BACHELOR IN COMPUTER APPLICATIONS

Term-End Examination

June, 2007

CS-60 (S) : FOUNDATION COURSE IN MATHEMATICS IN COMPUTING

Time : 3 hours *Maximum Marks : 75*

Note : *Question No. 1 is* ***compulsory****. Attempt any* ***two*** *questions from Questions No. 2 to 5. Calculators are not allowed.*

1. (a) Evaluate the following determinant without expansion

$$\begin{vmatrix} 1 & 2 & 3 \\ 3 & 5 & 7 \\ 8 & 14 & 20 \end{vmatrix}$$

(b) Examine whether $f(x) = x \dfrac{a^x + 1}{a^x - 1}$ is even or odd.

(c) Is the function $f(x) = |x|$ differentiable at $x = 0$?

(d) Prove that $f(x) = \dfrac{\sin x}{x}$ is a decreasing function in the range $0 < x < \pi/2$.

(e) Fill in the blanks with reference to the polar equation $r = f(\theta)$ of a curve :

(i) If the equation remains unchanged when θ is replaced by ___________, then the curve is symmetric with respect to the initial line.

(ii) If the equation does not change when r is replaced by $-r$, then the curve is symmetric about the ___________ .

(iii) If the equation does not change when θ is replaced by $\pi - \theta$, then the curve is symmetric with respect to the line ___________ .

(f) Evaluate :

$$\int \frac{e^x(1+x)}{\sin^2(xe^x)}\,dx$$

(g) If the sets A and B are defined as

$A = \{2, 5\}$ and $B = \{2, 3\}$;

find $A \times B$, $B \times A$, $A \times A$.

(h) Prove that : $(b + c)(c + a)(a + b) > 8abc$ if $a > 0$, $b > 0$, $c > 0$.

(i) Find the direction cosines of the y-axis.

(j) Show that the intersection with any plane parallel to the xy-plane of the paraboloid

$$x^2 + 2y^2 = 3z$$

is an ellipse. $4\frac{1}{2} \times 10 = 45$

2. (a) Find the equation to the pair of lines through the origin which are perpendicular to the lines represented by

$$ax^2 + 2hxy + by^2 = 0.$$

(b) If $y = \tan^{-1}\frac{1}{x^2+x+1} + \tan^{-1}\frac{1}{x^2+3x+3} + \tan^{-1}\frac{1}{x^2+5x+7} + \ldots$ upto n terms,

then prove that

$$\frac{dy}{dx} = \frac{1}{(x+n)^2+1} - \frac{1}{x^2+1}$$

(c) If α, β, γ are the roots of the cubic equation

$$x^3 - px^2 + qx - r = 0$$

find the value of $\Sigma\, \alpha^2 \beta \gamma$. *5+6+4=15*

3. (a) If a focal chord of the parabola $y^2 = 4ax$ meets the curve at $A(at_1^2, 2at_1)$ and $B(at_2^2, 2at_2)$ then show that $t_1 t_2 = -1$.

Hence, show that if S is the focus of the parabola, then

$$\frac{1}{SA} + \frac{1}{SB} = \text{a constant}$$

(b) If $I_n = \int_0^{\pi/4} \tan^n \theta \, d\theta$,

then $I_n = \frac{1}{n-1} - I_{n-2}$.

Hence, find the value of $\int_0^{\pi/4} \tan^4 \theta \, d\theta$.

(c) Show that the line

$$x - 1 = y - 2 = z + 1$$

lies entirely on the surface

$$xy - z^2 - 2x - y - 2z + 1 = 0$$

5+5+5=15

4. (a) Find the image of the point (– 3, 8, 4) on the plane $6x - 3y - 2z + 1 = 0$.

(b) If a point z moves on the Argand plane such that $\frac{z-i}{z-1}$ is always purely imaginary, then prove that the locus of z is a circle with centre at $\frac{1}{2}(1 + i)$ and radius $\frac{1}{\sqrt{2}}$.

(c) Prove that the condition for $ax + by + 1 = 0$ to touch

$$x^2 + y^2 + 2gx + 2fy + c = 0 \text{ is}$$

$$(ag + bf - 1)^2 = (a^2 + b^2)(g^2 + f^2 - c)$$

5+5+5=15

5. (a) Find the equation of the right circular cone which contains the three positive co-ordinate axes. 8

(b) Show that

$$\sqrt{1} + \sqrt{2} + \ldots + \sqrt{n} \leq n \sqrt{\frac{n+1}{2}}$$ 7