

**Entrance Test for B.Sc. Mathematics and Computing
Institute of Mathematics and Applications,
Bhubaneswar**

Full mark: 150 Time: 3.00 pm to 5.00 pm

Answer as many questions as you can.

1. What is the negation of the following statement ?
"If it does not rain then there would be crop failure."
2. How many relations are there between the sets $\{a, b\}$ and $\{1, 2, 3\}$?
How many of them are functions ?

3. Sum the series

$$1^2 - 2^2 + 3^2 - 4^2 + \dots + 2005^2 - 2006^2$$

4. Show that

$$\frac{1}{2} < \frac{1}{1001} + \dots + \frac{1}{2000} < 1$$

5. If z and w are two complex numbers with $|z| < 1, |w| < 1$ show that

$$\left| \frac{z - w}{1 - z\bar{w}} \right| < 1$$

6. What does

$$x^2 + 2xy + y^2 - 1 = 0$$

represent ? Justify your answer.

7. Find the radius of the circle of intersection of the plane $x + y + z = 1$ and the sphere $x^2 + y^2 + z^2 = 1$

8. Evaluate the limits (with out using L-Hopital rule)

$$(i) \lim_{\theta \rightarrow 0} \frac{1 - \cos \theta}{\theta^2}$$

$$(ii) \lim_{a \rightarrow 0} \frac{a^x - 1}{x}$$

9. Draw the graph of the function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = |x - 1|$.

10. Evaluate in the integral

$$\int_0^{\pi/2} \frac{\cos \theta}{\cos \theta + \sin \theta} d\theta$$

11. Solve the system of equations

$$x + y + z + w = 0$$

$$2x + y + 2z - w = 3$$

$$x + 3y + 3z + w = 2$$

$$5x + 6y + 6z + 7w = -3$$

12. What is the next term in the sequence 1,2,3,5,8 ?

13. What is the negation of the statement

"for every $\epsilon > 0$ we can find an n_0 such that $|x_n| < \epsilon$ when $n > n_0$ "

14. A group of 15 children consisting of 9 boys and 6 girls are made to sit in a row. In how many ways can they be seated so that no two girls sit next to each other.

15. What is the integral part of the following ?

$$(\sqrt{5} + 2)^{2006} + (\sqrt{5} - 2)^{2006}$$