SOLUTIONS TO IIT-JEE 2009
CHEMISTRY: Paper-I (Code: 06)

PART – I

SECTION – I
Single Correct Choice Type

This section contains 8 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), for its answer out of which **ONLY ONE** is correct.

Note: Questions with (*) mark are from syllabus of class XI.

1. Among the electrolytes Na_2SO_4, $CaCl_2$, $Al_2(SO_4)_3$ and NH_4Cl, the most effective coagulating agent for Sb_2S_3 sol is

(A) Na_2SO_4 (B) $CaCl_2$ (C) $Al_2(SO_4)_3$ (D) NH_4Cl

Sol.: According to Hardy–Schulze rule, the coagulating power of an ion is directly proportional to the magnitude of its charge.
∴ For negatively charged Sb_2S_3 sol, the most effective coagulating agent would be $Al_2(SO_4)_3$.
Correct choice: (C)

***2.** Given that the abundances of isotopes ^{54}Fe, ^{56}Fe and ^{57}Fe are 5%, 90% and 5%, respectively, the atomic mass of Fe is

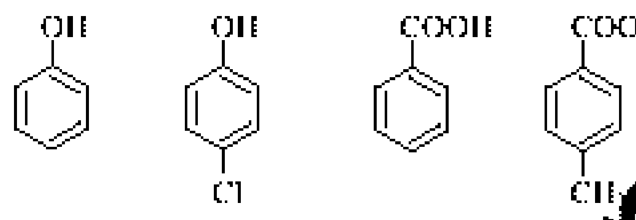
(A) 55.85 (B) 55.95 (C) 55.75 (D) 56.06

Sol.: Average atomic mass of an element = $\Sigma A_i X_i$
where A_i and X_i represents the atomic mass and mole fraction of the component in the mixture.

Average atomic mass of Fe = $\dfrac{(54\times 5)+(56\times 90)+(57\times 5)}{100} = 55.95$

Correct choice: (B)

***3.** The correct acidity order of the following is

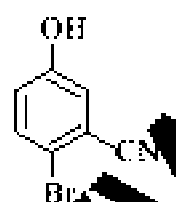


(I) (II) (III) (IV)

(A) (III) > (IV) > (II) > (I) (B) (IV) > (III) > (I) > (II) (C) (III) > (II) > (I) > (IV) (D) (II) > (III) > (IV) > (I)

Sol.: Carboxylic acid is stronger acid than phenol. The presence of electron donating methyl group decreases acidic strength while presence of electron withdrawing halogen increases acidic strength.
Correct choice: (A)

***4.** The IUPAC name of the following compound is



(A) 4-Bromo-3-cyanophenol (B) 2-Bromo-5-hydroxybenzonitrile
(C) 2-Cyano-4-hydroxybromobenzene (D) 6-Bromo-3-hydroxybenzonitrile

Sol.: –CN group is principal functional group.
Correct choice: (B)

***5.** The term that corrects for the attractive forces present in a real gas in the van der Waals equation is

(A) nb (B) $\dfrac{an^2}{V^2}$ (C) $-\dfrac{an^2}{V^2}$ (D) $-nb$

Sol.: The attractive forces in a real gas decreases its pressure relative to an ideal gas.

$$P_r = P_i - \frac{an^2}{V^2} \quad ; \quad P_i = \left(P_r + \frac{an^2}{V^2}\right)$$

The term that accounts for decrease in pressure due to attractive forces among molecules of a real gas must be added to the real gas pressure (observed) to get the ideal gas pressure.
Correct choice: (B)

6. Among cellulose, poly(vinyl chloride), nylon and natural rubber, the polymer in which the intermolecular force of attraction is weakest is

(A) Nylon (B) Poly(vinyl chloride) (C) Cellulose (D) Natural Rubber

Sol.: Nylon and cellulose, both have intermolecular hydrogen bonding whereas polyvinyl chloride has dipole-dipole interaction. Natural rubber will have London forces which are weakest.

Correct choice: (D)

7. The reaction of P_4 with **X** leads selectively to P_4O_6. The **X** is

(A) Dry O_2 (B) A mixture of O_2 and N_2

(C) Moist O_2 (D) O_2 in the presence of aqueous NaOH

Sol.: P_4 reacts with O_2 in limited supply of air (a mixture of O_2 and N_2) to give P_4O_6.

$$P_4 + 3O_2 \xrightarrow{\text{limited supply of air}} P_4O_6$$

Correct choice: (B)

8. The Henry's law constant for the solubility of N_2 gas in water at 298 K is 1.0×10^5 atm. The mole fraction of N_2 in air is 0.8. The number of moles of N_2 from air dissolved in 10 moles of water at 298 K and 5 atm pressure is

(A) 4.0×10^{-4} (B) 4.0×10^{-5} (C) 5.0×10^{-4} (D) 4.0×10^{-6}

Sol.: According to Henry's law

$$P_{N_2} = K_{N_2} X_{N_2}$$

Where K is the Henry's constant (in atm) and X_{N_2} is mole fraction of N_2.

$$P_{N_2} = X_{N_2} P_T = 0.8 \times 5 \text{ atm} = 4.0 \text{ atm.}$$

$$\therefore\ 4 \text{ atm} = 1.0 \times 10^5 \text{ atm} \times X_{N_2}$$

$$4 \times 10^{-5} = \frac{n_{N_2}}{n_{N_2} + n_{H_2O}} = \frac{n_{N_2}}{n_{N_2} + 10}$$

$$n_{N_2} = 4 \times 10^{-4} \text{ moles}$$

Correct choice: (A)

SECTION – II

Multiple Correct Choice Type

This section contains 4 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONE OR MORE** is/are correct.

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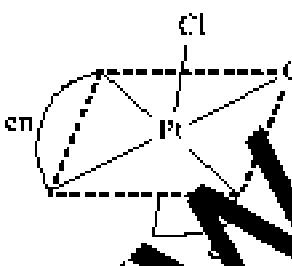
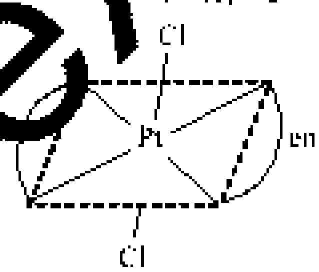
9. The compound(s) that exhibit(s) geometrical isomerism is(are)

(A) $[Pt(en)Cl_2]$ (B) $[Pt(en)_2]Cl_2$ (C) $[Pt(en)_2Cl_2]Cl_2$ (D) $[Pt(NH_3)_2Cl_2]$

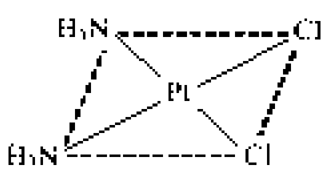
Sol.:



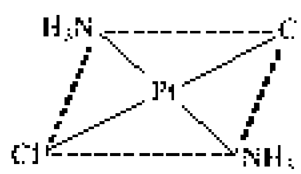
cis



trans



cis



trans

Correct choice: (C) and (D)

*10. The compound(s) formed upon combustion of sodium metal in excess air is(are)

(A) Na_2O_2 (B) Na_2O (C) NaO_2 (D) NaOH

Sol.: Combustion of sodium metal in excess of air yields Na_2O and Na_2O_2.

$$4Na + O_2 \xrightarrow{\Delta} 2Na_2O$$

$$2Na + O_2 \xrightarrow{\Delta} Na_2O_2$$

Correct choice: (A) and (B)

11. The correct statement(s) regarding defects in solids is(are)

(A) Frenkel defect is usually favoured by a very small difference in the sizes of cation and anion.

(B) Frenkel defect is a dislocation defect.

(C) Trapping of an electron in the lattice leads to the formation of F-center.

(D) Schottky defects have no effect on the physical properties of solids.

Sol.: Frenkel defect is a dislocation defect, observed when the size of cation and anion differ largely. F-center is created, when an anion is lost from the lattice and vacancy is filled by trapping of an electron. Schottky defect changes the density of a crystalline solid.

Correct choice: (B) and (C)

*12. The correct statement(s) about the compound $H_3C(HO)HC{-}CH{=}CH{-}CH(OH)CH_3$ **(X)** is(are)

(A) The total number of stereoisomers possible for **X** is 6.

(B) The total number of diastereomers possible for **X** is 3.

(C) If the stereochemistry about the double bond in **X** is *trans*, the number of enantiomers possible for **X** is 4.

(D) If the stereochemistry about the double bond in **X** is *cis*, the number of enantiomers possible for **X** is 2.

Sol.: $CH_3{-}\overset{*}{C}H(OH){-}CH{=}CH{-}\overset{*}{C}H(OH){-}CH_3$

I	d	cis	d
II	d	trans	d
III	*l*	cis	*l*
IV	*l*	trans	*l*
V	d	cis	*l*
VI	d	trans	*l*

I and III are enantiomers, II and IV are enantiomers, V is meso due to plane of symmetry while VI is meso due to center of symmetry. A total of 6 stereoisomers are possible while number of diastereomers are 4 (I or III, II or IV, V and VI). If double bond is trans, number of enantiomers is 2 (II and IV). If double bond is cis, number of enantiomers is 2 (I and III).

Correct choice: (A) and (D)

SECTION – III
Comprehension Type

This section contains 2 groups of questions. Each group has 3 multiple choice questions based on a paragraph. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which ONLY ONE is correct.

Paragraph for Question Nos. 13 to 15

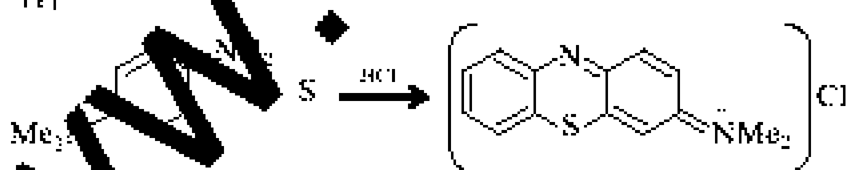
p-Amino-*N*,*N*-dimethylaniline is added to a strongly acidic solution of **X**. The resulting solution is treated with a few drops of aqueous solution of **Y** to yield blue coloration due to the formation of methylene blue. Treatment of the aqueous solution of **Y** with the reagent potassium hexacyanoferrate(II) leads to the formation of an intense blue precipitate. The precipitate dissolves on excess addition of the reagent. Similarly, treatment of the solution of **Y** with the solution of potassium hexacyanoferrate(III) leads to a brown coloration due to the formation of **Z**.

13. The compound **X** is

(A) $NaNO_3$ (B) NaCl (C) Na_2SO_4 (D) Na_2S

Sol.: $\underset{(X)}{Na_2S} + 2H^+ \longrightarrow H_2S + 2Na^+$

$\underset{(Y)}{FeCl_3} + H_2S \longrightarrow FeCl_2 + 2HCl + S$



Methylene blue

Correct choice: (D)

14. The compound **Y** is

(A) $MgCl_2$ (B) $FeCl_2$ (C) $FeCl_3$ (D) $ZnCl_2$

Sol.: Compound **Y** is $FeCl_3$, because when it treated with $K_4[Fe(CN)_6]$ gives intense blue precipitate of $Fe_4[Fe(CN)_6]_3$, which dissolves in excess of $K_4[Fe(CN)_6]$.

$4Fe^{3+} + 3[Fe(CN)_6]^{4-} \longrightarrow Fe_4[Fe(CN)_6]_3$

Correct choice: (C)

15. The compound **Z** is

(A) $Mg_2[Fe(CN)_6]$ (B) $Fe[Fe(CN)_6]$ (C) $Fe_4[Fe(CN)_6]_3$ (D) $K_2Zn_3[Fe(CN)_6]_2$

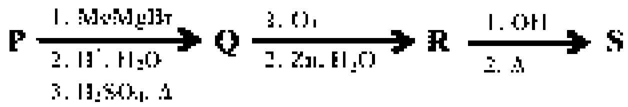
Sol.: Compound **Z** is $Fe[Fe(CN)_6]$ of brown colour, which is formed due to reaction of $K_3[Fe(CN)_6]$ and $FeCl_3$(**Y**)

$Fe^{+3} + [Fe(CN)_6]^{3-} \longrightarrow Fe[Fe(CN)_6]$

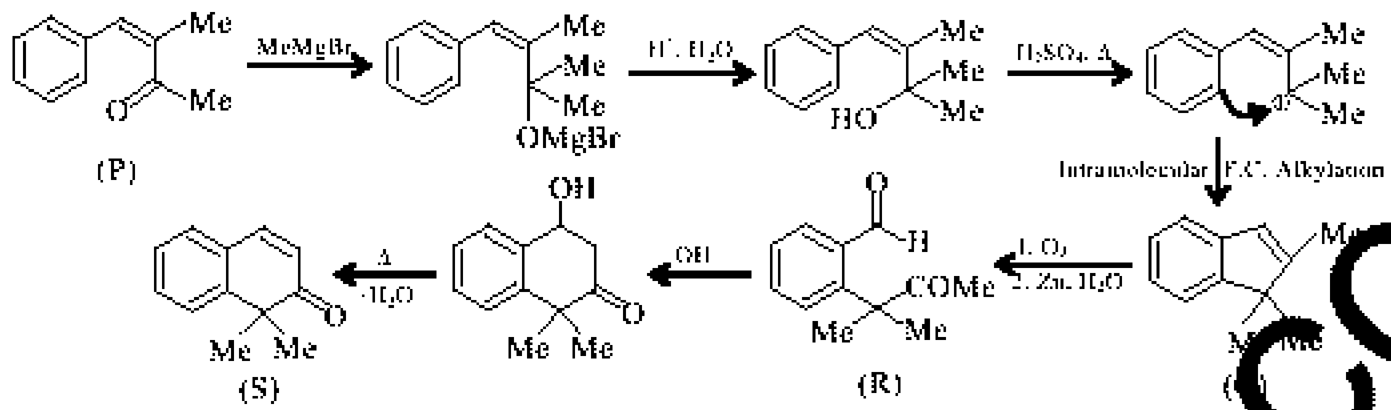
Correct choice: (B)

Paragraph for Question Nos. 16 to 18

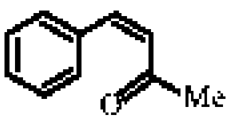
A carbonyl compound **P**, which gives positive iodoform test, undergoes reaction with MeMgBr followed by dehydration to give an olefin **Q**. Ozonolysis of **Q** leads to a dicarbonyl compound **R**, which undergoes intramolecular aldol reaction to give predominantly **S**.

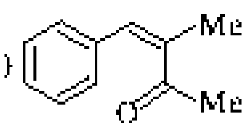


Sol.:

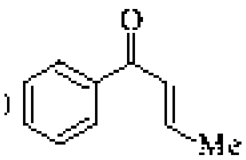


16. The structure of the carbonyl compound **P** is

(A) 

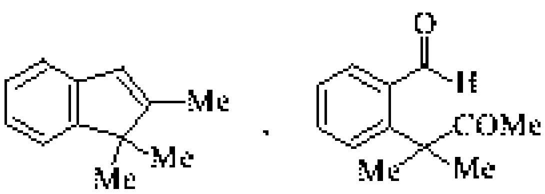
(B) 

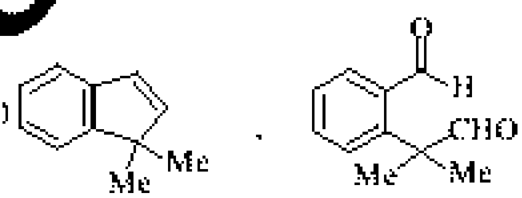
(C) 

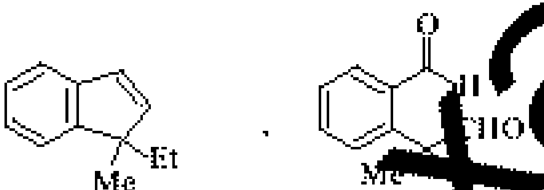
(D) 

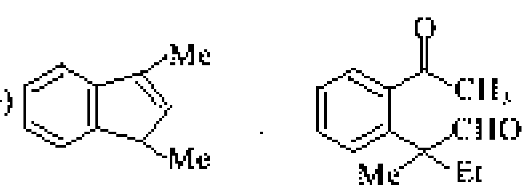
Correct choice: (B)

17. The structures of the products **Q** and **R**, respectively, are

(A) 

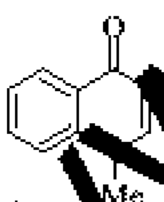
(B) 

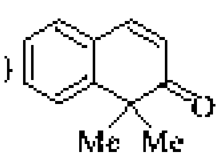
(C) 

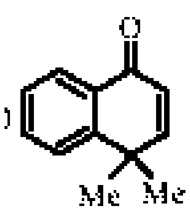
(D) 

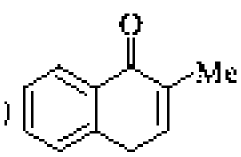
Correct choice: (A)

18. The structure of the product **S** is

(A) 

(B) 

(C) 

(D) 

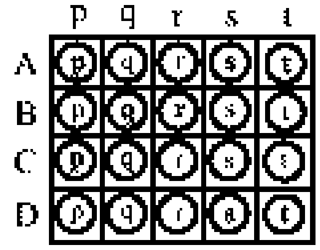
Correct choice: (B)

SECTION – IV

Matrix – Match Type

This section contains 2 questions. Each question contains statements given in two columns, which have to be matched. The statements in **Column I** are labelled A, B, C and D, while the statements in **Column II** are labelled p, q, r, s and t. Any given statement in **Column I** can have correct matching with **ONE OR MORE** statement(s) in **Column II**. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example:

If the correct matches are A–p, s and t; B–q and r; C–p and q; and D–s and t; then the correct darkening of bubbles will look like the following.



***19.** Match each of the diatomic molecules in **Column I** with its property/properties in **Column II**.

Column I	**Column II**
(A) B_2	(p) Paramagnetic
(B) N_2	(q) Undergoes oxidation
(C) O_2^-	(r) Undergoes reduction
(D) O_2	(s) Bond order ≥ 2
	(t) Mixing of 's' and 'p' orbitals

Sol.: **(A) – (p), (q), (r), (t) ; (B) – (q), (r), (s), (t) ; (C) – (p), (q), (r) ; (D) – (p), (q), (r), (s)**

20. Match each of the compound in **Column I** with its characteristic reaction(s) in **Column II**.

Column I	**Column II**
(A) $CH_3CH_2CH_2CN$	(p) Reduction with Pd–C/H_2
(B) $CH_3CH_2OCOCH_3$	(q) Reduction with $SnCl_2$/HCl
(C) CH_3–CH=CH–CH_2OH	(r) Development of foul smell on treatment with chloroform and alcoholic KOH
(D) $CH_3CH_2CH_2CH_2NH_2$	(s) Reduction with diisobutylaluminium hydride (DIBAL–H)
	(t) Alkaline hydrolysis

Sol.: **(A) – (p), (q), (s), (t) ; (B) – (p), (s), (t) ; (C) – (p), (s) ; (D) – (r)**

SOLUTIONS TO IIT-JEE 2009
MATHEMATICS: Paper-I (Code: 06)

PART – II

SECTION – I

Single Correct Choice Type

This section contains 8 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONLY ONE** is correct.

Note: Questions with (*) mark are from syllabus of class XI.

21. Let f be a non-negative function defined on the interval $[0, 1]$. If $\int_0^x \sqrt{1-(f'(t))^2}\,dt = \int_0^x f(t)\,dt,\ 0 \le x \le 1$ and $f(0) = 0$, then

(A) $f\left(\frac{1}{2}\right) < \frac{1}{2}$ and $f\left(\frac{1}{3}\right) > \frac{1}{3}$

(B) $f\left(\frac{1}{2}\right) > \frac{1}{2}$ and $f\left(\frac{1}{3}\right) > \frac{1}{3}$

(C) $f\left(\frac{1}{2}\right) < \frac{1}{2}$ and $f\left(\frac{1}{3}\right) < \frac{1}{3}$

(D) $f\left(\frac{1}{2}\right) > \frac{1}{2}$ and $f\left(\frac{1}{3}\right) < \frac{1}{3}$

Sol.: Differentiate both sides with respect to x

$$\sqrt{1-(f'(x))^2} = f(x)$$

Now $1-(f'(x))^2 \ge 0$

$$|f'(x)| \le 1$$

Apply L.M.V.T on $f(x)$ in $\left[0, \frac{1}{2}\right]$

$$|f'(x)| = \left|\frac{f\left(\frac{1}{2}\right) - f(0)}{\frac{1}{2} - 0}\right| < 1$$

$$f\left(\frac{1}{2}\right) < \frac{1}{2}$$

Similarly, apply L.M.V.T on $f(x)$ in $\left[0, \frac{1}{3}\right]$

$$|f'(x)| = \left|\frac{f\left(\frac{1}{3}\right) - f(0)}{\frac{1}{3}}\right| < \blacklozenge \Rightarrow \quad f\left(\frac{1}{3}\right) < \frac{1}{3}$$

Alternative Solution:

$\sqrt{1-(f'(x))^2} = f(x)$ By Newton Leibnitz Rule

$\Rightarrow (f(x))^2 + (f'(x))^2 = 1 \Rightarrow (f'(x))(f(x) + f''(x)) = 0 \Rightarrow f''(x) < 0$ as $f(x)$ is non negative.

Also $f(0) = 0$ and $f'(0) = 1$ (as $f'(0) = -1$ makes $f(x)$ negative in RHS of $x = 0$)

$\Rightarrow$ The graph of $f(x)$ has $y = x$ as tangent and is concave downwards i.e. lies below $y = x \Rightarrow f(x) < x$

Correct choice: (C)

22. Let $P(3, 2, 6)$ be a point in space and Q be a point on the line $\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu(-3\hat{i} + \hat{j} + 5\hat{k})$. Then the value of μ for which the vector $\overrightarrow{PQ}$ is parallel to the plane $x - 4y + 3z = 1$ is

(A) $\frac{1}{4}$ (B) $-\frac{1}{4}$ (C) $\frac{1}{8}$ (D) $-\frac{1}{8}$

Sol.: Let any point $Q \equiv (1-3\mu, \mu-1, 5\mu+2)$; $P \equiv (3, 2, 6)$

So dr's of PQ is $-2-3\mu, \mu-3, 5\mu-4$

Now PQ is parallel to $x-4y+3z=1$

$\Rightarrow \quad -2-3\mu-4(\mu-3)+3(5\mu-4)=0 \Rightarrow -2-3\mu-4\mu+12+15\mu-12=0 \quad \Rightarrow \quad \mu=\dfrac{1}{4}$

Correct choice: (A)

23. If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are unit vectors such that $(\vec{a}\times\vec{b}).(\vec{c}\times\vec{d})=1$ and $\vec{a}.\vec{c}=\dfrac{1}{2}$, then

(A) $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar

(B) $\vec{b}, \vec{c}, \vec{d}$ are non-coplanar

(C) $\vec{b}, \vec{d}$ are non-parallel

(D) $\vec{a}, \vec{d}$ are parallel and $\vec{b}, \vec{c}$ are parallel

Sol.: $\vec{a}\times\vec{b}=\sin\theta_1.\hat{n}_1$

$\vec{c}\times\vec{d}=\sin\theta_2.\hat{n}_2$

$(\vec{a}\times\vec{b}).(\vec{c}\times\vec{d})=\sin\theta_1\sin\theta_2\,\hat{n}_1.\hat{n}_2 = \sin\theta_1.\sin\theta_2.\cos\theta_3$

Now $\sin\theta_1.\sin\theta_2.\cos\theta_3=1 \Rightarrow \theta_1=\dfrac{\pi}{2}, \theta_2=\dfrac{\pi}{2}, \theta_3=0^\circ \quad \Rightarrow \quad \vec{a}\perp\vec{b}, \vec{c}\perp\vec{d}, (\vec{a}\times\vec{b}) \parallel \vec{c}\times\vec{d}$

Let $\vec{a}\times\vec{b}=\lambda(\vec{c}\times\vec{d}) \Rightarrow \quad (\vec{a}\times\vec{b}).\vec{c}=\lambda(\vec{c}\times\vec{d}).\vec{c}=0$

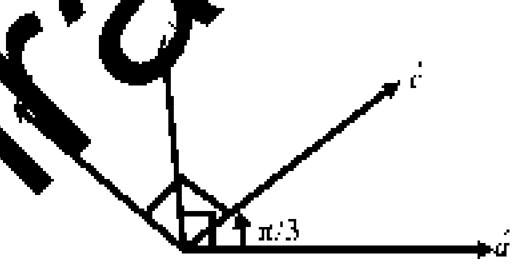
$\Rightarrow \quad \vec{a}, \vec{b}, \vec{c}$ are coplanar ...(i)

Similarly, $\vec{b}, \vec{c}, \vec{d}$ are coplanar ...(ii)

From (i) and (ii) $\Rightarrow \vec{a}, \vec{b}, \vec{c}, \vec{d}$ are coplanar

Now so angle between $\vec{b}$ and $\vec{d} = \dfrac{\pi}{3}$

Correct choice: (C)



***24.** The number of seven digit integers, with sum of the digits equal to 10 and formed by using the digits 1, 2 and 3 only, is

(A) 55 (B) 66 (C) 77 (D) 88

Sol.: Possible ways are

(i) 2 three times, 1 four times. Number of ways $=\dfrac{7!}{3!\,4!}$

(ii) 2 once, 3 once and 1 five times. Number ways $=\dfrac{7!}{5!}$

Required number = 77

Correct choice: (C)

***25.** Tangents drawn from the point $P(1, 8)$ to the circle $x^2+y^2-6x-4y-11=0$ touch the circle at the points A and B. The equation of the circumcircle of the triangle PAB is

(A) $x^2+y^2+4x-6y+19=0$

(B) $x^2+y^2-4x-10y+19=0$

(C) $x^2+y^2-2x+6y-29=0$

(D) $x^2+y^2-6x-4y+19=0$

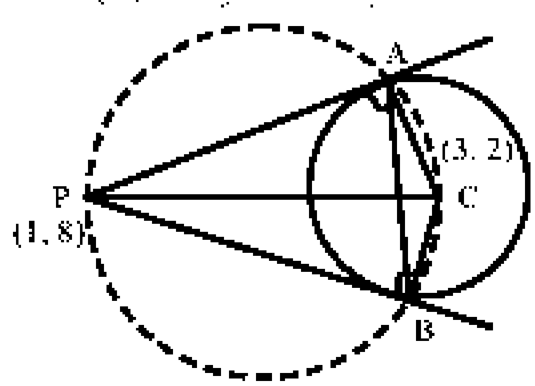
Sol.: Since PC subtends angle $\dfrac{\pi}{2}$ at A and B both.

So required circle is the circle with PC as diameter.

Circle will be $(x-1)(x-3)+(y-8)(y-2)=0$

$\Rightarrow \quad x^2+y^2-4x-10y+19=0$

Correct choice: (B)



*26. Let $z = \cos\theta + i\sin\theta$. Then the value of $\sum_{m=1}^{15} \text{Im}\left(z^{2m-1}\right)$ at $\theta = 2^\circ$ is

(A) $\frac{1}{\sin 2^\circ}$ (B) $\frac{1}{3\sin 2^\circ}$ (C) $\frac{1}{2\sin 2^\circ}$ (D) $\frac{1}{4\sin 2^\circ}$

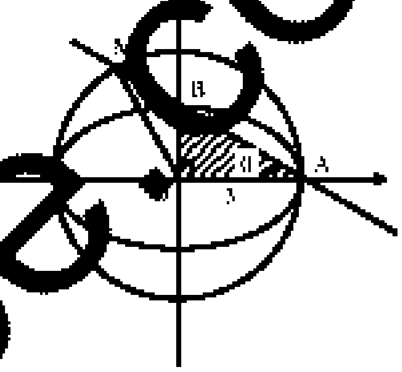
Sol.: $\sum_{m=1}^{15} \text{Im}\left(z^{2m-1}\right) = \sum_{m=1}^{15} \sin((2m-1)\theta) = \frac{\sin\left(\frac{29\theta+\theta}{2}\right).\sin\left(\left(\frac{2\theta}{2}\right).15\right)}{\sin\left(\frac{2\theta}{2}\right)} = \frac{\sin 30^\circ . \sin 30^\circ}{\sin 2^\circ} = \frac{1}{4\sin 2^\circ}$

Correct choice: (D)

*27. The line passing through the extremity A of the major axis and extremity B of the minor axis of the ellipse $x^2 + 9y^2 = 9$ meets its auxiliary circle at the point M. Then the area of the triangle with vertices at A, M and the origin O is

(A) $\frac{31}{10}$ (B) $\frac{29}{10}$ (C) $\frac{21}{10}$ (D) $\frac{27}{10}$

Sol.: If $\angle OAB = \theta$, then $\tan\theta = \frac{1}{3}$



Area $(OAM) = \frac{1}{2} r^2 \sin 2\theta = \frac{27}{10}$

Correct choice: (D)

*28. Let $z = x + iy$ be a complex number where x and y are integers. Then the area of the rectangle whose vertices are the roots of the equation $z\bar{z}^3 + \bar{z}z^3 = 350$ is

(A) 48 (B) 32 (C) 40 (D) 80

Sol.: $z\bar{z}^3 + \bar{z}z^3 = 350$

$|z|^2\left(\bar{z}^2 + z^2\right) = 350$

$\left(x^2 + y^2\right)\left(x^2 - y^2\right) = 175 \Rightarrow \left(x^2 + y^2\right)\left(x^2 - y^2\right) = 25 \times 7$

So vertices of rectangle are $(4, 3), (-4, 3), (4, -3), (-4, -3)$. Required area $= 8 \times 6 = 48$

Correct choice: (A)

SECTION – II
Multiple Correct Choice Type

This section contains 4 multiple correct questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONE OR MORE** is/are correct.

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29. Let $L = \lim_{x \to 0} \frac{a - \sqrt{a^2 - x^2} - \frac{x^2}{4}}{x^4}$, $a > 0$. If L is finite, then

(A) $a = 2$ (B) $a = 1$ (C) $L = \frac{1}{64}$ (D) $L = \frac{1}{32}$

Sol.: $\lim_{x \to 0} \frac{a - \sqrt{a^2 - x^2} - \frac{x^2}{4}}{x^4}$, $a > 0$ Put $x = a\sin\theta$

$\lim_{\theta \to 0} \frac{a - a|\cos\theta| - \frac{a^2}{4}\sin^2\theta}{\theta^4 a^4} = \lim_{\theta \to 0} \frac{a\sin\theta - \frac{2a^2}{4}\sin\theta\cos\theta}{4\theta^3 a^4} = \lim_{\theta \to 0} \frac{1 - \frac{2a}{4}\cos\theta}{4\theta^2 a^3}$

L is finite so numerator should be zero at $\theta \to 0$

$\Rightarrow a = 2 \Rightarrow L = \lim_{\theta \to 0} \frac{1 - \cos\theta}{4\theta^2 \times 8} = \frac{1}{64}$

Alternative Solution:

$$\lim_{x\to 0}\frac{a-a\sqrt{1-\left(\frac{x}{a}\right)^2}-\frac{x^2}{4}}{x^4}$$

$$\lim_{x\to 0}\frac{a-a\left\{1-\frac{1}{2}\left(\frac{x}{a}\right)^2-\frac{\frac{1}{2}\times\frac{1}{2}}{2}\left(\frac{x}{a}\right)^4 \ldots\ldots\right\}-\frac{x^2}{4}}{x^4}\ ;\quad \lim_{x\to 0}\frac{\left(\frac{1}{2}\frac{x^2}{a}+\frac{1}{8}\frac{x^4}{a^3}+\ldots\ldots\right)-\frac{x^2}{4}}{x^4}$$

It has finite limit if $a=2$ and value of limit $L=\frac{1}{8a^3}=\frac{1}{64}$

Correct choice: (A) and (C)

*30. In a triangle ABC with fixed base BC, the vertex A moves such that $\cos B+\cos C=4\sin^2\frac{A}{2}$. If a, b and c denote the lengths of the sides of the triangle opposite to the angles A, B and C, respectively, then

(A) $b+c=4a$ (B) $b+c=2a$

(C) locus of point A is an ellipse (D) locus of point A is a pair of straight lines

Sol.: $a\cos B+a\cos C=2a(1-\cos A)$

$\Rightarrow\quad a\cos B+b\cos A+a\cos C+c\cos A=2a(1-\cos A)+b\cos A+c\cos A$

$\Rightarrow\quad (b+c-2a)=(b+c-2a)\cos A \Rightarrow b+c=2a \Rightarrow AC+AB>BC \Rightarrow$ So locus of point A is ellipse.

Alternative Solution:

$\cos B+\cos C=2(1-\cos A)\Rightarrow\quad 2\cos\left(\frac{B+C}{2}\right).\cos\left(\frac{B-C}{2}\right)=4\sin^2\frac{A}{2}\Rightarrow\sin\frac{A}{2}=0$ (not possible in a triangle)

So $\cos\left(\frac{B-C}{2}\right)=2\sin\left(\frac{A}{2}\right)\Rightarrow\frac{\cos\left(\frac{B-C}{2}\right)}{\cos\left(\frac{B+C}{2}\right)}=2\Rightarrow\frac{\sin\frac{B}{2}\sin\frac{C}{2}}{\cos\frac{B}{2}\cos\frac{C}{2}}=\frac{1}{3}\Rightarrow\tan\frac{B}{2}\tan\frac{C}{2}=\frac{1}{3}$

$\Rightarrow\quad\sqrt{\frac{(s-c)(s-a)}{s(s-b)}}\times\sqrt{\frac{(s-b)(s-a)}{s(s-c)}}=\frac{1}{3}\Rightarrow\frac{s-a}{s}=\frac{1}{3}\Rightarrow 2s=3a\Rightarrow b+c=2a\Rightarrow AC+AB>BC$

So locus of point A is ellipse.

Correct choice: (B) and (C)

31. Area of the region bounded by the curve $y=e^x$ and lines $x=0$ and $y=e$ is

(A) $e-1$ (B) $\int_1^e \ln(e+1-y)\,dy$ (C) $e-\int_0^1 e^x\,dx$ (D) $\int_1^e \ln y\,dy$

Sol.: Given curve $C_1: y=e^x$

$C_2: x=0$

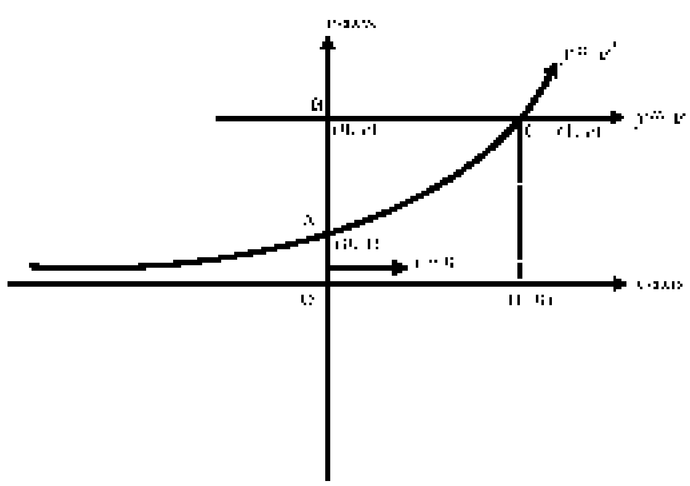
$C_3: y=e$

Intersection point for the curve C_1 and C_3 is $(1,e)$

Point $A(0,1)$ and $B(0,e)$

Required area $=\int_1^e x\,dy=\int_1^e \log y\,dy$

or



Required area of the region $ABC=\int_0^1(e-e^x)\,dx=e-\int_0^1 e^x\,dx$

Correct choice: (B), (C) and (D)

***32.** If $\dfrac{\sin^4 x}{2}+\dfrac{\cos^4 x}{3}=\dfrac{1}{5}$, then

(A) $\tan^2 x=\dfrac{2}{3}$

(B) $\dfrac{\sin^8 x}{8}+\dfrac{\cos^8 x}{27}=\dfrac{1}{125}$

(C) $\tan^2 x=\dfrac{1}{3}$

(D) $\dfrac{\sin^8 x}{8}+\dfrac{\cos^8 x}{27}=\dfrac{2}{125}$

Sol.: $\dfrac{\sin^4 x}{2}+\dfrac{\cos^4 x}{3}=\dfrac{1}{5}$

$\Rightarrow \quad 3\sin^4 x+2\left(1-\sin^2 x\right)^2=\dfrac{6}{5} \Rightarrow 5\sin^4 x-4\sin^2 x+2-\dfrac{6}{5}=0 \Rightarrow 25\sin^4 x-20\sin^2 x+4=0$

$\Rightarrow \quad \left(5\sin^2 x-2\right)^2=0 \Rightarrow \sin^2 x=\dfrac{2}{5}$

So $\cos^2 x=\dfrac{3}{5} \Rightarrow \tan^2 x=\dfrac{2}{3}$ and $\dfrac{\sin^8 x}{8}+\dfrac{\cos^8 x}{27}=\dfrac{\left(\frac{2}{5}\right)^4}{8}+\dfrac{\left(\frac{3}{5}\right)^4}{27}=\dfrac{2}{(5)^4}+\dfrac{3}{(5)^4}=\dfrac{5}{5^4}=\dfrac{1}{5^3}=\dfrac{1}{125}$

Correct choice: (A) and (B)

SECTION – III
Comprehension Type

This section contains 2 groups of questions. Each group has 3 multiple choice questions based on a paragraph. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONLY ONE** is correct.

Paragraph for Question Nos. 33 to 35

Let $\mathcal{A}$ be the set of all 3×3 symmetric matrices all of whose entries are either 0 or 1. Five of these entries are 1 and four of them are 0.

33. The number of matrices in $\mathcal{A}$ is

(A) 12 (B) 6 (C) 9 (D) 3

34. The number of matrices A in $\mathcal{A}$ for which the system of linear equations $A\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1\\0\\0\end{bmatrix}$ has a unique solution, is

(A) less than 4

(B) at least 4 but less than 7

(C) at least 7 but less than 10

(D) at least 10

35. The number of matrices A in $\mathcal{A}$ for which the system of linear equations $A\begin{bmatrix}x\\y\\x\end{bmatrix}=\begin{bmatrix}1\\0\\0\end{bmatrix}$ is inconsistent, is

(A) 0 (B) more than 2 (C) 2 (D) 1

Sol.: **Case I:** When diagonal is having two zeros and one-1.

$$\underset{(i)}{\begin{pmatrix}0&1&1\\1&0&0\\1&0&1\end{pmatrix}}\underset{(ii)}{\begin{pmatrix}0&1&0\\1&0&1\\0&1&1\end{pmatrix}}\underset{(iii)}{\begin{pmatrix}0&0&1\\0&0&1\\1&1&1\end{pmatrix}}$$

$$\underset{(iv)}{\begin{pmatrix}0&1&1\\1&1&0\\1&0&0\end{pmatrix}}\underset{(v)}{\begin{pmatrix}0&1&0\\1&1&1\\0&1&0\end{pmatrix}}\underset{(vi)}{\begin{pmatrix}0&0&1\\0&1&1\\1&1&0\end{pmatrix}}$$

$$\underset{(vii)}{\begin{pmatrix}1&1&1\\1&0&0\\1&0&0\end{pmatrix}}\underset{(viii)}{\begin{pmatrix}1&1&0\\1&0&1\\0&1&0\end{pmatrix}}\underset{(ix)}{\begin{pmatrix}1&0&1\\0&0&1\\1&1&0\end{pmatrix}}$$

Case II: When all diagonal elements are one and non diagonal elements having two zeros and one-1 on either side.

$$\begin{pmatrix}1&0&0\\0&1&1\\0&1&1\end{pmatrix}\quad\begin{pmatrix}1&0&1\\0&1&0\\1&0&1\end{pmatrix}\quad\begin{pmatrix}1&1&0\\1&1&0\\0&0&1\end{pmatrix}$$
(x) (xi) (xii)

33. Clearly 12 matrices are possible.

Correct choice: (A)

34. For (i), (ii), (iv), (vi), (viii), (ix) determinant value is non-zero.

Correct choice: (B)

35. (iii), (v), (xi), (xii) never give solution.

Correct choice: (B)

Alternative solution:

33. **Case I:** All diagonal elements are 1's

Two 0's, one 1's, are non diagonal elements on either side. Number of ways = 3

Case II: Two 0's and one 1's in diagonal. Number of ways = ${}^3C_2 \times 3 = 9$. Total number of ways $= 3 + 9 = 12$

Correct choice: (A)

34. For $|A| = 0$

Case I: All diagonal elements are 1

Number of ways = 3

Case II: $a_{11}, a_{22} = 0,\ a_{33} = 1$

Along with $a_{12} = a_{21} = 0$

Cyclically 3 such cases are there

$\therefore$ Total number of ways $= 3 + 3 = 6$. For $|A| \neq 0$, number of ways $= 12 - 6 = 6$

Correct choice: (B)

35. For infinite solutions row, II should be same as row III

Only two cases are there i.e., with elements $(1, 0, 0)$ or $(0, 1, 1)$

For inconsistent system; number of ways = total number of ways − ways of unique solution − ways of infinite solution

$= 12 - 6 - 2 = 4$

Correct choice: (B)

Paragraph for Question Nos. 36 to 38

A fair die is tossed repeatedly until a six is obtained. Let X denote the number of tosses required.

***36.** The probability that $X = 3$ equals

(A) $\frac{25}{216}$ (B) $\frac{25}{36}$ (C) $\frac{5}{36}$ (D) $\frac{125}{216}$

Sol.: $P(X = 3) = \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} = \frac{25}{216}$ ($\because$ all are independent)

Correct choice: (A)

***37.** The probability that $X \geq 3$ equals

(A) $\frac{125}{216}$ (B) $\frac{25}{36}$ (C) $\frac{5}{36}$ (D) $\frac{25}{216}$

Sol.: $P(X \geq 3) = 1 - P(X < 3)$

$= 1 - [P(X=1) + P(X=2)] = 1 - \left[\frac{1}{6} + \frac{5}{6} \times \frac{1}{6}\right] = \frac{25}{36}$

Correct choice: (B)

*38 The conditional probability that $X \geq 6$ given $X > 3$ equals

(A) $\frac{125}{216}$ (B) $\frac{25}{216}$ (C) $\frac{5}{36}$ (D) $\frac{25}{36}$

Sol.: Let A and B be the events of getting $X > 3$ and $X \geq 6$

$$P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)} = \frac{P(X \geq 6)}{1 - P(X \leq 3)} = \frac{\left(\frac{5}{6}\right)^5 \left[\frac{1}{6} + \frac{5}{6^2} + \ldots\ldots \infty\right]}{1 - \left[\frac{1}{6} + \frac{5}{6^2} + \frac{25}{216}\right]} = \frac{25}{36}$$

Correct choice: (D)

SECTION – IV
Matrix–Match Type

This section contains 2 questions. Each question contains statements given in two columns, which have to be matched. The statements in **Column I** are labelled A, B, C and D, while the statements in **Column II** are labelled p, q, r, s and t. Any given statement in **Column I** can have correct matching with **ONE OR MORE** statement(s) in **Column II**. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example:

If the correct matches are A – p, s and t; B – q and r; C – p and q; and D – s and t; then the correct darkening of bubbles will look like the following.

39. Match the statements/ expressions in **Column I** with the open intervals in **Column II**.

Column I	**Column II**
(A) Interval contained in the domain of definition of non-zero solutions of the differential equation $(x-3)^2 y' + y = 0$	(p) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
(B) Interval containing the value of the integral $\int_1^5 (x-1)(x-2)(x-3)(x-4)(x-5)dx$	(q) $\left(0, \frac{\pi}{2}\right)$
(C) Interval in which at least one of the points of local maximum of $\cos^2 x + \sin x$ lies	(r) $\left(\frac{\pi}{8}, \frac{5\pi}{4}\right)$
(D) Interval in which $\tan^{-1}(\sin x + \cos x)$ is increasing	(s) $\left(0, \frac{\pi}{8}\right)$
	(t) $(-\pi, \pi)$

Sol.: **A-p, q, s** $(x-3)^2 y' + y = 0$

or $(x-3)^2 \frac{dy}{dx} = -y$ or $-\int \frac{1}{y} dy = \int \frac{dx}{(x-3)^2}$ or $-\log y = -\frac{1}{(x-3)} + \log c$

or $\log y = \left(\frac{1}{x-3} - \log c\right)$ or $\log yc = \frac{1}{x-3}$ or $yc = e^{\frac{1}{x-3}}$, $\therefore$ $x \neq 3$

B-p, t Put $x - 3 = t$

$$\int_{-2}^{2}(t+2)(t+1)t(t-1)(t-2)dt = 0$$

C-p, q, r, t $\quad f(x) = -\sin^2 x + \sin x + 1 \qquad$ put $\sin x = t$

$$f(t) = -t^2 + t + 1 = -\left[\left(t - \frac{1}{2}\right)^2\right] + \frac{5}{4}$$

It gives maximum at $t = \frac{1}{2}$ or $\sin x = \frac{1}{2}$

$$x = n\pi + (-1)^n \frac{\pi}{6}$$

D-s $\quad f(x) = \tan^{-1}(\sin x + \cos x)$; $\quad f(x) = \tan^{-1}\left\{\sqrt{2}\left[\sin\left(x + \frac{\pi}{4}\right)\right]\right\}$

Function $f(x)$ is increasing when $\sin\left(x + \frac{\pi}{4}\right)$ is increasing thus

$$2n\pi - \frac{\pi}{2} < x + \frac{\pi}{4} < 2n\pi + \frac{\pi}{2} \Rightarrow 2n\pi - \frac{3\pi}{4} < x < 2n\pi + \frac{\pi}{4}$$

***40.** Match the conics in **Column I** with the statements/ expressions in **Column II**.

Column I	Column II
(A) Circle	(p) The locus of the point (h, k) for which the line $hx + ky = 1$ touches the circle $x^2 + y^2 = 4$
(B) Parabola	(q) Points z in the complex plane satisfying $\lvert z+2\rvert - \lvert z-2\rvert = \pm 3$
(C) Ellipse	(r) Points of the conic have parametric representation $x = \sqrt{3}\left(\frac{1-t^2}{1+t^2}\right), y = \frac{2t}{1+t^2}$
(D) Hyperbola	(s) The eccentricity of the conic lies in the interval $1 \le x < \infty$
	(t) Points z in the complex plane satisfying $\text{Re}(z+1)^2 = \lvert z\rvert^2 + 1$

Sol.: **A-p** $\quad$ Perpendicular distance from centre (0, 0) to $hx + ky = 1$ should be 2.

$$\left|\frac{-1}{\sqrt{h^2+k^2}}\right| = 2 \Rightarrow h^2 + k^2 = \frac{1}{4}$$. So locus is a circle.

B-s, t $\quad \lvert z+2\rvert - \lvert z-2\rvert = \pm 3$. Equation of hyperbola is $\lvert z - z_1\rvert - \lvert z - z_2\rvert = k$ where $\lvert k\rvert < \lvert z_1 - z_2\rvert$.

So locus is a hyperbola.

C-r $\quad \frac{x}{\sqrt{3}} = \frac{1-t^2}{1+t^2} \quad$ ...(i); $\quad y = \frac{2t}{1+t^2} \quad$ ...(ii)

Squaring and adding $\Rightarrow \frac{x^2}{3} + y^2 = 1$, which is an ellipse.

D-q, s $\quad$ If eccentricity is 1, locus is a parabola. If eccentricity is > 1, locus is hyperbola.

Let $z = x + iy$; $\qquad \text{Re}(x+iy)^2 = x^2 + y^2 + 1$

$(x+1)^2 - y^2 = x^2 + y^2 + 1 \qquad \Rightarrow \qquad y^2 = x$, which is a parabola.

SOLUTIONS TO IIT-JEE 2009
PHYSICS: Paper-I (Code: 06)

PART – III

SECTION – I

Single Correct Choice Type

This section contains 8 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONLY ONE** is correct.

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41. A ball is dropped from a height of 20 m above the surface of water in a lake. The refractive index of water is 4/3. A fish inside the lake, in the line of fall of the ball, is looking at the ball. At an instant, when the ball is 12.8 m above the water surface, the fish sees the speed of ball as [Take $g = 10$ m/s^2.]

(A) 9 m/s (B) 12 m/s (C) 16 m/s (D) 21.33 m/s

Sol.: Let speed of ball in air at height of 12.8 m from water surface be u.

$u = \sqrt{2gh} = \sqrt{2 \times 10 \times 7.2} = 12$ m/s

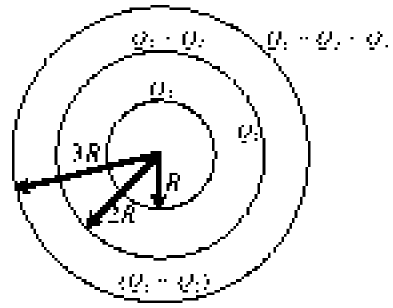
Speed of ball as observed by fish is $\frac{4}{3}u = 16$ m/s

Correct choice: (C)

42. Three concentric metallic spherical shells of radii R, $2R$, $3R$, are given charges Q_1, Q_2, Q_3, respectively. It is found that the surface charge densities on the outer surfaces of the shells are equal. Then, the ratio of the charges given to the shells, $Q_1 : Q_2 : Q_3$, is

(A) 1 : 2 : 3 (B) 1 : 3 : 5 (C) 1 : 4 : 9 (D) 1 : 8 : 18

Sol.: $\frac{Q_1}{4\pi R^2} = \frac{Q_1 + Q_2}{4\pi(2R)^2} = \frac{Q_1 + Q_2 + Q_3}{4\pi(3R)^2}$

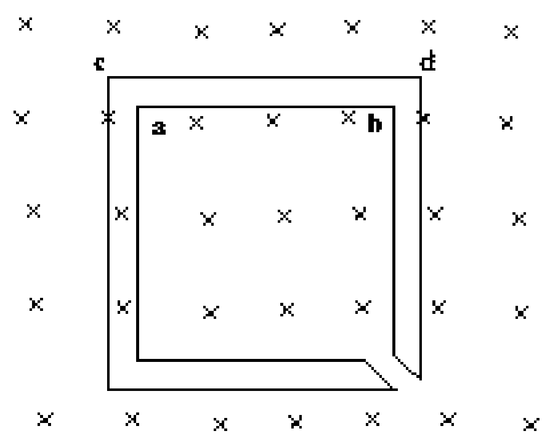


$Q_1 = \frac{Q_1 + Q_2}{4} = \frac{Q_1 + Q_2 + Q_3}{9}$

$\Rightarrow \quad Q_1 : Q_2 : Q_3 = 1 : 3 : 5$

Correct choice: (B)

43. The figure shows certain wire segments joined together to form a coplanar loop. The loop is placed in a perpendicular magnetic field in the direction going into the plane of the figure. The magnitude of the field increases with time. I_1 and I_2 are the currents in the segments **ab** and **cd**. Then,



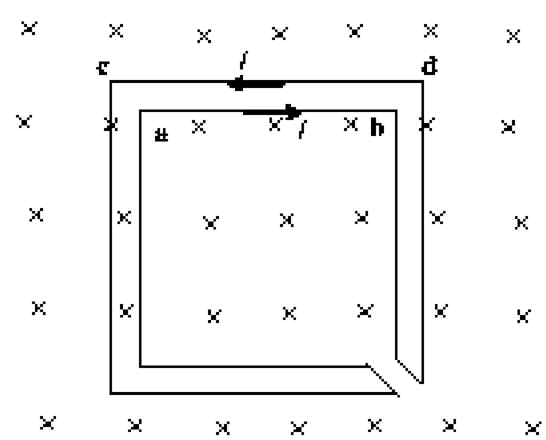
(A) $I_1 > I_2$

(B) $I_1 < I_2$

(C) I_1 is in the direction **ba** and I_2 is in the direction **cd**

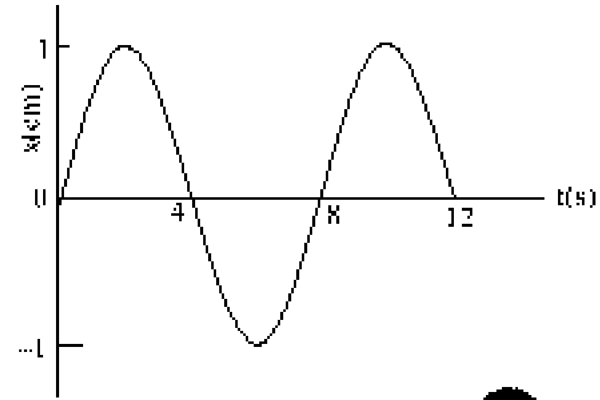
(D) I_1 is in the direction **ab** and I_2 is in the direction **dc**

Sol.: Using Lenz's law, the current is as shown in the figure



Correct choice: (D)

***44.** The x-t graph of a particle undergoing simple harmonic motion is shown below. The acceleration of the particle at $t = 4/3$ s is



(A) $\frac{\sqrt{3}}{32}\pi^2$ cm/s^2 (B) $-\frac{\pi^2}{32}$ cm/s^2

(C) $\frac{\pi^2}{32}$ cm/s^2 (D) $-\frac{\sqrt{3}}{32}\pi^2$ cm/s^2

Sol.: $x = A\sin\omega t$; $T = 8s$ and A = 1 cm

$$a = -A\omega^2\sin\omega t = -1\frac{4\pi^2}{8^2}\sin\left(\frac{2\pi}{8}\frac{4}{3}\right) = -\frac{\pi^2}{16}\times\frac{\sqrt{3}}{2} = -\frac{\sqrt{3}\pi^2}{32}\text{ cm/s}^2$$

Correct choice: (D)

***45.** A block of base 10 cm × 10 cm and height 15 cm is kept on an inclined plane. The coefficient of friction between them is $\sqrt{3}$. The inclination θ of this inclined plane from the horizontal plane is gradually increased from 0°. Then

(A) at $\theta = 30°$, the block will start sliding down the plane

(B) the block will remain at rest on the plane up to certain θ and then it will topple

(C) at $\theta = 60°$, the block will start sliding down the plane and continue to do so at higher angles

(D) at $\theta = 60°$, the block will start sliding down the plane and on further increasing θ, it will topple at certain θ

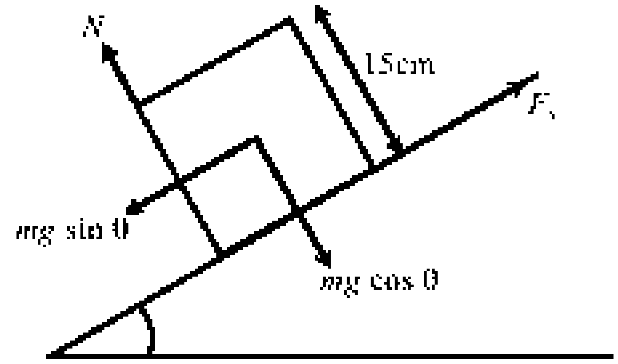
Sol.: The block will start sliding down (if it does not topple) at angle θ_s where

i.e. $\mu = \tan\theta$ ∴ $\theta_s = 60^0$

The block will start toppling (if it does not slide) at angle θ_t if

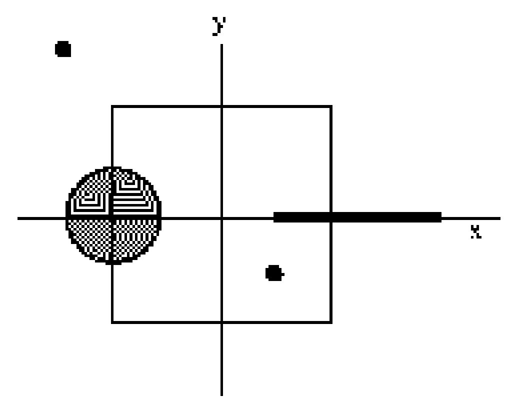
$mg\sin\theta\left(\frac{15}{2}\right) = mg\cos\theta\left(\frac{10}{2}\right)$ ⇒ $\theta_t = \tan^{-1}\left(\frac{2}{3}\right)$

∴ As $\theta_t < \theta_s$ block will first topple before it slides.



Correct choice: (B)

46. A disc of radius $a/4$ having a uniformly distributed charge 6C is placed in the x-y plane with its centre at $(-a/2, 0, 0)$. A rod of length a carrying a uniformly distributed charge 8C is placed on the x-axis from $x = a/4$ to $x = 5a/4$. Two point charges −7C and 3C are placed at $(a/4, -a/4, 0)$ and $(-3a/4, 3a/4, 0)$, respectively. Consider a cubical surface formed by six surfaces $x = \pm a/2$, $y = \pm a/2$, $z = \pm a/2$. The electric flux through this cubical surface is



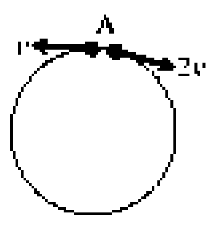
(A) $-\frac{2C}{\varepsilon_0}$ (B) $\frac{2C}{\varepsilon_0}$

(C) $\frac{10C}{\varepsilon_0}$ (D) $\frac{12C}{\varepsilon_0}$

Sol.: $\phi = \frac{q_{encl.}}{\varepsilon_0} = \frac{1}{\varepsilon_0}(3C + 2C - 7C) = \frac{-2C}{\varepsilon_0}$

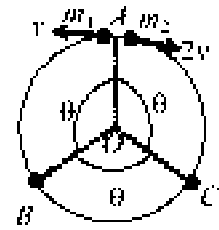
Correct choice: (A)

***47.** Two small particles of equal masses start moving in opposite directions from a point A in a horizontal circular orbit. Their tangential velocities are v and $2v$, respectively, as shown in the figure. Between collisions, the particles move with constant speeds. After making how many elastic collisions, other than that at A, these two particles will again reach the point A?



(A) 4 (B) 3 (C) 2 (D) 1

Sol.: Let first collision be at an angle θ,



$$\therefore \quad \frac{\theta r}{v} = \frac{(2\pi - \theta)r}{2v}$$

$$2\theta = 2\pi - \theta$$

$$\theta = \frac{2\pi}{3} = 120^0$$

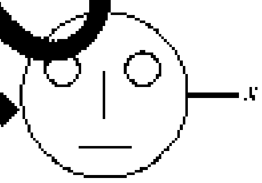
$\therefore$ After first collision at B m_2 will move back with speed v and make collision with m_1 at C.

(again at $\theta = \frac{2\pi}{3}$ anticlockwise from OB)

Now, again m_1 will move back with speed v and meet m_2 at A (at $\theta = \frac{2\pi}{3}$ anticlockwise from OC)

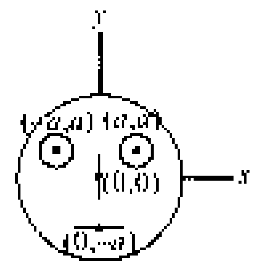
Correct choice: (C)

*48. Look at the drawing given in the figure which has been drawn with ink of uniform line-thickness. The mass of ink used to draw each of the two inner circles, and each of the two line segments is m. The mass of the ink used to draw the outer circle is $6m$. The coordinates of the centres of the different parts are: outer circle (0, 0), left inner circle ($-a$, a), right inner circle (a, a), vertical line (0, 0) and horizontal line (0, $-a$). The y-coordinate of the centre of mass of the ink in this drawing is



(A) $\frac{a}{10}$ (B) $\frac{a}{8}$ (C) [illegible] (D) $\frac{a}{3}$

Sol.: $$y_{cm} = \frac{6m(0) + m(a) + m(a) + m(0) + m(-a)}{6m + m + m + m + m} = \frac{a}{10}$$



Correct choice: (A)

SECTION – II
Multiple Correct Choice Type

This section contains 4 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONE OR MORE** is/are correct.

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*49. A student performed the experiment of determination of focal length of a concave mirror by u-v method using an optical bench of length 1.[illegible] meter. The focal length of the mirror used is 24 cm. The maximum error in the location of the image can be 0.2 cm. The 5 sets of (u, v) values recorded by the student (in cm) are: (42, 56), (48, 48), (60, 40), (66, 33), (78, 39). The data set(s) that [illegible] come from experiment and is (are) incorrectly recorded, is (are)

(A) (42, 56) (B) (48, 48) (C) (66, 33) (D) (78, 39)

Sol.: By [illegible] mirror formula $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

For set 1 $\rightarrow$ u = −42 cm, f = −24 cm $\Rightarrow$ v = −56 cm

For set 2 $\rightarrow$ u = −48 cm, f = −24 cm $\Rightarrow$ v = −48 cm

For set 3 $\rightarrow$ u = −60 cm, f = − 24cm $\Rightarrow$ v = −40 cm

For set 4 $\rightarrow$ u = − 66 cm, f = −24 cm $\Rightarrow$ v = −37.7 ± 0.2 ≠ 33 cm

For set 5 $\rightarrow$ u = − 78cm, f = −24 cm $\Rightarrow$ v = −34.67 ± 0.2 ≠ 39 cm

Correct choice: (C), (D)

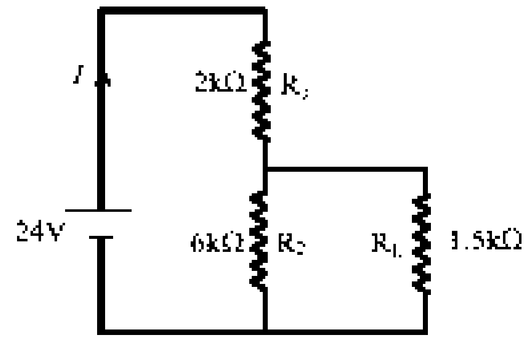
50. For the circuit shown in the figure

(A) the current I through the battery is 7.5 mA

(B) the potential difference across R_L is 18V

(C) ratio of powers dissipated in R_1 and R_2 is 3

(D) if R_1 and R_2 are interchanged, magnitude of the power dissipated in R_L will decrease by a factor of 9



Sol.: $R_{eq} = \dfrac{6\times 1.5}{6+1.5} + 2 = 3.2\,\text{k}\Omega$; $\qquad I = \dfrac{24}{3.2} = 7.5\,\text{mA}$

$V_{R_2} = V_{R_L} = \dfrac{6\times 1.5}{6+1.5}\times 7.5 = 9\text{V}$; $V_{R_1} = 24 - 9 = 15\text{V}$; $\dfrac{P_{R_1}}{P_{R_2}} = \dfrac{V_{R_1}^2 / R_1}{V_{R_2}^2 / R_2} = \dfrac{25}{3}$

Initially $P_{R_L} = \dfrac{9\times 9}{1.5\times 10^3} = 54\times 10^{-3}$ J

When R_1 and R_2 an interchanged $R_{eq} = \dfrac{2\times 1.5}{2+1.5} + 6 = \dfrac{48}{7}\,\text{k}\,\Omega$

$I = 3.5$ mA

$P'_{R_L} = \left(3.5\times \dfrac{2}{3.5}\times 10^{-3}\right)^2 \times 1.5\times 10^3 = 6\times 10^{-3}$ J

$\dfrac{P_{R_L}}{P'_{R_L}} = 9$

Correct choice: (A, D)

***51.** If the resultant of all the external forces acting on a system of particles is zero, then from an inertial frame, one can surely say that

(A) linear momentum of the system does not change in time

(B) kinetic energy of the system does not change in time

(C) angular momentum of the system does not change in time

(D) potential energy of the system does not change in time

Sol.: $\therefore$ if $\vec{F}_{ext} = 0$ $\qquad \therefore$ $\vec{P}$ = constant

But $\vec{\tau}_{ext}$ may be non-zero $\qquad \therefore$ $\vec{L} \neq$ constant

For example in case of a non-rigid body, $(\text{K.E})_{system} \neq$ constant

Correct choice: (A)

***52.** C_V and C_P denote the molar specific heat capacities of a gas at constant volume and constant pressure, respectively. Then

(A) $C_P - C_V$ is larger for a diatomic ideal gas than for a monoatomic ideal gas

(B) $C_P + C_V$ is larger for a diatomic ideal gas than for a monoatomic ideal gas

(C) C_P/C_V is larger for a diatomic ideal gas than for a monoatomic ideal gas

(D) $C_P \cdot C_V$ is larger for a diatomic ideal gas than for a monoatomic ideal gas

Sol.: $C_P - C_V = R$ same for all the ideal gases

$C_P + C_V = R(1+f)$ $\qquad \therefore \qquad (C_P + C_V)_{dia} > (C_P + C_V)_{mono}$ because $f_{dia} > f_{mono}$

$\dfrac{C_P}{C_V} = \gamma = 1 + \dfrac{2}{f}$ $\qquad \gamma_{dia} < \gamma_{mono}$

$C_P . C_V = \left(1 + \dfrac{f}{2}\right)\dfrac{f}{2} R^2$ $\qquad \therefore \qquad (C_P C_V)_{dia} > (C_P C_V)_{mono}$

Correct choice: (B), (D)

SECTION – III
Comprehension Type

This section contains 2 groups of questions. Each group has 3 multiple choice questions based on a paragraph. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONLY ONE** is correct.

Paragraph for Question Nos. 53 to 55

Scientists are working hard to develop nuclear fusion reactor. Nuclei of heavy hydrogen, ${}_1^2\text{H}$, known as deuteron and denoted by D, can be thought of as a candidate for fusion reactor. The D-D reaction is ${}_1^2\text{H} + {}_1^2\text{H} \to {}_2^3\text{He} + n$ + energy. In the core of fusion reactor, a gas of heavy hydrogen is fully ionized into deuteron nuclei and electrons. This collection of ${}_1^2\text{H}$ nuclei and electrons is known as plasma. The nuclei move randomly in the reactor core and occasionally come close enough for nuclear fusion to take place. Usually, the temperatures in the reactor core are too high and no material wall can be used to confine the plasma. Special techniques are used which confine the plasma for a time t_0 before the particles fly away from the core. If n is the density (number/volume) of deuterons, the product nt_0 is called Lawson number. In one of the criteria, a reactor is termed successful if Lawson number is greater than 5×10^{14} s/cm^3.

It may be helpful to use the following: Boltzmann constant $k = 8.6 \times 10^{-5}$ eV/K; $\dfrac{e^2}{4\pi\varepsilon_0} = 1.44 \times 10^{-9}$ eVm.

53. In the core of nuclear fusion reactor, the gas becomes plasma because of

(A) strong nuclear force acting between the deuterons

(B) Coulomb force acting between the deuterons

(C) Coulomb force acting between deuteron – electron pairs

(D) the high temperature maintained inside the reactor core

Sol.: Due to high temperature every molecule dissociates into nucleus and electron.

Correct choice: (D)

54. Assume that two deuteron nuclei in the core of fusion reactor at temperature T are moving towards each other, each with kinetic energy $1.5\ kT$, when the separation between them is large enough to neglect Coulomb potential energy. Also neglect any interaction from other particles in the core. The minimum temperature T required for them to reach a separation of 4×10^{-15} m is in the range

(A) $1.0 \times 10^9\ \text{K} < T < 2.0 \times 10^9\ \text{K}$ (B) $2.0 \times 10^9\ \text{K} < T < 3.0 \times 10^9\ \text{K}$

(C) $3.0 \times 10^9\ \text{K} < T < 4.0 \times 10^9\ \text{K}$ (D) $4.0 \times 10^9\ \text{K} < T < 5.0 \times 10^9\ \text{K}$

Sol.: $3kT = \dfrac{e^2}{4\pi\varepsilon_0 r}$; $\quad 3kT = \dfrac{1.44 \times 10^{-9}}{r}$

$$T = \frac{1.44 \times 10^{-9}\ \text{eVm}}{4 \times 10^{-15}\,m \times 3 \times 8.6 \times 10^{-5}\ \text{eV/K}} = \frac{0.12}{8.6} \times 10^{11}\ \text{K} = 1.4 \times 10^9\ \text{K}$$

Correct choice: (A)

55. Results of calculations for four different designs of a fusion reactor using D-D reaction are given below. Which of these is most promising based on Lawson criterion?

(A) deuteron density = 2.0×10^{12} cm^{-3}, confinement time = 5.0×10^{-3}s

(B) deuteron density = 8.0×10^{14} cm^{-3}, confinement time = 9.0×10^{-1}s

(C) deuteron density = 4.0×10^{23} cm^{-3}, confinement time = 1.0×10^{-11}s

(D) deuteron density = 1.0×10^{24} cm^{-3}, confinement time = 4.0×10^{-12}s

Sol.: For (A) $nt_0 = 10 \times 10^9 = 10^{10} < 5 \times 10^{14}$ s/cm^3

For (B) $nt_0 = 72 \times 10^{13} = 7.2 \times 10^{14} > 5 \times 10^{14}$ s/cm^3

For (C) $nt_0 = 4 \times 10^{12} < 5 \times 10^{14}$ s/cm^3

For (D) $nt_0 = 4 \times 10^{12} < 5 \times 10^{14}$ s/cm^3

Correct choice: (B)

Paragraph for Question Nos. 56 to 58

When a particle is restricted to move along x-axis between $x = 0$ and $x = a$, where a is of nanometer dimension, its energy can take only certain specific values. The allowed energies of the particle moving in such a restricted region, correspond to the formation of standing waves with nodes at its ends $x = 0$ and $x = a$. The wavelength of this standing wave is related to the linear momentum p of the particle according to the de Broglie relation. The energy of the particle of mass m is related to its linear momentum as $E = \frac{p^2}{2m}$. Thus, the energy of the particle can be denoted by a quantum number 'n' taking values 1, 2, 3, ... ($n = 1$, called the ground state) corresponding to the number of loops in the standing wave.

Use the model described above to answer the following three questions for a particle moving in the line $x = 0$ to $x = a$. Take $h = 6.6 \times 10^{-34}$ Js and $e = 1.6 \times 10^{-19}$C.

56. The allowed energy for the particle for a particular value of n is proportional to

(A) a^{-2} (B) $a^{-3/2}$ (C) a^{-1} (D) a^2

Sol.: $\lambda_1 = 2a$;

$$\lambda_n = \frac{2a}{n};$$

$$p = \frac{h}{\lambda_n} = \frac{nh}{2a};$$

$$E = \frac{p^2}{2m} = \frac{n^2h^2}{8ma^2}$$

Correct choice: (A)

57. If the mass of the particle is $m = 1.0 \times 10^{-30}$ kg and $a = 6.6$ nm, the energy of the particle in its ground state is closest to

(A) 0.8 meV (B) 8 meV (C) 80 meV (D) 800 meV

Sol.: $$E = \frac{n^2h^2}{8a^2m} = \frac{1^2\left(6.6\times10^{-34}\right)^2}{8\times\left(6.6\times10^{-9}\right)^2\times1\times10^{-30}}\text{J}$$

$$= \frac{0.125\times10^{-20}}{1.6\times10^{-19}}\text{eV}$$

$$= 8 \text{ meV}$$

Correct choice: (B)

58. The speed of the particle, that can take discrete values, is proportional to

(A) $n^{-3/2}$ (B) n^{-1} (C) $n^{1/2}$ (D) n

Sol.: $$p = \frac{h}{\lambda_n} = \frac{nh}{2a};$$

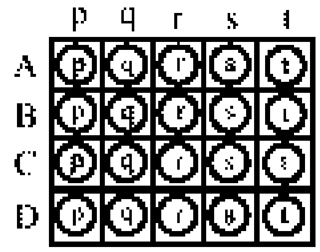
$$v = \frac{nh}{2am}$$

Correct choice: (D)

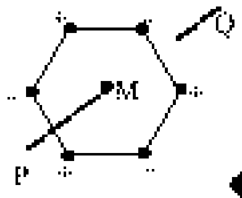
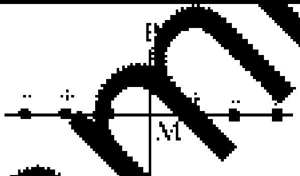
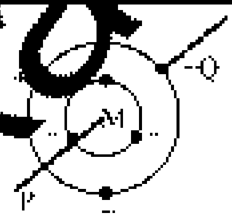
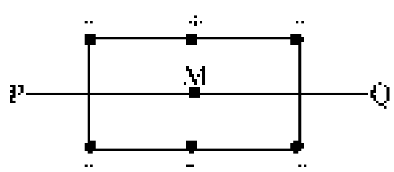
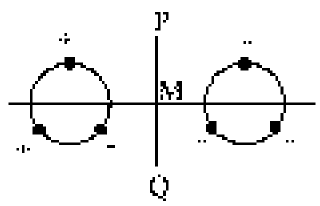
SECTION – IV
Matrix–Match Type

This section contains 2 questions. Each question contains statements given in two columns, which have to be matched. The statements in **Column I** are labelled A, B, C and D, while the statements in **Column II** are labelled p, q, r, s and t. Any given statement in **Column I** can have correct matching with **ONE OR MORE** statement(s) in **Column II**. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example:

If the correct matches are A – p, s and t; B – q and r; C – p and q; and D – s and t; then the correct darkening of bubbles will look like the following.

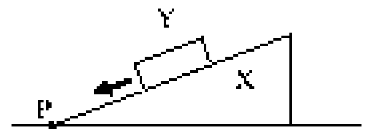
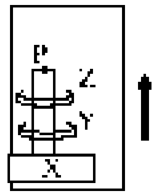
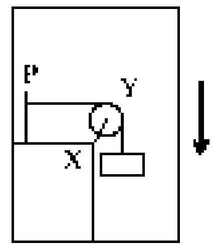
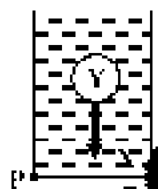


59. Six point charges, each of the same magnitude q, are arranged in different manners as shown in Column II. In each case, a point M and a line PQ passing through M are shown. Let E be the electric field and V be the electric potential at M (potential at infinity is zero) due to the given charge distribution when it is at rest. Now, the whole system is set into rotation with a constant angular velocity about the line PQ. Let B be the magnetic field at M and μ be the magnetic moment of the system in this condition. Assume each rotating charge to be equivalent to a steady current.

Column–I	Column–II	
(A) $E = 0$	(p) 	Charges are at the corners of a regular hexagon. M is at the centre of the hexagon. PQ is perpendicular to the plane of the hexagon
(B) $V \neq 0$	(q) 	Charges are on a line perpendicular to PQ at equal intervals. M is the mid-point between the two innermost charges.
(C) $B = 0$	(r) 	Charges are placed on two coplanar insulating rings at equal intervals. M is the common centre of the rings. PQ is perpendicular to the plane of the rings.
(D) $\mu \neq 0$	(s) 	Charges are placed at the corners of a rectangle of sides a and $2a$ and at the mid points of the longer sides. M is at the centre of the rectangle. PQ is parallel to the longer sides
	(t) 	Charges are placed on two coplanar, identical insulating rings at equal intervals. M is the mid-point between the centres of the rings. PQ is perpendicular to the line joining the centres and coplanar to the rings.

Sol.: **(A) –(p,r,s); (B) –(r,s); (C) – (p,q,t); (D) –(r,s)**

*60. **Column II** shows five systems in which two objects are labelled as X and Y. Also in each case a point P is shown. **Column I** gives some statements about X and/or Y. Match these statements to the appropriate system(s) from **Column II.**

Column–I		**Column–II**	
(A) The force exerted by X on Y has a magnitude Mg.	**(p)**		Block Y of mass M left on a fixed inclined plane X, slides on it with a constant velocity
(B) The gravitational potential energy of X is continuously increasing.	**(q)**		Two ring magnets Y and Z, each of mass M, are kept in frictionless vertical plastic stand so that they repel each other. Y rests on the base X and Z hangs in air in equilibrium. P is the topmost point of the stand on the common axis of the two rings. The whole system is in a lift that is going up with a constant velocity.
(C) Mechanical energy of the system X + Y is continuously decreasing.	**(r)**		A pulley Y of mass m_0 is fixed to a table through a clamp X. A block of mass M hangs from a string that goes over the pulley and is fixed at point P of the table. The whole system is kept in a lift that is going down with a constant velocity.
(D) The torque of the weight of Y about point P is zero.	**(s)**		A sphere Y of mass M is put in a nonviscous liquid X kept in a container at rest. The sphere is released and it moves down in the liquid.
	(t)		A sphere Y of mass M is falling with its terminal velocity in a viscous liquid X kept in a container.

Sol.: **(A) –(p,t); (B) –(q,s,t); (C) –(p,r,t); (D) –(q)**