

# Entrance Test -2007

B.Sc.(Mathematics and Computing )

Institute of Mathematics and Applications

Bhubaneswar

Full Marks :150

Time-11.30 A.M. - 1.30 P.M.

The questions are of equal value

Answer as many questions as you can

1. Examine whether the proposition  $p \implies (p \vee q)$  is a tautology .
2. If  $3 + 4i$  is a root of the quadratic equation  $x^2 + bx + c = 0$  , then find the values of  $b$  and  $c$  .
3. if  $A = \{x \in \mathbb{R} : x^2 - 3x + 2 > 0\}$  , then find  $A \cap B$  .
4. In the complex plane, find the set of points defined by the equation  $|z - i| = |z + i|$ .
5. Prove that  $101^{50} > 99^{50} + 100^{50}$  .
6. Find the value(s) of  $\lambda$  for which the following system of equations has a unique solution.

$$\lambda x + y + z = 1$$

$$x + \lambda y + z = 1$$

$$x + y + \lambda z = 1$$

7. If  $A = \begin{pmatrix} 3x & 0 \\ x & x \end{pmatrix}$  and  $A^{-1} = \begin{pmatrix} 1 & 0 \\ -1 & 3 \end{pmatrix}$ , then find the value of  $x$ .
8. Evaluate the following :
- (i)  $\lim_{x \rightarrow \infty} \left(\frac{x}{1+x}\right)^x$       (ii)  $\lim_{x \rightarrow \infty} \frac{\tan x - \sin x}{x^3}$
9. The function  $f(x) = \frac{\sqrt{1+x}-1}{x}$  is not defined at  $x = 0$ . What should be its value at  $x = 0$ , so that  $f$  becomes continuous?
10. Find  $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\cot x}}{\sqrt{\cot x} + \sqrt{\tan x}} dx$ .
11. Sketch the region bounded by the curve  $|x| + |y| = 1$  and find its area.
12. Find the number of common tangents to the circle  $x^2 + y^2 - x = 0$  and  $x^2 + y^2 + x = 0$ .
13. Find the image of the point  $(4, -13)$  with respect to the lines  $5x + y + 6 = 0$ .
14. In how many ways can a committee of 5 members be selected from 6 men and 5 ladies consisting of 3 men and 2 ladies? Justify your answer.
15. A bag contains 8 white and 6 red balls. Find the probability of drawing two balls of the same color.