INSTITUTE OF MATHEMATICS & APPLICATIONS, BHUBANESWAR
ENTRANCE TEST-2010

B.Sc.(Honours): Mathematics & Computing

Maximum Marks :100 **Time Alloted: 2 Hr.**

(Multiple choice questions)

All questions are compulsory. Each question has 4 choices $(A), (B), (C), (D)$, out of which *ONLY ONE* is correct. Choose the correct answer. Each question carries $+4$ marks for correct answer and -1 mark for wrong answer.

1. Which one of the following is a function from $X = \{1, 2, 3, 4\}$ to itself ?

   (A) $f_1 = \{(x, y) : y - x = 1\}$ (B) $f_2 = \{(x, y) : y + x > 4\}$

   (C) $f_3 = \{(x, y) : y - x < 0\}$ (D) $f_4 = \{(x, y) : y + x = 5\}$

2. The image of the interval $[-1, 3]$ under the mapping $f(x) = 4x^3 - 12x$ is

   (A) $[-2, 0]$ (B) $[-8, 72]$

   (C) $[-8, 0]$ (D) none of these.

3. If S_1, S_2, S_3 are the sum of $n, 2n, 3n$ terms respectively of an A.P., then

   (A) $S_3 = 2(S_1 + S_2)$ (B) $S_3 = S_1 + S_2$

   (C) $S_3 = 3(S_2 - S_1)$ (D) $S_3 = 3(S_2 + S_1)$

4. The coefficients of $\lambda^n \mu^n$ in the expansion of $[(1 + \lambda)(1 + \mu)(\lambda + \mu)]^n$ is

   (A) $\sum_{k=0}^{n} C_k^2$ (B) $\sum_{k=0}^{n} C_{k+2}^2$

   (C) $\sum_{k=0}^{n} C_{k+3}^2$ (D) $\sum_{k=0}^{n} C_k^3$

5. The number of ways in which 13 gold coins can be distributed among three persons such that each one gets at least two gold coins is

   (A) 36 (B) 24

   (C) 12 (D) 6.

6. The total number of non-differentiability points of the function $f(x) = \min\left\{|\sin x|, |\cos x|, \dfrac{1}{4}\right\}$ in $(0, 2\pi)$ is

   (A) 7 (B) 10

   (C) 11 (D) 13.

7. If $\begin{vmatrix} x & 3 & 6 \\ 3 & 6 & x \\ 6 & x & 3 \end{vmatrix} = \begin{vmatrix} 2 & x & 7 \\ x & 7 & 2 \\ 7 & 2 & x \end{vmatrix} = \begin{vmatrix} 4 & 5 & x \\ 5 & x & 4 \\ x & 4 & 5 \end{vmatrix} = 0$, then x is equal to

   (A) 9 (B) -9

   (C) 0 (D) none of these.

8. The equation of the tangent to the hyperabola $3x^2 - y^2 = 3$ parallel to the line $y = 2x + 4$ is

(A) $y = 2x + 3$ (B) $y = 2x + 1$

(C) $y = 2x + 2$ (D) $y = 3x + 1$.

9. If $|z - 1 - i| = 1$, then the locus of points represented by the complex number $5(z - i) - 7$ is a circle with

(A) centre $(1, 0)$ (B) radius 2 units

(C) radius 5 units (D) centre $(2, 0)$.

10. $\int (2x^2 + 3x + 6)^{10}(x^{12} + x^{11} + x^{10})dx$ is equal to

(A) $\dfrac{x^{12} + x^{11} + x^{10}}{66}$ + constant (B) $\dfrac{x^6(2x^2 + 3x + 6)^{11}}{66}$ + constant

(C) $\dfrac{x^{11}(2x^2 + 3x + 6)^{11}}{66}$ + constant (D) $\dfrac{(2x^2 + 3x + 6)^{11}}{66}$ + constant

11. Let x, y, z be three real and distinct numbers satisfying the equation $8(4x^2 + y^2) + 2z^2 - 4(4xy + yz + 2xz) = 0$. Then

(A) $\dfrac{x}{y} = \dfrac{1}{2}$ (B) $\dfrac{y}{z} = \dfrac{1}{4}$

(C) $\dfrac{x}{y} = \dfrac{1}{3}$ (D) x, y, z are in G.P.

12. The function $f : [0, \infty) \longrightarrow [0, \infty)$ defined by $f(x) = \dfrac{x}{1 + x}$ is

(A) one-one and onto (B) one-one, but not onto

(C) not one-one, but onto (D) neither one-one nor onto

13. Which one of the following numbers is rational ?

(A) $\sin 15°$ (B) $\cos 15°$

(C) $\sin 15° \cos 15°$ (D) $\sin 15° \cos 75°$

14. The smallest positive root of the equation $\tan x = x$ lies in the interval

(A) $\left(0, \dfrac{\pi}{2}\right)$ (B) $\left(\dfrac{\pi}{2}, \pi\right)$

(C) $\left(\pi, \dfrac{3\pi}{2}\right)$ (D) $\left(\dfrac{3\pi}{2}, 2\pi\right)$

15. The number of real solutions of the equation $|x|^2 - 3|x| + 2 = 0$ is

(A) 1 (B) 2

(C) 3 (D) 4

16. The coefficient of x^{99} in the expansion of $(x-1)(x-2)\cdots(x-100)$ is

(A) -5150 (B) -5050

(C) -5051 (D) -5151

17. The area enclosed by the curves $y = 1 + |x|$ and $y = 1 - |x|$ is

(A) 1 sq. unit (B) 2 sq. units

(C) $2\sqrt{2}$ units (D) 4 sq. units

18. If $\lim_{n\to\infty} \dfrac{1}{1 + \left(\dfrac{x^2+a}{x^2+1}\right)^n} = 1$, then the value of a lies in the interval

(A) $(-1, 1)$ (B) $(0, 1)$

(C) $(-1/2, 1)$ (D) $(-1, 0)$

19. a and b are two solutions of the equation $e^x \cos x - 1 = 0$. The minimum number of solution(s) of the equation $e^x \sin x - 1 = 0$ lying between a and b is

(A) 0 (B) 1

(C) 3 (D) none of these

20. If the parabola $y^2 = 4ax$ and the circle $x^2 + y^2 + 2bx = 0$ have more than two common tangents, then ab may be equal to $(ab \neq 0)$

(A) $\dfrac{-5}{2}$ (B) -3

(C) 2 (D) none of these

21. A point on the curve $x^2 + 2y^2 = 6$ whose distance from the line $y = 7 - x$ is minimum is

(A) $(\sqrt{6}, 0)$ (B) $(0, \sqrt{3})$

(C) $(2, 1)$ (D) $(\sqrt{2}, \sqrt{2})$

22. The function $f(x) = (\cos x)^{\frac{1}{x}}$ is not defined at $x = 0$. The value which should be assigned to f at $x = 0$, so that f is continuous at $x = 0$, is

(A) 0 (B) -1

(C) 1 (D) e

23. If $f(x) = \min\{1, x^2, x^3\}$, then

(A) f is continuous & differentiable everywhere.
(B) f is continuous everywhere but not differentiable at two points.
(C) f is continuous everywhere but not differentiable at one point.
(D) none of these.

24. If $u = f(\tan x), v = g(\sec x), f'(x) = \tan^{-1} x$, and $g'(x) = \text{cosec}^{-1}x$, then the value of $\dfrac{du}{dv}$ at $\dfrac{\pi}{4}$ is

(A) 1 (B) $\sqrt{2}$

(C) 2 (D) $\dfrac{1}{\sqrt{2}}$

25. Consider the following statements:

S_1: Both $\sin x$ and $\cos x$ are decreasing functions in the interval $\left(\dfrac{\pi}{2}, \pi\right)$.

S_2: If a differentiable function decreases in the interval (a, b), then its derivative also decreases in (a, b).

Then which one of the following is true ?

(A) Both S_1 and S_2 are wrong.

(B) Both S_1 and S_2 are correct, but S_2 is not the correct explanation for S_1.

(C) S_1 is correct and S_2 is the correct explanation for S_1.

(D) S_1 is correct, but S_2 is wrong.