ADMISSION TEST-2009

B. Sc. (Hons.) in Mathematics and Computing
INSTITUTE OF MATHEMATICS AND APPLICATIONS
BHUBANESWAR

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FULL MARKS : 200 TIME : 2 Hours

Answer as many questions as you can. Circle the correct answer(s) in the answer book. (Do not guess as there is a penalty for wrong answer.)

1. Which of the following are correct ?

   (a) $A \subseteq A^c$, if and only if $A = \varnothing$.

   (b) $A^c \subseteq A$, if and only if $A = X$, where X is the universal set.

   (c) If $A \cup B = A \cup C$, then $B = C$.

   (d) $A = B$ is equivalent to $A \cup C = B \cup C$ and $A \cap C = B \cap C$.

2. For real numbers x and y, define a relation R by xRy, if and only if $x - y + \sqrt{2}$ is an irrational number. Then the relation R is

   (a) reflexive.

   (b) symmetric.

   (c) transitive.

   (d) an equivalence relation.

3. If $A = B = [-1, 1],\ C = [0, \infty),\ R_1 = \{(x, y) \in A \times B : x^2 + y^2 = 1\}$ and $R_2 = \{(x, y) \in A \times C : x^2 + y^2 = 1\}$, then

   (a) R_1 defines a function from A into B.

   (b) R_2 defines a function from A into C.

   (c) R_2 defines a function from A onto C.

   (d) R_2 defines a one-one function from A onto C.

4. The locus of the points z satisfying the condition $|z + i| + |z - i| = k$ is an ellipse, provided

   (a) $k \in (-2, 2)$.

   (b) $k \in (-2, 0) \cup (0, 2)$.

   (c) $k \in (0, 2)$.

   (d) $k \in (2, \infty)$.

5. If $\dfrac{(1+i)x - 2i}{3+i} + \dfrac{(2-3i)y + i}{3-i} = i$, then the values of x and y are given by

   (a) $x = -3,\ y = -1$.

   (b) $x = 3,\ y = -1$.

(c) $x = 3,\ y = 1$.

(d) $x = -1,\ y = 3$.

6. If z is a complex number, then the system of equations $|z + 1 - i| = \sqrt{2}$ and $|z| = 3$ has

(a) no solution.

(b) one solution.

(c) two solutions.

(d) none of these.

7. Two students while solving a quadratic equation in the variable x, one copied the constant term incorrectly and got the roots 3 and 2. The other copied the constant term and the coefficient of x^2 correctly and got the roots as -6 and 1, respectively. The correct roots of the equation are

(a) 3 and -2.

(b) -3 and 2.

(c) -6 and -1.

(d) 6 and -1.

8. If A is an $n \times n$ non-singular matrix, then $\text{adj}(\text{adj}(A)) =$

(a) $|A|^{n-1}A$.

(b) $|A|^{n-2} - A$.

(c) $|A|^{n-1} - A$.

(d) $|A|^{n-2}A$.

9. If $a,\ b,\ c$ are non-zero real numbers such that $\begin{vmatrix} bc & ca & ab \\ ca & ab & bc \\ ab & bc & ca \end{vmatrix} = 0$, then

(a) $\dfrac{1}{a} + \dfrac{1}{b\omega} + \dfrac{1}{c\omega^2} = 0$.

(b) $\dfrac{1}{a} + \dfrac{1}{b\omega^2} + \dfrac{1}{c\omega} = 0$.

(c) $\dfrac{1}{a\omega} + \dfrac{1}{b\omega^2} + \dfrac{1}{c} = 0$.

(d) All the above are true.

10. The system of equations: $x + y + z = 6,\ x + 2y + 3z = 10,\ x + 2y + mz = n$ have infinite number of solutions, if

(a) $m = 3$ and $n \in \mathbb{R}$.

(b) $m = 3$ and $n \neq 10$.

(c) $m = 3$ and $n = 10$.

(d) none of these.

11. If $x,\ y$ and z are positive real numbers such that $x + y + z = \alpha$, then

(a) $\dfrac{1}{x} + \dfrac{1}{y} + \dfrac{1}{z} \geq \dfrac{9}{\alpha}$.

(b) $(\alpha - x)(\alpha - y)(\alpha - z) \geq 8\,xyz$.

(c) $(\alpha - x)(\alpha - y)(\alpha - z) \leq \dfrac{8}{27}\alpha^3$.

(d) All the above are true.

12. If $a,\ b,\ c \in \mathbb{R}$ and $a+b+c=0$, then the quadratic equation: $4ax^2+3bx+2c=0$ has

    (a) one positive and one negative root.

    (b) imaginary roots.

    (c) real roots.

    (d) None of these.

13. If a function f satisfies the condition $f\left(x+\frac{1}{x}\right) = x^2 + \frac{1}{x^2}$ $(x \neq 0)$, then $f(x)$ equals

    (a) $x^2 - 2$ for all $x \in \mathbb{R}$.

    (b) $x^2 - 2$ for all $x \neq 0$.

    (c) $x^2 - 2$ for all x satisfying $|x| \geq 2$.

    (d) $x^2 - 2$ for all x satisfying $|x| < 2$.

14. Two non-zero distinct numbers $a,\ b$ are used as elements to make determinants of third order. The number of determinants whose value is zero for all $a,\ b$ is

    (a) 24.

    (b) 32.

    (c) $a+b$.

    (d) none of these.

15. If the sum of the coefficients in the expansion of $(\alpha\, x^2 - 2x + 1)^{37}$ is equal to the sum of the coefficients in the expansion of $(x - \alpha\, y)^{37}$, then α is equal to

    (a) 0.

    (b) 1.

    (c) may be any real number.

    (d) no such value exists.

16. $\lim_{x\to 0}\left(1^{\csc^2 x} + 2^{\csc^2 x} + 3^{\csc^2 x} + \cdots + n^{\csc^2 x}\right)^{\sin^2 x} =$

    (a) 0.

    (b) $\dfrac{n}{2}$.

    (c) n.

    (d) none of these.

17. If $f(x) = \begin{cases} \sin[x], & [x] \neq 0 \\ 0, & [x] = 0 \end{cases}$, where $[x]$ is the greatest integer $\leq x$, then $\lim_{x\to 0} f(x) =$

    (a) 0.

    (b) 1.

    (c) -1.

    (d) does not exist.

18. The values of α and β such that $\lim_{x\to 0} \dfrac{x(1+\alpha\,\cos x)-\beta\,\sin x}{x^3}=1$ are

(a) $\frac{5}{2}, \frac{3}{2}$.

(b) $\frac{5}{2}, -\frac{3}{2}$.

(c) $-\frac{5}{2}, -\frac{3}{2}$.

(d) $-\frac{5}{2}, \frac{3}{2}$.

19. If $f(x)=\sqrt{1-\sqrt{1-x^2}}$, then f is

(a) continuous on $[-1,1]$ and differentiable on $(-1,1)$.

(b) continuous on $[-1,1]$ and differentiable on $(-1,0)\cup(0,1)$.

(c) continuous and differentiable on $[-1,1]$.

(d) None of these.

20. If $f(x)=\begin{cases} x^2\sin\frac{1}{x}, & x\neq 0 \\ 0, & x=0 \end{cases}$, then

(a) f and f' are continuous at $x=0$.

(b) f is differentiable at $x=0$.

(c) f is differentiable at $x=0$ and f' is not continuous at $x=0$.

(d) (b) and (c) are true.

21. If $x+|y|=2y$, then y as a function of x is

(a) defined for all x.

(b) continuous at $x=0$.

(c) such that $\dfrac{dy}{dx}=\dfrac{1}{3}$ for $x<0$.

(d) such that (a), (b) and (c) are true.

22. On which of the following intervals is the function $f(x)=2x^2-\log|x|$ $(x\neq 0)$ increasing ?

(a) $\left(\frac{1}{2},\infty\right)$.

(b) $\left(-\infty,-\frac{1}{2}\right)\cup\left(\frac{1}{2},\infty\right)$.

(c) $\left(-\infty,-\frac{1}{2}\right)\cup(0,\infty)$.

(d) $\left(-\frac{1}{2},0\right)\cup\left(\frac{1}{2},\infty\right)$.

23. All points on the curve $y^2=4a\left(x+a\sin\frac{x}{a}\right)$ at which the tangents are parallel to the X-axis, lie

(a) on a circle.

(b) on a parabola.

(c) on a straight line.

(d) on an ellipse.

24. The value of $\tan 1^\circ \tan 2^\circ \tan 3^\circ \cdots \tan 89^\circ$ is

(a) 0.

(b) $\frac{1}{2}$.

(c) 1.

(d) -1.

25. The value of θ for which $\cos\theta + \sqrt{3}\sin\theta = 2$ is

(a) $\frac{\pi}{3}$.

(b) $\frac{2\pi}{3}$.

(c) $\frac{4\pi}{3}$.

(d) $\frac{5\pi}{3}$.

26. $2\,Tan^{-1}\left(\frac{1}{3}\right) + Tan^{-1}\left(\frac{1}{7}\right) =$

(a) $Tan^{-1}\left(\frac{49}{29}\right)$.

(b) $\frac{\pi}{2}$.

(c) $\frac{\pi}{4}$.

(d) 0.

27. The largest term in the sequence $a_k = \dfrac{k}{k^2+100}$ is

(a) a_5.

(b) a_7 or a_8.

(c) a_{10}.

(d) a_{99}.

28. The number of positive unequal integral solutions of the equation $x + y + z = 6$ is

(a) 3!.

(b) 4!.

(c) 5!.

(d) $2 \times 4!$.

29. The number of ways in which 6 red roses and 3 white roses can form a garland so that all the white roses come together is

(a) 2170 (b) 2165 (c) 2160 (d) 2155

30. A point is selected at random from the interior of a circle. The probability that the point is closer to the centre than the circumference of the circle is

(a) $\frac{1}{4}$.

(b) $\frac{1}{2}$.

(c) $\frac{3}{4}$.

(d) none of these.

31. For two events A and B, if $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{4}$ and $P(A \cup B) = \frac{1}{2}$, then $P\left(\frac{\bar{A}}{\bar{B}}\right)$ is

(a) $\frac{3}{4}$.

(b) $\frac{2}{3}$.

(c) $\frac{1}{6}$.

(d) $\frac{1}{8}$.

32. $\displaystyle\int \frac{\sin 2x}{\sin^4 x + \cos^4 x}\, dx =$

(a) $Tan^{-1}(\tan^2 x)$+constant.

(b) $Tan^{-1}(\cot^2 x)$+constant.

(c) $Cot^{-1}(\tan^2 x)$+constant.

(d) $Cot^{-1}(\cot^2 x)$+constant.

33. The value of the integral $\int_0^{3/2} [x^2]\, dx$ is

(a) $2 + \sqrt{2}$.

(b) $2 - \sqrt{2}$.

(c) $4 + 2\sqrt{2}$.

(d) $4 - 2\sqrt{2}$.

34. The area of the region bounded by the curve $|x| + |y| = 1$ is

(a) $\frac{1}{2}$ sq. unit.

(b) 1 sq. unit.

(c) $\frac{3}{2}$ sq. unit.

(d) 2 sq. unit.

35. The differential equation for all family of lines which are at a unit distance from the origin is

(a) $\left(y - x\frac{dy}{dx}\right)^2 = 1 - \left(\frac{dy}{dx}\right)^2$.

(b) $\left(y + x\frac{dy}{dx}\right)^2 = 1 + \left(\frac{dy}{dx}\right)^2$.

(c) $\left(y - x\frac{dy}{dx}\right)^2 = 1 + \left(\frac{dy}{dx}\right)^2$.

(d) $\left(y + x\frac{dy}{dx}\right)^2 = 1 - \left(\frac{dy}{dx}\right)^2$.

36. If the axes are rotated through an angle of $45°$ in clockwise direction, then the new equation of $x^2 - y^2 = a^2$ is

(a) $xy - a^2 = 0$.

(b) $xy - 2a^2 = 0$.

(c) $2xy - a^2 = 0$.

(d) $2xy + a^2 = 0$.

37. Consider the circles $x^2 + (y-1)^2 = 9$ and $(x-1)^2 + y^2 = 25$. They are such that

(a) these circles touch each other.

(b) one of the circle lies entirely inside the other.

(c) each of these circles lies outside the other.

(d) they intersect in two points.

38. A line is such that it is inclined to the Y-axis and Z-axis at $60°$, then at what angle is it inclined to the X-axis ?

(a) $45°$.

(b) $30°$.

(c) $75°$.

(d) $60°$.

39. The equation of the plane which passes through the points $(2,1,-1)$, $(-1,3,4)$ and perpendicular to the plane $x - 2y + 4z = 0$ is

(a) $18x + 17y + 4z = 49$.

(b) $18x - 17y + 4z = 49$.

(c) $18x + 17y - 4z = -49$.

(d) $18x + 17y + 4z = -49$.

40. If $(2,3,5)$ is one end of the diameter of the sphere $x^2 + y^2 + z^2 - 6x - 12y - 2z + 20 = 0$, then the co-ordinates of the other end of the diameter are

(a) $(4,3,5)$.

(b) $(4,3,-3)$.

(c) $(4,9,-3)$.

(d) $(3,9,-3)$.