Find the foci, vertices and the eccentricity of the ellipse $16x^2 + 25y^2 = 400$.

b) Obtain the equation of the hyperbola with foci (0, ±3) and vertices $(0, \pm\frac{\sqrt{11}}{2})$.

c) Determine the equation of the straight line passing through the point (-2, -3) and inclined at 60° to the line $x + \sqrt{3y} = 2$.

(5+6+7)