

## Mathematical Sciences

### Paper II

**Time Allowed : 75 Minutes]**

**[Maximum Marks : 100**

**Note :** This Paper contains **Fifty (50)** multiple choice questions. Each question carries **Two (2)** marks. Attempt *All* questions.

1. Let  $f(z) = \sin z, z \in \mathbb{C}$ . Then  $f(z)$  :

- (A) is bounded in the complex plane
- (B) assumes all complex numbers
- (C) assumes all complex numbers except  $i$
- (D) assumes all complex numbers except  $i$  and  $-i$

2. The radius of convergence of the series

$$\sum \frac{n!z^n}{n^n}$$

is :

- (A) 1
- (B)  $\infty$
- (C)  $1/4$
- (D)  $e$

3. Let the sequence  $\{a_n\}$  be given by

$$1, 2, 3, 1 + \frac{1}{2}, 2 + \frac{1}{2}, 3 + \frac{1}{2}, \\ 1 + \frac{1}{3}, 2 + \frac{1}{3}, 3 + \frac{1}{3}, \dots \dots .$$

Then

$$\limsup_{n \rightarrow \infty} a_n$$

is :

- (A) 3
- (B)  $\infty$
- (C) 1
- (D)  $-1$

4. If  $\emptyset \neq E \subset F \subset \mathbb{R}$ , then :

- (A)  $\inf E \leq \inf F$
- (B)  $\inf E > \inf F$
- (C)  $\inf E \geq \inf F$
- (D)  $\inf E < \inf F$

5. If  $a$  and  $b$  are real numbers, then  
 $\inf \{a, b\} =$

(A)  $\frac{a + b - |a - b|}{2}$

(B)  $\frac{a + b + |a - b|}{2}$

(C)  $\frac{a - b + |a + b|}{2}$

(D)  $\frac{a - b - |a - b|}{2}$

6. The dimension of the space of  
 $n \times n$  matrices all of whose  
 components are 0 except possibly  
 the diagonal components is :

(A)  $n^2$

(B)  $n - 1$

(C)  $n^2 - 1$

(D)  $n$

7. The matrix  $R(\theta)$  associated with the  
 rotation by  $\theta = \pi/4$  is :

(A)  $\begin{bmatrix} -\sqrt{2}/2 & \sqrt{2}/2 \\ -\sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix}$

(B)  $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

(C)  $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

(D)  $\begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 \\ \sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix}$

8. Let  $W$  be a subspace of a vector  
 space  $V$  and let  $T : W \rightarrow V'$  be a  
 linear map, then :

(A)  $T$  can be extended to a linear  
 transformation from  $V$  to  $V'$

(B)  $T$  is necessarily linear map from  
 $V$  to  $V'$

(C)  $\text{Ker } T$  is not a subspace of  $V$

(D)  $\text{Im } T$  is a subspace of  $V$

9. Let  $S, T \in L(V_1, V_2)$ ,  $V_1, V_2$  are finite dimensional vector spaces. Then :
- (A)  $\text{rank } S + \text{rank } T \leq \text{rank } (S + T)$
- (B)  $\text{Im } (S + T) = \text{Im } S + \text{Im } T$
- (C)  $\text{Im } (S + T) \subseteq \text{Im } S + \text{Im } T$
- (D)  $\min\{\text{rank } S, \text{rank } T\} \leq \text{rank } ST$ ,  
where  $V_1 = V_2$
10. Let  $A$  be a matrix similar to a square matrix  $B$ . Then which one of the following is *false* ?
- (A) If  $A$  is self-adjoint then so is  $B$
- (B) If  $A$  is non-singular then so is  $B$
- (C) Determinant of  $A$  is the same as the determinant of  $B$
- (D) Trace of  $A$  is the same as the trace of  $B$
11. A sample space consists of five simple events  $E_1, E_2, E_3, E_4$  and  $E_5$ . If  $P(E_1) = P(E_2) = 0.1$ ,  $P(E_3) = 0.4$  and  $P(E_4) = 3P(E_5)$ . Then  $P(E_4)$  and  $P(E_5)$  :
- (A) are 0.06 and 0.02, respectively
- (B) cannot be determined from the given information
- (C) are 0.6 and 0.2, respectively
- (D) are 0.3 and 0.1, respectively
12. If independent binomial experiments are conducted with  $n = 10$  trials. If the probability of success in each trial is  $P = 0.6$ , then the average number of successes per experiment is :
- (A) 4
- (B) 6
- (C) 8
- (D) 10

13. The joint distribution of r.v.s X and Y is given by :

X\Y	-2	0	2
0	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
2	$\frac{1}{8}$	$\frac{1}{8}$	0

Then  $P[X + Y = 0]$  is :

- (A)  $3/4$   
 (B)  $1/4$   
 (C)  $3/8$   
 (D)  $5/8$
14. Let X and Y be two r.v.s such that  $E[X] = 2$ ,  $E[X^2] = 5$ ,  $E[Y] = 4$ ,  $E[Y^2] = 16$  and  $E[XY] = 12$ .

Consider the statements :

- (I) The r.v.s X and Y are independent  
 (II) The r.v. Y is degenerate  
 (III) Given expected values are not compatible

Then :

- (A) Only statements I and II are correct  
 (B) Only statements II and III are correct  
 (C) Only statements I and III are correct  
 (D) All three statements are correct

15. The objective 'Linear' in Linear Programming Problem implies :

- (A) The objective function and the constraints are both linear in the variables  
 (B) The constraints are linear but not the objective function  
 (C) The objective function alone is linear in the variables  
 (D) None of the above

16. The optimal solution to the Linear Programming Problem :

Maximize :  $Z = 2x_1 + x_2$

Subject to :  $x_1 + x_2 \leq 1 \rightarrow (1)$

$3x_1 + x_2 \leq 2 \rightarrow (2)$

$x_1 + 2x_2 \leq 3 \rightarrow (3)$

$x_1, x_2 \geq 0 \rightarrow (4)$

- (A) lies at the intersection of (1) and (3)  
 (B) lies at the intersection of (2) and (3)  
 (C) lies at the intersection of (1) and (2)  
 (D) cannot be determined

17. If  $a_n = (-1)^n n$ , for  $n = 1, 2, 3, 4, \dots$ , then :

$$\lim_{n \rightarrow \infty} \sup a_n$$

is equal to :

- (A) 1  
(B)  $\infty$   
(C) -1  
(D)  $-\infty$

Or

Let F and G be probability distribution functions. Which of the following may *not* be a probability distribution function ?

- (A)  $H(x) = F^2(x)G^3(x)$   
(B)  $H(x) = F(G(x))$   
(C)  $H(x) = \int_{-\infty}^x G(u) dF(u)$   
(D)  $H(x) = \frac{F(x) + G(x)}{2}$

18. If  $\phi \neq E \subset F \subset \mathbb{R}$ , then :

- (A)  $\sup E \leq \sup F$   
(B)  $\sup E \geq \sup F$   
(C)  $\sup E < \sup F$   
(D)  $\sup E > \sup F$

Or

The characteristic function of a standard Cauchy distribution :

- (A) is  $e^{-|t|}$   
(B) is  $e^{it}$   
(C) is  $e^{\pi t}$   
(D) does not exist

19. Radius of convergence of the series :

$$\sum \frac{n! z^n}{n^n}$$

is :

- (A) 1  
(B)  $\frac{1}{4}$   
(C)  $e$   
(D)  $\infty$

Or

The p.d.f. of the random variable X follows  $f(x)$ , where :

$$f(x) = \frac{1}{2\theta} \exp\left[-\frac{|x - \theta|}{\theta}\right],$$

$$-\infty < x < \infty$$

Mean of the distribution is :

- (A)  $\frac{1}{\theta}$
- (B)  $2\theta$
- (C)  $\theta$
- (D) 0

20. The interval of convergence of the series

$$x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$$

is :

- (A)  $-1 < x \leq 1$
- (B)  $-1 < x < 1$
- (C)  $-1 \leq x < 1$
- (D)  $0 \leq x \leq 2$

Or

Let the random variables  $X_1, X_2$  be distributed as  $N(\theta, 1)$ . Then number of unbiased estimators of  $\theta$  is :

- (A) infinity
- (B) 3
- (C) 2
- (D) 4

21. In order that  $f$  defined by  $f(x) = (1 + x)^{\cot x}$   $x \neq 0$  be continuous at  $x = 0$ , how  $f(0)$  be defined ?

- (A)  $f(0) = 0$
- (B)  $f(0) = \frac{1}{e}$
- (C)  $f(0) = e$
- (D)  $f(0) = 1$

Or

Let the random variables  $X_1, X_2$  are Poisson variates with parameter  $\lambda$ . Then which of the statements is true ?

- (A)  $X_1$  is sufficient for  $\lambda$
- (B)  $X_1 - X_2$  is sufficient for  $\lambda$
- (C)  $X_1 + 2X_2$  is not sufficient for  $\lambda$
- (D)  $X_2$  is sufficient for  $\lambda$

22. If  $g(x) = \sin x$  and  $f(x) = \cos x$ , then the point at which the conclusion of Cauchy's Mean Value Theorem holds in  $[-\pi/4, \pi/4]$  is :

- (A)  $\pi/6$
- (B) 0
- (C)  $\pi/4$
- (D)  $-\pi/6$

Or

For testing  $H_0 : F_X(x) = H_Y(y)$ , when two independent random observations on X and Y are available, which of the following non-parametric test cannot be used ?

- (A) Mann-Whitney test
- (B) Sign test
- (C) Kolmogorov-Smirnov test
- (D) Wald-Wolfowitz run test

23. If  $f(x)$  is continuous on  $[a, b]$ , then the *incorrect* statement among the following is :

- (A)  $f(x)$  is bounded on  $[a, b]$
- (B)  $f(x)$  assumes all values between  $f(a)$  and  $f(b)$
- (C)  $f(x)$  is increasing on  $[a, b]$
- (D)  $f(x)$  is uniformly continuous on  $[a, b]$

Or

Let  $X_1, X_2, \dots, X_n$  be a sample from  $N(\mu, 1)$ . Then the shortest confidence interval based on  $T_\mu(X) = \sqrt{n}(\bar{X} - \mu)$  is :

(A)  $\left( \bar{X} - \frac{Z_{1-\alpha/2}}{\sqrt{n}}, \bar{X} + \frac{Z_{1-\alpha/2}}{\sqrt{n}} \right)$

(B)  $\left( \bar{X} - \frac{Z_{1-\alpha/2}}{\sqrt{n}}, \bar{X} + \frac{Z_{\alpha/2}}{\sqrt{n}} \right)$

(C)  $\left( \bar{X} - \frac{Z_{\alpha/2}}{\sqrt{n}}, \bar{X} + \frac{Z_{\alpha/2}}{\sqrt{n}} \right)$

(D)  $\left( \bar{X} - \frac{Z_{\alpha/2}}{\sqrt{n}}, \bar{X} + \frac{Z_{1-\alpha/2}}{\sqrt{n}} \right)$

24. Let  $f: \mathbb{C} \rightarrow \mathbb{C}$  be a complex valued function. If  $f(z)$  and  $\overline{f(z)}$  are both analytic, then :

- (A)  $f(z)$  is a constant function  
 (B)  $f(z)$  is the identity function  
 (C)  $f(z)$  is unbounded  
 (D)  $f(z)$  is a non-constant entire function

Or

If, for a given  $\alpha$ ,  $0 \leq \alpha \leq 1$ , non-randomized Neyman-Pearson and likelihood ratio tests of a simple hypothesis against a simple alternative exist, then :

- (A) They are equivalent  
 (B) They are one and the same  
 (C) They are exactly opposite  
 (D) One can't say anything about it

25. Let

$$x = \frac{1}{2}(-1 + i\sqrt{3}), y = -\frac{1}{2}(1 + i\sqrt{3}).$$

Then :

- (A)  $x^2 + y^2 = 1$   
 (B)  $x^2 - y^2 = x - y$   
 (C)  $(xy)^2 = xy$   
 (D)  $x^3 = x, y^3 = y$

Or

Let  $y_1, y_2, y_3$  be three independent observations having expectations,  $E(y_1) = \beta_0 - \beta_1 + \beta_2$ ,  $E(y_2) = \beta_0 - 2\beta_2$ ,  $E(y_3) = \beta_0 + \beta_1 + \beta_2$  and  $V(y_i) = \sigma^2$  for  $i = 1, 2, 3$ . The least squares estimates of  $\beta_0, \beta_1$  and  $\beta_2$  are  $\hat{\beta}_0 = (y_1 + y_2 + y_3)/3$ ,  $\hat{\beta}_1 = (-y_1 + y_3)/2$ ,  $\hat{\beta}_2 = (y_1 - 2y_2 + y_3)/6$ . Which of the following statements is false ?

- (A) Variances of  $\hat{\beta}_i$  are unequal
- (B)  $\hat{\beta}_i$ 's are unbiased estimators of  $\beta_i$
- (C) It is possible to obtain unbiased estimator of  $\sigma^2$
- (D)  $\text{cov}(\hat{\beta}_i, \hat{\beta}_j) = 0$

26. If  $|z_1 + z_2| = |z_1| + |z_2|$  for  $z_1, z_2 \in \mathbb{C}$ , then :

- (A) One of  $z_1$  or  $z_2$  is positive multiple of the other
- (B)  $z_1$  and  $z_2$  have the same length
- (C)  $z_1$  is real and  $z_2$  is imaginary
- (D)  $z_1, z_2$  are real

Or

For the two way ANOVA model :

$$y_{ijk} = \mu + \alpha_i + \beta_j + \gamma_{ij} + e_{ijk}$$

$$i = 1, \dots, 4; j = 1, \dots, 5; k = 1, 2,$$

the error degrees of freedom are :

- (A) 20
- (B) 24
- (C) 12
- (D) 27

27. For  $z \in \mathbb{C}$  the inequality :

$$|z + 1| > |z - 1|$$

is :

- (A) always true
- (B) never true
- (C) true iff  $\text{Re } z > 0$
- (D) true iff  $\text{Im } z > 0$

Or

Let  $y_1, y_2$  be two independent observations having expectations  $E(y_1) = \theta_1 + \theta_2$ ,  $E(y_2) = \theta_1 - \theta_2$  and  $V(y_1) = V(y_2) = \sigma^2$ . Let  $\hat{\theta}_i$  be least squares estimate of  $\theta_i$ . Which of the following statements is *correct* ?

- (A)  $\hat{\theta}_1 = \frac{y_1 - y_2}{2}$  and  $V(\hat{\theta}_1) = \frac{\sigma^2}{2}$
- (B)  $\hat{\theta}_2 = \frac{y_1 + y_2}{2}$  and  $V(\hat{\theta}_2) = \frac{\sigma^2}{4}$
- (C)  $\hat{\theta}_1 = \frac{y_1 + y_2}{2}$  and  $V(\hat{\theta}_1) = \frac{\sigma^2}{2}$
- (D)  $\hat{\theta}_2 = \frac{(y_1 - y_2)}{2}$  and  $V(\hat{\theta}_2) = \frac{\sigma^2}{4}$

28. Consider the statements :

- (a) If a function is analytic in a bounded domain, then it is bounded.
- (b) If  $u(x, y)$  is harmonic in a domain  $D$ , then there exists a harmonic function  $v(x, y)$  such that  $u(x, y) + iv(x, y)$  is analytic.

Then :

- (A) both (a) and (b) are true
- (B) both (a) and (b) are false
- (C) only (a) is true
- (D) only (b) is true

Or

A block design is said to be connected if :

- (A)  $R(C) < v - 1$
- (B)  $R(C) = v$
- (C)  $R(C) < v - 2$
- (D)  $R(C) = v - 1$

29. The value of  $(i)^i - (-i)^{-i}$  is :

- (A) zero
- (B) non-zero real
- (C) purely imaginary
- (D) simply a chaos

*Or*

A RBD is :

- (A) Connected and balanced
- (B) Connected but not balanced
- (C) Not connected but balanced
- (D) Not connected not balanced

30. Define a function  $f : \mathbb{C} \rightarrow \mathbb{C}$  by  $f(z) = |z|$ . Then  $f(z)$  is :

- (A) continuous everywhere but not differentiable at the origin
- (B) continuous everywhere but differentiable only at the origin
- (C) continuous and differentiable everywhere
- (D) analytic at the origin

*Or*

Layout of a block design with 4 blocks and four treatments A, B, C, D is given below :

Block 1 : A, B, C,  $x$

Block 2 : A, B, D,  $y$

Block 3 : A, C, D,  $w$

Block 4 : B, C, D,  $z$

Identify  $x, y, w$  and  $z$  so that given design is RBD.

- (A)  $x = D, y = C, w = B, z = A$
- (B)  $x = B, y = A, w = B, z = A$
- (C)  $x = B, y = C, w = B, z = A$
- (D)  $x = D, y = B, w = B, z = A$

31. Which of the following ring is isomorphic to the field of complex numbers ?

- (A)  $\mathbf{R}[x]$
- (B)  $\mathbb{C}[\sqrt{5}i]$
- (C)  $\mathbb{Q}[i]$
- (D)  $\mathbb{C}[x]$

*Or*

For any two events A and B, which of the following are always true ?

- (A)  $P(A \text{ or } B) = P(A) + P(B)$
  - (B)  $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$
  - (C)  $P(A \text{ or } B) = P(A) \cdot P(B)$
  - (D)  $P(A \text{ and } B) = P(A) \cdot P(B)$
32. Let A, B be ideals of a ring R :
- (A)  $AB = A \cap B$
  - (B)  $AB \subseteq A \cap B$
  - (C)  $AB \subset A \cap B$
  - (D)  $AB = A \cap B$  if  $A = B$

*Or*

Given that we have collected pairs of observations on two variables X and Y, we would consider fitting a straight line with X as an explanatory variable if :

- (A) the change in Y is an additive constant
- (B) the change in Y is a constant for each unit change in X
- (C) the change in Y is a fixed percent of Y
- (D) the change in Y is exponential

33. Let R, S be rings and  $R \otimes S$  be the direct product of R and S. Then which of the following is *not* true ?

- (A) If R and S are commutative then so is  $R \otimes S$
- (B) If R and S are non-commutative then so is  $R \otimes S$
- (C) If R and S are with identity then so is  $R \otimes S$
- (D) If R and S are integral domains then so is  $R \otimes S$

*Or*

The least squares regression line is the line :

- (A) for which the sum of the residuals about the line is zero
- (B) which has the largest sum of the squared residuals of any line through the data values
- (C) which is determined by use of a function of the distance between the observed Y's and the predicted Y's
- (D) which has the smallest sum of the squared residuals of any line through the data values

34. Let  $G$  be a group,  $a \in G$  a fixed element and  $f: G \rightarrow G$  be a mapping given by  $f(x) = axa^{-1}$ . Which of the following is *not* true ?

- (A)  $f$  is an automorphism
- (B)  $f$  is not a homomorphism
- (C)  $f$  is onto
- (D)  $f$  is one-to-one

*Or*

In a single-factor ANOVA problem involving five treatments, with a random sample of four observations from each one, it is found that :

$$SST_r = 16.1408 \text{ and } SSE = 37.3801.$$

Then the value of test statistic is :

- (A) 1.522
- (B) 1.619
- (C) 2.316
- (D) 0.432

35. Let  $I$  be an ideal of a ring  $R$ , such that  $\frac{R}{I} \cong \mathbf{Z}_2$ , Then which of the following is *true* ?

- (A) Both  $R$  and  $I$  are finite
- (B)  $R$  is finite iff  $I$  is finite
- (C) Both  $R$  and  $I$  are infinite
- (D) If  $R$  is infinite, then  $I$  is finite

*Or*

A survey of college students taking the professional exam to be certified as public school teacher shows that 15 percent fail. On a national exam day, 12,000 students take the test. Let  $X$  denote the number who fail.

The mean value of  $X$  is :

- (A) 12,000
- (B) 1,800
- (C) 1,500
- (D) 10,200

36. Let  $A, B$  be ideals of a ring  $R$ .  
Suppose that  $A + B = A \cup B$ . Then  
which of the following is *true* ?

- (A)  $A + B = A$
- (B)  $AB = A \cap B$
- (C)  $AB = A \cup B$
- (D) Either  $A \subseteq B$  or  $B \subseteq A$

*Or*

Which of the following is *not*  
example of a variable ?

- (A) Gender of a high school graduate
- (B) Number of major credit cards a person has
- (C) Type of automobile transmission
- (D) Capital city of a country

37. Let  $v$  and  $w$  be eigen-vectors of  $T$  corresponding to two distinct eigen-values  $\lambda_1$  and  $\lambda_2$  respectively.  
Then :

- (A) for non-zero scalars  $\alpha_1, \alpha_2$ , the vector  $\alpha_1 v + \alpha_2 w$  is not an eigen-vector of  $T$
- (B) for all scalars  $\alpha_1, \alpha_2$ , the vector  $\alpha_1 v + \alpha_2 w$  is not an eigen-vector of  $T$
- (C)  $\alpha_1 v + \alpha_2 w$  is an eigen-vector of  $T$  if  $\alpha_1 = \alpha_2$
- (D)  $\alpha_1 v + \alpha_2 w$  is an eigen-vector of  $T$  if  $\alpha_1 = -\alpha_2$

*Or*

The initial solution of a Transportation problem can be obtained by applying any known method. However, the only condition is that :

- (A) The rim conditions are satisfied
- (B) The solution must be optimum
- (C) The solution should be non-degenerate
- (D) All of the above

38. Let S and T be two diagonalizable linear operator that are similar. Then :

- (A) Eigen-values of S and T are conjugates
- (B) S and T have same characteristic polynomial
- (C) Eigen-vectors of S and T are conjugates
- (D) S and T are triangulable

Or

You are given a two server queueing system, in a steady-state condition where the number of customers in the system varies between 0 to 4. The probability that there are exactly  $n$  customers in the system is  $P_n$  and the values of  $P_n$  are  $P_0 = \frac{1}{16} = P_4$ ;  $P_1 = \frac{4}{16} = P_3$  and  $P_2 = \frac{6}{16}$  then Expected number of customers in the queue  $L_q$  is :

- (A) 1
- (B)  $\frac{3}{8}$
- (C)  $\frac{3}{4}$
- (D) 2

39. Let W be a subspace of a finite dimensional vector space V. Then :

- (A) Linearly independent subset of W need not be Linearly independent in V
- (B) W need not be finite dimensional
- (C)  $\dim W = \dim W^0$ , where  $W^0$  is the annihilator of W
- (D)  $\dim V = \dim W + \dim W^0$ , where  $W^0$  is the annihilator of W

Or

The problem of replacement in the replacement theory is *not* concerned about the :

- (A) items that deteriorate graphically
- (B) items that fail suddenly
- (C) determination of optimum replacement interval
- (D) maintenance of an item to work out profitability

40. Let  $N$  be a nilpotent matrix :

- (A)  $I-N$  is invertible
- (B) Eigen-values of  $N$  are non-zero
- (C)  $N$  is diagonalizable
- (D) Eigen-vectors are orthogonal

*Or*

A sequencing problem involving six jobs and three machines requires evaluation of :

- (A)  $(6!)^3$  sequences
- (B)  $(6! + 6! + 6!)$  sequences
- (C)  $(6 \times 6 \times 6)$  sequences
- (D)  $(6 + 6 + 6)$  sequences

41. Let  $D$  be the derivative  $d/dt$ ,  $f(t) = 3\sin t + 5\cos t$ . The co-ordinates of  $Df(t)$  with respect to the basis  $\{\sin t, \cos t\}$  are :

- (A)  $(-5, 3)$
- (B)  $(5, 3)$
- (C)  $(3, 5)$
- (D)  $(3, -5)$

*Or*

Probability sampling is the procedure that gives all units :

- (A) an equal, calculable and non-zero chance to be selected
- (B) a chance to be included in the study
- (C) an equal chance to be selected
- (D) an equal chance to be selected or not to be selected

42. Let  $T$  be a linear operator on a finite dimensional vector space  $V$  and let  $W$  be a subspace of  $V$ . Then :

- (A)  $\dim W = \dim V = \dim V/W$
- (B) every basis of  $V/W$  can be constituted with only elements of  $W$
- (C)  $\dim V \geq \dim W + \dim V/W$

(D)  $\frac{W}{W \cap \ker T} \cong T(W)$

Or

In stratified sampling, the strata :

- (A) are equal in size to each other
- (B) are proportionate to the units in the target population
- (C) are disproportionate to the units in the target population
- (D) can be proportionate or disproportionate to the units in the target population

43. Let  $V$  be an inner product space over  $F$  and  $u, v \in V$ , then which one of the following is *false* ?

- (A)  $\|u + v\|^2 + \|u - v\|^2 = 2\|u\|^2 + 2\|v\|^2$
- (B)  $|\langle u, v \rangle| \leq \|u\| \cdot \|v\|$
- (C)  $\|u\| - \|v\| \leq \|u - v\|$
- (D)  $\|u - v\| \geq \|u\| + \|v\|$

Or

A researcher chose the respondents of his study by interviewing a few available couples and by obtaining names of new couples from the previous respondents. This procedure is called :

- (A) Systematic Sampling
- (B) Convenient Sampling
- (C) Quota Sampling
- (D) Snowball Sampling

44. If the number of constants to be eliminated from the given relation is greater than the number of independent variables, then the equation obtained is a :

- (A) linear equation of first order
- (B) non-linear equation of second order
- (C) second order partial differential equation
- (D) non-linear partial differential equation of first order

Or

A wholesale distributor has found that the amount of a customer's order is a normal random variable with a mean of Rs. 2,000 and a standard deviation of Rs. 500. What is the probability that the total amount in a random sample of 20 orders is greater than Rs. 45,000 ?

- (A) 0.1915
- (B) 0.3085
- (C) 0.0125
- (D) 0.0228

45. The order of the differential equation of the family of all ellipses is :

- (A) 1
- (B) 2
- (C) 3
- (D) 4

Or

The sampling distribution of the mean refers to :

- (A) the distribution of various sample sizes which might be used in a given study
- (B) the distribution of different possible values of the sample mean together with their respective probabilities of occurrence
- (C) the distribution of the values of the items in the population
- (D) the distribution of the values of the items actually selected in a given sample

46. Let  $W(\phi_1, \phi_2)(x)$  be the Wronskian of two linearly independent solution  $\phi_1, \phi_2$  of constant coefficient equation  $L(y) = 0$  on  $I$ . Then :

- (A)  $W(\phi_1, \phi_2)(x) = 0 \Rightarrow W(\phi_1, \phi_2)(x_0) = 0$  for some  $x_0 \in I$
- (B)  $W(\phi_1, \phi_2)(x_0) = 0 \Rightarrow W(\phi_1, \phi_2)(x) = 0 \forall x \in I$
- (C)  $W(\phi_1, \phi_2)(x) = 0 \Leftrightarrow W(\phi_1, \phi_2)(x_0) = 0$
- (D) (A), (B) and (C) are not true

*Or*

Principal component analysis will *not* be affected if :

- (A) variables are standardized
  - (B) variables are studentized
  - (C) there is a change of origin but not a change of scale
  - (D) variables are subjected to a linear transformation
47. The solution of the differential equation  $y'' + 4y' + 3y = 0$  is :
- (A) continuous and infinitely differentiable
  - (B) continuous and unbounded
  - (C) continuous but not differentiable
  - (D) unbounded but infinitely differentiable

*Or*

Hotelling  $T^2$  test can be used for testing :

- (A) a general linear hypothesis in the mean vector
- (B) an arbitrary hypothesis about the mean vector
- (C) simultaneous hypotheses about the mean vectors
- (D) two-sided hypothesis in the mean vector

48. Let  $y = x^2 \sin x$  be a solution of a homogeneous initial value problem. Then the least possible order of the differential equation is :

- (A) 3
- (B) 4
- (C) 5
- (D) 6

*Or*

Which of the following statements is *not* true about a Poisson Probability Distribution with parameter  $\lambda$  ?

- (A) The mean of the distribution is  $\lambda$
- (B) The standard deviation of the distribution is the positive square root of  $\lambda$
- (C) The parameter  $\lambda$  must be greater than zero
- (D) The parameter  $\lambda$  is coefficient of variation

49. Let  $f(x, y, z, p, q) = 0$  be a first order partial differential equation. Then the relation between the variables involving as many arbitrary constants as there are independent variables is called :

- (A) general integral
- (B) singular integral
- (C) complete integral
- (D) particular solution

Or

Suppose the sequences  $\{X_n\}$  and  $\{Y_n\}$  of r.v.s are such that  $X_n \xrightarrow{d} X$ ,  $Y_n \xrightarrow{P} Y$  as  $n \rightarrow \infty$  and suppose  $P[Y = 3] = 1$ . Which of the following is correct ?

- (A)  $X_n + Y_n \xrightarrow{P} X + 3$  as  $n \rightarrow \infty$
- (B)  $X_n Y_n \xrightarrow{P} 3X$  as  $n \rightarrow \infty$
- (C)  $Y_n \cos(X_n) \xrightarrow{d} 3\cos(X)$  as  $n \rightarrow \infty$
- (D)  $Y_n X_n^2 \xrightarrow{d} 3X^2$  as  $n \rightarrow \infty$

50. For the differential equation :

$$(1 - x^2) y'' - 2xy' + y = 0$$

- (A)  $x = 1$  is a regular singular point and  $x = -1$  is irregular singular point
- (B)  $x = 1$  and  $x = -1$  are regular singular points
- (C)  $x = -1$  is a regular singular point and  $x = 1$  is irregular singular point
- (D)  $x = 1$  and  $x = -1$  are ordinary points

Or

The mean and variance of 25 is :

- (A) Mean = 25, Variance = 1
- (B) Mean = 25, Variance = 0
- (C) Mean = 25, Variance =  $\frac{1}{25}$
- (D) Mean = 25 and Variance cannot be found

**ROUGH WORK**

**NOV - 30211/II**

**ROUGH WORK**

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Test Booklet No.

प्रश्नपत्रिका क्र.

**F**

Signature of Invigilators

1. ....

2. ....

Seat No.

**MATHEMATICAL SCIENCES** (In figures as in Admit Card)

**Paper II**

Seat No. (In words) .....

**NOV- 30211**

Answer Sheet No.

**Time Allowed : 75 Minutes]**

**[Maximum Marks : 100**

Number of Pages in this Booklet : 24

**Instructions for the Candidates**

1. Write your Seat Number in the space provided on the top of this page. Write your Answer Sheet No. in the space provided for Answer Sheet No. on the top of this page.
2. Write and darken Test Booklet No. on OMR Answer Sheet.
3. This paper consists of **Fifty (50)** multiple choice type of questions.
4. Each item has four alternative responses marked (A), (B), (C) and (D). You have to darken the responses as indicated below on the correct response against each item.  
**Example :** (A) (B) (C) (D)  
Where (C) is the correct response.
5. Your responses to the items for this paper are to be indicated on the Answer Sheet only. Responses like (x) (□) (/) and **light shaded responses will not be considered/evaluated.**
6. Read instructions given inside carefully.
7. One Sheet is attached at the end of the booklet for rough work.
8. You should return the test booklet and answer sheet **both** to the invigilator at the end of the paper and should not carry any paper with you outside the examination hall.
9. Answers marked on the body of the question paper will not be evaluated.

**परीक्षार्थीसाठी सूचना**

1. या पानावरील वरच्या कोपऱ्यात आपला आसन क्रमांक तसेच आपणास दिलेल्या उत्तरपत्रिकेचा क्रमांक त्याखाली लिहावा.
2. प्रश्नपत्रिका क्रमांक OMR उत्तरपत्रिकेवर दिलेल्या रकान्यात लिहून त्याप्रमाणे काळा करावा.
3. या प्रश्नपत्रिकेत **पन्नास** बहुनिवड प्रश्न आहेत.
4. प्रत्येक प्रश्नासाठी (A), (B), (C) आणि (D) अशी चार विकल्प उत्तरे दिली आहेत. त्यातील योग्य उत्तराचा रकाना खाली दर्शविल्याप्रमाणे ठळकपणे काळा करावा.  
**उदा.** (A) (B) (C) (D)  
जर (C) हे योग्य उत्तर असेल तर.
5. या प्रश्नपत्रिकेतील प्रश्नांची उत्तरे उत्तरपत्रिकेमध्येच द्यावीत. उत्तराच्या रकान्यामध्ये (x) (□) (/) व **अस्पष्टपणे काळे केलेले उत्तर ग्राह्य धरले जाणार नाही.**
6. आत दिलेल्या सूचना काळजीपूर्वक वाचाव्यात.
7. कच्च्या कामासाठी प्रश्नपत्रिकेच्या शेवटी कोरे पान जोडले आहे.
8. या पेपरची परीक्षा संपल्यानंतर प्रश्नपत्रिका व उत्तरपत्रिका **दोन्ही** पर्यवेक्षकांना परत करावी. यातील कोणताही कागद तुमच्या बरोबर परीक्षा केंद्राबाहेर नेण्यास सक्त मनाई आहे.
9. प्रश्नपत्रिकेवर दर्शविलेली उत्तरे तपासली जाणार नाहीत.