Actuarial Society of India

EXAMINATIONS

24th May 2006

Subject ST6 – Finance and Investment B

Time allowed: Three Hours (2.15* - 5.30 pm)

INSTRUCTIONS TO THE CANDIDATE

1. ***You have 15 minutes at the start of the examination in which to read the questions. You are strongly encouraged to use this time for reading only but notes may be made. You then have three hours to complete the paper.***
2. ***You must not start writing your answers until instructed to do so by the supervisor.***
3. ***The answers are not expected to be any country or jurisdiction specific. However, if examples/illustrations are required for any answer, the country or jurisdiction from which they are drawn should be mentioned.***
4. ***Mark allocations are shown in brackets.***
5. ***Attempt all questions, beginning your answer to each question on a separate sheet.***
6. ***Fasten your answer sheets together in numerical order of questions. This, you may complete immediately after expiry of the examination time.***

Professional Conduct:

"It is brought to your notice that in accordance with provisions contained in the Professional Conduct Standards, If any candidate is found copying or involved in any other form of malpractice, during or in connection with the examination, Disciplinary action will be taken against the candidate which may include expulsion or suspension from the membership of ASI."

Candidates are advised that a reasonable standard of handwriting legibility is expected by the examiners and that candidates may be penalized if undue effort is required by the examiners to interpret scripts.

AT THE END OF THE EXAMINATION
Hand in BOTH your answer script and this question paper to the supervisor.

1. a) Briefly discuss the role of arbitrageurs in the derivatives market (4)

   b) Discuss the following statements:

   (i) It is only possible to create arbitrages if the markets allow short selling. (3)

   (ii) Government bonds provide a risk free rate of return. (4)

[11]

2. a) Brownian motion is the only process with stationary, independent increments and continuous sample paths. Briefly explain the three underlined terms. (3)

   b) Let W_t be standard Brownian motion. Let X_t be an Ito process defined by the stochastic differential equation:

$$dX_t = 2\,dt - dW_t$$

   i) Define $R_t = \dfrac{1}{X_t}$

   Find the stochastic differential equation for the process R_t. Express the coefficients in your answer in terms of R_t. (2)

   ii) Determine the range of values of R_t for which the process R_t has positive drift. (1)

   iii) Briefly explain the term "mean-reverting". Is the process R_t, a mean reverting process? Why or why not? (2)

   iv) Assume that the process R_t will be used to model a real world quantity. State one undesirable property of the process R_t that might make this on inappropriate model. (2)

[10]

3. a) Suppose you are required to evaluate an expression of the form $E_Q[X_t | F_s]$ where $s < t$, by first changing the measure from Q to P.

   Briefly explain how you would do this.

   Provide all the relevant formulae in your answer; and define all the symbols you use. (4)

   b) A random variable X has a N (μ, σ^2) distribution under the probability measure P and a N (μ+r, σ^2) distribution under Q.

   (i) Prove that:

$$\frac{dP}{dQ} = \exp\left[-\frac{r}{s^2}(x - m - \frac{1}{2}r)\right]$$

(3)

(ii) By applying a change of measure from P to Q and by choosing an appropriate value for 'r' prove that:

$$E_p\,[\exp(k\,X) \bullet I\,(a < X < b)]$$

$$= \exp\left[km + \frac{1}{2}k^2 s^2\right] \bullet$$

$$\left[\Phi\left\{\frac{b-m}{s} - ks\right\} - \Phi\left\{\frac{a-m}{s} - ks\right\}\right]$$

where Φ denotes cumulative distribution function of the standard normal distribution and "•" denotes multiplicative sign. (7)

[14]

4. a) The current price of a European call option is Rs.0.32 and the risk free rate is 5% pa. Suppose that a 0.5% increase in the risk free rate causes the option price to jump instantly to Rs.0.33 (all else remaining the same). Estimate the rho of the option. (2)

b) Is theta positive or negative for a European call option? (1)

c) The current price of a share is 100. Match the following options on this share with their correct deltas.

i)	Call option (strike price 50)	-0.30
ii)	Put option (strike price 50)	+0.21
iii)	Put option (strike price 55)	-0.79
iv)	Put option (strike price 120)	+0.41
v)	Call option (strike price 95)	- 0.70

(5)

d) Explain why the value of a call option is usually higher for a 6 month option than for a 3 month option. (1)

e) Explain whether this would also be true for a put option. (1)

[10]

5. Consider a call option on a stock whose current price is Rs.100. The exercise price is Rs.110, the continuously compounded risk free rate of interest is 3% per annum, the volatility is 35% per annum and the time to maturity 2 months. Using the Black-Scholes model, calculate the price of the option if:

(a) there are no dividends over the next two months

(b) there is a dividend of Rs.13 to be paid in one month's time.

[5]

6. An exotic derivative on a non-dividend paying share has a payoff function of $(S_T - 500)^2$, where T denotes the share price at the expiry time T, which is one year from now. The market price of the share at the current time t is $S_t = 480$.

The annual risk-free force of interest r is such that $e^r = 1.05$.

It is required to estimate the Greeks Kappa, lambda and rho, using a one step binomial free with share price ratios $u = \frac{1}{d} = 1.25$

(i) (a) Construct the tree based on the current parameter values and hence show that the fair price of the derivative is 10,987.
(2)

(b) Explain why it would not be advisable to use this tree to estimate the derivative's delta.
(3)

(ii) (a) Use a perturbation approach to estimate the value of rho, by recalculating the derivative price assuming that $e^r = 1.06$. Quote your answer to two significant figures. (2)

(b) Use a similar approach, modifying your tree as necessary, to estimate the value of kappa, given that the price ratio u is calculated from the volatility using the formula $u = e^s$. (2)

(c) Estimate the value of lambda, assuming that a dividend rate q would reduce the final share price S_T by a factor of e^{-q}. (2)

(iii) Estimate the new price of the derivative if the risk-free force of interest immediately increases by 0.25%, the volatility increase by 1% and the share price drops by 2. Assume that the derivative has a delta of 5,500.
(2)

(iv) State three problems that a derivatives fund manager might experience with this type of derivative. (2)
[15]

7. Suppose that zero interest rates with continuous compounding are as follows:

Maturity (Years)	Rate (% Per Annum)
0.5	8
1	7.5
1.5	7.2
2	7.0

For bond A that lasts for 30 months and pays a coupon of 8% per annum semiannually, the yield is 7 % per annum (with continuous compounding). For bond B that lasts for 3 years and pays coupon of 10% per annum semiannually, the yield is 6.9 % per annum (with continuous compounding). Assume the face value of each bond as Rs.1,000.

a. What are the prices of bonds A and B? (2)

b. Calculate zero rates (with continuous compounding) for maturities of 30 months and 36 months. (2)

c. What are the 6-month, 12-month, 18-month, 24-month, 30-month and 36-month par yields for bonds that provide semi-annual coupon payments? (3)

d. What are the forward rates (with continuous compounding) for the periods: 6 months to 12 months, 12 months to 18 months, 18 months to 24 months, 24 months to 30 months, 30 months to 36 months? (3)

[10]

8 (a) Companies A and B have been offered the following rates per annum on a $50 million five-year loan:

	Fixed Rate (Per Annum)	Floating Rate (Per Annum)
Company A	11.4%	LIBOR - 0.1%
Company B	12.8%	LIBOR + 0.4%

Company A requires a floating rate loan; Company B requires a fixed rate loan. Design a swap that will net a bank, acting as intermediary, 0.1% and that will appear equally attractive to both companies. Show full working. (3)

(b) A $200 million interest swap has a remaining life of 16 months. Under the terms of the swap, six-month LIBOR is exchanged for 10% per annum (compounded semiannually). The average of the bid-offer rate being exchanged for six-month LIBOR in swaps of all maturities is currently 9% per annum with continuous compounding. The six-month LIBOR rate was 8.6% per annum two months ago. What is the current value of the swap to the party paying floating? What is its value to the party paying fixed?

(3)

(c) The party with the short position in treasury bond futures has decided to deliver and is trying to choose among the four bonds in table below:

Bond	Quoted Price	Conversion Factor
1	124.25	1.2232
2	139.50	1.3992
3	114.00	1.1250
4	143.00	1.4127

Suppose that the current quoted futures price is Rs.94.25. Which of the above four bonds is cheapest to deliver? The quoted price is for a bond with a face value of Rs.100.

(3)

[9]

9 (a) Consider a 16-month European put option on a 10.75 year bond with a face value of Rs.1,000. Suppose that the current cash bond price is Rs.980, the strike price is Rs.1,000, the 16-month risk-free interest rate is 8% per annum (with continuous compounding), and the volatility of the forward bond price in 16 months is 7% per annum. The bond pays a semi-annual coupon of 8% per annum. The three-month, nine-month and fifteen-month risk-free interest rates are 7%, 7.5% and 7.8% (all with continuous compounding) respectively. What is the value of the European put option as per Black's model? (3)

(b) Consider a contract that caps the interest rate on a Rs. 1 million loan at 9% per annum (with quarterly compounding) for three months starting in fifteen months. This is caplet and could be one element of a cap. Suppose that the zero curve is flat at 8% per annum with quarterly compounding and the 15-month volatility for the three-month rate underlying the caplet is 18% per annum. What is the price of the caplet? (3)

[6]

10 (a) Suppose that á = 0.1 and ì = 0.1 in both the Vasicek and the Cox-Ingersoll-Ross model. In both models, the initial short rate is 10% and the initial standard deviation of the short rate change in a short time ät is 0.02 ät. Compare the prices given by the two models for a zero coupon bond with a principal of Rs. 100 that matures in 10 years. (4)

(b) Define market risk. How is market risk measured? How large could the market risk be for the writer of a call option on a share? (3)

(c) Define credit risk. (1)

(d) Explain the following statement:
"The purchaser of the option has credit exposure to the counter party where as option writer does not have credit exposure" (2)

[10]

****