MCA (Revised)

Term-End Examination

December, 2006

MCS-013 : DISCRETE MATHEMATICS

Time : 2 hours *Maximum Marks : 50*

Note : *Question number 1 is* ***compulsory****. Attempt any* ***three*** *questions from the rest.*

1. (a) Find a formula for f^{-1} when $f(x) = 2x - 3$, $x \in \mathbf{R}$. Also tell the domain and codomain of f^{-1}. *3*

(b) If A, B, C are pairwise intersecting sets, draw a Venn diagram to represent $A \cup (B \cap C)$. *3*

(c) Use a Truth Table to show that $p \wedge q \wedge (\sim p)$ is a contradiction. *3*

(d) If 11 shoes are to be selected from 10 pairs of shoes, prove that there must be a pair of matching shoes amongst the selection. *2*

(e) Find the number of permutations that can be formed from all the letters of the word MISSISSIPPI. *2*

(f) Write down all the partitions of 7. Also find P_7^4 and P_7^5. *3*

(g) Find the logic circuit corresponding to the Boolean expression

$$(x_1 \vee x_2)' \wedge x_3$$ 4

2. (a) Consider the functions $f(x) = x^2 + 1$ and $g(x) = 2x$. Find formulae for the composite functions $g\circ f$ and $f\circ g$. Do we have $g\circ f = f\circ g$? 2

(b) By constructing Truth Tables, show that the two propositions

$$(p \wedge q) \vee (p \wedge r)$$

and

$$(\bar{p} \vee (\bar{q} \wedge \bar{r}))$$

are negation of each other. (Read $\bar{p} = \sim p$) 4

(c) Prove by induction that for $n \geq 1$,

$$1.2 + 2.3 + \ldots + n(n + 1) = \frac{n(n+1)(n+2)}{3}$$ 4

3. (a) What is the sum of the coefficients of all the terms in the expansion of $(x + y + z)^7$? 3

(b) Define Generalised Pigeon Hole Principle. Show that in a group of 35 people, one can always find 6 people who were home on the same day of the week. 3

(c) How many numbers from 0 to 999 are not divisible by either 5 or 7 ? *4*

4. (a) Out of 200 students, 50 of them take the course Discrete Mathematics, 140 of them take the course Economics, and 24 of them take both the courses. Use Venn diagram to find the number of students who are not in either one of these courses. *2*

(b) An urn contains 15 balls, 8 of which are red and 7 are blue. In how many ways can (i) 5 balls be chosen so that all 5 are red, and (ii) 7 balls be chosen so that at least 5 are red ? *3*

(c) Let A = {a, b, c, d}, B = {1, 2, 3}, and R = {(a, 2), (b, 1), (c, 2), (d, 3)}. Show that R is a function from A to B. Is R one-one ? Is R onto ? *3*

(d) Suppose 25 people have exactly the same briefcases, which they leave at a counter. The cases are handed back to the people randomly. What is the probability that no one gets the right case ? *2*

5. (a) Find the Boolean expression in DNF for the function defined in tabular form below : *4*

x_1	x_2	x_3	$f(x_1, x_2, x_3)$
1	0	1	0
0	1	0	1
1	1	0	0
0	0	1	1
0	1	1	0
1	0	0	1
0	0	0	0
1	1	1	1

(b) Simplify the following Boolean expression : 2

$$(\bar{x}_1 \wedge \bar{x}_2 \wedge x_3) \vee (x_1 \wedge x_3)$$

(c) Find the Boolean expression represented by the following circuit : 4