S-E (Comp) Sem III (R)

29/05/09

756 : H-m.

Discrete Structure & Draft Theory

Con. 2739-09. (REVISED COURSE) VR-3330

16

(3 Hours) [Total Marks : 100

3 P.m. to 6 p.m.

N.B. (1) Question No. **1** is **compulsory**.
(2) Attempt any **four** questions out of remaining **six** questions.
(3) **Assumptions** made should be **clearly** stated.
(4) **Figures** to the **right** indicate **full** marks.

1. (a) Prove that $A \times (B \cap C) = (A \times B) \cap (A \times C)$. 4
(b) Prove there is no rational number p/q whose square is 2. 4
(c) Show that $n^3 + 2n$ is divisible by 3 for all $n \geq 1$. 6
(d) Among the integers 1 and 300, 6
(i) How many of them are divisible by 3,5 or 7 and are not divisible by 3 nor by 5 nor by 7 ?
(ii) How many of them are divisible by 3 but not by 5 nor by 7 ?

2. (a) Prove that if any 14 integers from 1 to 25 are chosen, then one of them is a multiple of another. 4
(b) Solve the recurrence relation $d_n = 2d_{n-1} - d_{n-2}$ with initial conditions $d_1 = 1{\cdot}5$ and $d_2 = 3$. 4
(c) Let A = Z , the set of integers and let R be the relation less than. Is R Transitive ? 6
(d) Negate the statement. 6
For all real numbers x, if $x > 3$ then $x^2 > 9$.

3. (a) Let A = { a, b, c, d, e } and 6
R = { (a,a), (a,b), (b,c), (c,e), (c,d), (d,e) }
Compute (i) R^2 and R^∞.
(b) Let A = { 1, 2, 3, 4 } and let R = { (1,2), (2,3), (3,4), (2,1) } 6
Find Transitive Closure of R using Warshall's algorithms.
(c) Explain the Equivalence Class with an Example. 4
(d) Explain with an Example dual of the poset. 4

4. (a) Show that in a bounded distributive lattice, if a complement exists, it is unique. 6
(b) Determine whether the following posets are Boolean algebras. Just your answers 6
(i) A = { 1, 2, 3, 6 } with divisibility.
(ii) D_{20} : divisors of 20 with "divisibility".
(c) Explain Primitive Recursive Function. Every primitive recursive function is a total function, Justify. 4
(d) (i) Is Every Eulerian graph a Hamiltonian ? 4
(ii) Is every Hamiltonian graph a Eulerian ?
Justify with the necessary graph.

5. (a) Show that if set A has 3 elements, then we can find 8 relations on A that all have the same symmetric closure. 6
(b) Draw the Hasse diagram of the poset A = { 2, 3, 6, 12, 24, 36, 72 } 6
Under the relation of divisibility.
Is this Poset a lattice ? Justify.
(c) Let A = { 0, −1, 1 } and B = { 0, 1 }, Let $f : A \to B$ where $f(a) = |a|$. Is f onto ? 4
(d) State and prove right or left cancellation property for a group. 4

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6. (a) Prove that every field is an integral domain. 6
(b) Consider the chains of divisors of 4 and 9 i. e., $L_1 = \{1, 2, 4\}$ 6
And $L_2 = \{1, 3, 9\}$.
Find partial ordering relation of division on L_1 and L_2.
Draw lattice of $L_1 \times L_2$.
(c) Explain the linear recurrence relations with constant co-efficeints. 4
(d) Explain the types of generating function with an example. 4

7. (a) Consider the (3, 5) group encoding function $e : B^3 \to B^5$ defined by— 6

$$
\begin{array}{ll}
e(000) = 00000 & e(100) = 10011 \\
e(001) = 00110 & e(101) = 10101 \\
e(010) = 01001 & e(110) = 11010 \\
e(011) = 01111 & e(111) = 11100
\end{array}
$$

Decode the following words relative to a maximum likelihood decoding function—
(i) 11001 (ii) 01010 (iii) 00111
(b) Let G be the set of all nonzero real numbers and let 6
$a * b = ab / 2$. Show that (G, *) is an Abelian group.
(c) Let m = 2, n = 5 and 4

$$
H = \begin{matrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}
$$

Determine the group code $e_H : B^2 \to B^5$
(d) Explain congruence relation with an example. 4